Misalignment Retrieval of an Off-axis Parabolic Mirror using Kalman Filtering

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Outline

• Introduction
• Model
• Image processing and reconstruction
• State estimation and control
• Simulation result
• Conclusion
Optical misalignments introduce wavefront aberration

- Wavefront aberration - *Any deviation of the wavefront formed by an optical system from perfect spherical*
- Main aberrations

Ideal  Defocus  Astigmatism  Coma
Model

DOF:
- Decenter $D_x$
- Decenter $D_y$
- Decenter $D_z$
- Tilt $T_x$
- Tilt $T_y$

http://assets.newport.com/
Misalignment Effect

Decenter $\mathbf{Dx}$

Decenter $\mathbf{Dy}$

Decenter $\mathbf{Dz}$

Tilt $\mathbf{Tx}$

Tilt $\mathbf{Ty}$
Gaussian Fitting and Image Decomposition

• Gaussian fitting to get center position $C_x$ and $C_y$

$$F(x, y) = G_1 + G_2 \exp \left( -\frac{(x' / a)^2 - (y' / b)^2}{2} \right)$$

$$x' = (x - C_x) \cos \phi - (y - C_y) \sin \phi$$

$$y' = (x - C_x) \sin \phi + (y - C_y) \cos \phi$$

• Vector-mean-subtracted image vector

$$\bar{v}_i = v_i - \mu(v_i)$$

\[ \text{collect image dataset} \rightarrow \bar{V} = [\bar{v}_1, \bar{v}_2, ..., \bar{v}_n] \]

• Image-to-image covariance

$$S = \frac{1}{p-1} \bar{V}^T \bar{V}$$

• Eigen decomposition

$$S \Phi = \Phi \Lambda$$

• Karhunen-Loève modes (eigen images)

$$Z = \bar{V} \Phi$$
Karhunen-Loève Modes

Weights:
w1, w2, w3, w4

Measurements:
w2/w1, w3/w1, w4/w1
Measurement Function

- Measurements in terms of states (Dx, Dy, Dz, Tx, Ty).

\[
\begin{align*}
C_x &= a_1x_1 + a_2x_2 \\
C_y &= b_1x_2 + b_2x_3 + b_3x_4 + b_4x_1^2 + b_5x_5^2 \\
w_2 &= c_1x_1 + c_2x_5 \\
w_1 &= \frac{w_2}{w_1} \\
w_3 &= d_1x_2 + d_2x_4 + d_3x_1^2 + d_4x_3^2 + d_5x_5^2 \\
w_1 &= \frac{w_3}{w_1} \\
w_4 &= e_1x_3 + e_2x_4 + e_3x_2^2 + e_4x_5^2 \\
w_1 &= \frac{w_4}{w_1}
\end{align*}
\]
Measurement Function Error

Test set 5000 images

\[
\begin{bmatrix}
8.6\mu m \\
7.7\mu m \\
4.4 \times 10^{-3} \\
3.9 \times 10^{-3} \\
4.3 \times 10^{-3}
\end{bmatrix}
\]
Image noise

- **Shot noise** – The random arrival of photons → **Poisson distribution**
- **CCD read noise** – Noise generated by electronics as the charge present in the pixels is transferred to the camera → **Gaussian distribution**
Image Noise Effects on Center Position and KL weight deviation

- 2000 images with random misalignment and noise
- The effect of random noise (both Poisson and Gaussian noise) on the measurements are normal distribution
Estimation and Control

Plant
- collimated laser beam

Optical System (Lenses and Stages)

Camera
- focused beam
- images

stage command $u_k$

Kalman Filter
- $F_k, B_k$
- system function
- $h_k$
- measurement function
- $\hat{x}_{k|k-1}$
- $ \hat{x}_{k|k}$
- $K_k$
- Kalman gain

Kalman Filter
- $K_k$
- KL Modes Decomposition
- KL coefficients
- $y_k$
Kalman Filtering

- Given variables: control input $u_k$, measurements $y_k$
- State space representation:
  \[
  x_k = F_k x_{k-1} + B_k u_k + q_k \\
  y_k = h_k(x_k) + r_k
  \]
- $q_k \sim N(0, Q_k)$
- $r_k \sim N(0, R_k)$
- $R_k = R_{model} + R_{meas}$
Simulation Result - State (IEKF)

- Shift Error - log(mm)
  - $D_x$
  - $D_y$
  - $D_z$

- Tilt Error - log(degree)
  - $T_x$
  - $T_y$

0.5 $\mu$m

1 arcsec
Simulation Result - Measurement (IEKF)
Before and After
Conclusion and Future Work

• Image processing and optimal estimation achieve automated alignment of an off-axis parabolic mirror without using a wavefront sensor.
• Other estimation and control technique can be applied (such as particle filter, robust control).
• An experiment will be conducted on an optical bench.

https://github.com/JFgithubJF/oap_model
Thank You