Orbital Design and Scheduling Techniques for Space Science and Exoplanet-Finding Missions

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A Exam
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Overview

- **Part I**: Trajectory Design for Zodiacal Light Imaging Mission

- **Part II**: Starshade Observation Scheduling
  - Starshade Dynamics
  - Observation Scheduling

- **Part III**: Future Work
Part I: Trajectory Design for a Zodiacal Light Imaging Mission
Motivation


ZODI – ZOdiacal Dust Imager

Before Flyby
Before Flyby

Inputs

\[ u = \begin{bmatrix} t_D \\ t_E \end{bmatrix} \]
Before Flyby

Inputs

\[ \mathbf{u} = \begin{bmatrix} t_D \\ t_E \end{bmatrix} \]

\[ \mathbf{r}_D = \mathbf{r}_{\text{spacecraft}}(t_D) \]

\[ \mathbf{r}_B = \mathbf{r}_{\text{planet}}(t_E) + \mathbf{r}_E \]
Before Flyby (Planetary Frame)

\[
\mathbf{u} = \begin{bmatrix} t_D \\ t_E \\ \eta \\ \zeta \end{bmatrix}
\]

\[||\mathbf{r}_E|| = R_{SOI}\]

\[R_{SOI} = ||\mathbf{r}_{\text{planet}}(t_E)|| \left(\frac{\mu_P}{\mu_S}\right)^{0.4}\]
Before Flyby (Planetary Frame)

\[ u = \begin{bmatrix} t_D \\ t_E \\ \eta \\ \zeta \end{bmatrix} \]

\[ \hat{q}_2 = \hat{v}_P(t) \]

\[ \hat{q}_3 = \frac{\dot{r}_P(t) \times \hat{q}_2}{\cos \phi_P(t)} \]

\[ \hat{q}_1 = \hat{q}_2 \times \hat{q}_3 \]

\[ \cos \phi_P(t) = \frac{r_{planet} \times v_{planet}}{||r_{planet}|| \cdot ||v_{planet}||} \]

Before Flyby

\[ TOF = t_E - t_D \]

Lambert’s Theorem

\[ \mathbf{u} = \begin{bmatrix} t_D \\ t_E \\ \eta \\ \zeta \end{bmatrix} \]
Before Flyby

$$\Delta v(t_D) = ||v_D - v_{D,\text{spacecraft}}||$$

Constraints

$$\Delta v(t_D) < \Delta v_{\text{max}}$$

Inputs

$$u = \begin{bmatrix} t_D \\ t_E \\ \eta \\ \zeta \end{bmatrix}$$
During Flyby

Constraints
\[ \Delta v(t_D) < \Delta v_{max} \]
\[ \gamma < \gamma_{max} \]
\[ r_{per} > R_P + h_{min} \]

Inputs
\[ \mathbf{u} = \begin{bmatrix} t_D \\ t_E \\ \eta \\ \zeta \end{bmatrix} \]

Velocity Turning Angle
\[ \sin \frac{\delta_v}{2} = \frac{\mu_P}{\mu_P + r_{min} v_\infty^2} \]

Position Turning Angle
\[ \delta_r = \delta_v - 2\gamma - \pi \]

After Flyby

Constraints

\[ \Delta v(t_D) < \Delta v_{\text{max}} \]
\[ \gamma < \gamma_{\text{max}} \]
\[ r_{\text{per}} > R_P + h_{\text{min}} \]

Inputs

\[ \mathbf{u} = \begin{bmatrix} t_D \\ t_E \\ \eta \\ \zeta \end{bmatrix} \]
After Flyby

**Goal**

\[
\text{maximize } \left| z \right|_{u}
\]

**Constraints**

\[
\Delta v(t_D) < \Delta v_{\text{max}} \quad \gamma < \gamma_{\text{max}} \quad r_{\text{per}} > R_P + h_{\text{min}}
\]

**Inputs**

\[
u = \begin{bmatrix} t_D \\ t_E \\ \eta \\ \zeta \end{bmatrix}
\]
After Flyby

**Goal**

\[
\max_{\mathbf{u}} |z|
\]

**Constraints**

\[
\begin{align*}
\Delta v(t_D) &< \Delta v_{max} \\
\gamma &< \gamma_{max} \\
r_{per} &> R_P + h_{min}
\end{align*}
\]

\[
\mathbf{u} = \begin{bmatrix} t_D \\ t_E \\ \eta \\ \zeta \end{bmatrix}
\]

**Inputs**

1. Limit Fuel Mass
2. Achieve Proper Field of View
3. Retrieve Data \[ f_T = \frac{T_A}{T_{Earth}} \]
Case Study with Europa Clipper

Elapsed Time in years = 0.01

SPK kernel taken from Acton, C. and is found at https://naif.jpl.nasa.gov/pub/naif/EUROPACLIPPER/
Second Earth Flyby – 10/21/2024

\[ z_{\text{max}} = 0.2215\text{AU} \]

\[ \Delta v = 37.21\text{m/s} \]

Next Closest Earth Approach = \(2.13 \times 10^5\text{km}\)

10/22/2025, 1yr later
Conclusions

- Developed algorithm for optimizing flyby based on mission requirements
  - Achieves goal: 0.22 AU orbital height
  - Small impulsive correction
- Requires no integration, low computational cost
- Applicable to any payload of opportunity
Part II: Starshade Observation Scheduling for Exoplanet Imaging

Starshade Dynamics
Motivation

Data taken from NASA Exoplanet Archive found at
http://exoplanetarchive.ipac.caltech.edu/cgi-bin/TblView/nph-tblView?app=ExoTbls&config=planets
Starshade Configuration

Exoplanet

Target Star

Starshade

Telescope

No starlight enters telescope directly
Starshade Configuration

Maintain constant separation during observation

Shaklan et al (2011) "A starshade petal error budget for exo-earth detection and characterization" SPIE
Starshade Configuration

- Exoplanet
- Target Star
- Starshade
- Telescope

Limited fuel on board
On a halo orbit
Starshade in the CR3BP Frame

- Target Star
- Starshade
- Telescope
- Sun
- Earth
- Halo Orbit
- L2
- X
- Y
- R
Starshade in the CR3BP Frame

\[ \ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} + f_{SRP} \cdot \hat{x} \]
\[ \ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} + f_{SRP} \cdot \hat{y} \]
\[ \ddot{z} = \frac{\partial \Omega}{\partial z} + f_{SRP} \cdot \hat{z} \]

\[ \Omega(x, y, z) = \frac{1}{2}(x^2 + y^2) + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2}, \]
\[ r_1 = \sqrt{(\mu - x)^2 + y^2 + z^2}, \]
\[ r_2 = \sqrt{(1 - \mu - x)^2 + y^2 + z^2} \]
Solar Radiation Pressure

\[ f_{SRP} = 2PA \cos \alpha \left[ b_1 \hat{p}_1 + (b_2 \cos \alpha + b_3) \hat{n} \right] \]

\[ \hat{n} = \cos \alpha \hat{p}_1 + \sin \alpha \cos \delta \hat{p}_2 + \sin \alpha \sin \delta \hat{p}_3 \]

Glassman et al (2011) “Creating optimal observation schedules for a starshade planet-finding mission” IEEE
Flight Modes

Target Star “A”

Telescope
Flight Modes
Flight Modes
Flight Modes

Station-Keeping
Flight Modes

Target Star “A”

Station-Keeping

Target Star “B”
Flight Modes

Target Star “A”

Station-Keeping

Target Star “B”
Flight Modes

Target Star “A”

Retargeting

Target Star “B”
Flight Modes
Flight Modes
Retargeting Trajectories

\[ \mathbf{v}_{RT}(t_A) \]

\[ \mathbf{v}_{RT}(t_B) \]

\[ \mathbf{r}(t_A) \]

\[ \mathbf{r}(t_B) \]
Retargeting Trajectories

\[ \Delta v(t_A) = \| \mathcal{I}v_{RT}(t_A) - \mathcal{I}v_{SK}(t_A) \| \]
\[ \Delta v(t_B) = \| \mathcal{I}v_{RT}(t_B) - \mathcal{I}v_{SK}(t_B) \| \]

Retargeting Trajectories

<table>
<thead>
<tr>
<th>Diameter</th>
<th>26 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>IWA</td>
<td>72 mas</td>
</tr>
<tr>
<td>Separation Distance</td>
<td>37,242.26 km</td>
</tr>
<tr>
<td>Dry Mass</td>
<td>1250 kg</td>
</tr>
<tr>
<td>Total Mass</td>
<td>3500 kg</td>
</tr>
<tr>
<td>$I_{sp}$</td>
<td>308 s</td>
</tr>
<tr>
<td>Total $\Delta v$</td>
<td>2094.33 m/s</td>
</tr>
</tbody>
</table>

Shaklan et al (2011) "A starshade petal error budget for exo-earth detection and characterization" SPIE
Part II: Starshade Observation Scheduling for Exoplanet Imaging

Observation Scheduling
Parameterizing Fuel Costs

\[ \Delta v = f(i, j, \Delta t, t_0, T_{halo}, d) \]
Parameterizing Fuel Costs

- Stars arranged by ecliptic longitude
- Constant slew time of 20 days
- 3D cost matrix for multiple slew times

Parameterizing Fuel Costs
Parameterizing Fuel Costs

\[ \Delta v = f(i, j, \Delta t, t_0) \]

\[ \Delta v = f(\psi, \Delta t) \]

Parameterizing Fuel Costs - Errors

\[ \Delta u_{INT} = f(\Delta t)|_{(\psi_0, t_0)} \]

\[ \Delta u_{BVP} = f(\Delta t, t_0)|_{\psi_0} \]
Parameterizing Fuel Costs - Errors

![Graph showing the frequency of errors in \( \Delta v \)]
Parameterizing Fuel Costs

- Assume constant halo and separation distance
- Before: 12 minutes to compute map at every decision step
  - 5 day time step
- Now: single map generated offline for any target list

Keepout Constraints
Keepout Constraints
Keepout Constraints
Keepout Constraints
Keepout Constraints

![Graph showing Keepout Constraints with labeled celestial objects and concentric circles representing keepout zones.](image-url)
Keepout Constraints

Keepout Constraints
Cost Function

\[ c = c_1 \Delta v_{min} + c_2 (1 - C_O) - c_3 f_{unv} + c_4 f_{rev} \]

Size 1 × \( j \) for stars \( j \in \mathbb{T} \)

Target List \( \mathbb{T} \subset \mathbb{Z} \)

\[ J = \arg \min_{j} (c) \]
Cost Function

\[ c = c_1 \Delta v_{min} + c_2 (1 - C_O) - c_3 f_{unv} + c_4 f_{rev} \]

\[ f_{unv}(j) = \begin{cases} 
0 & N(j) > 0 \\
\left(\frac{t}{t_F}\right)^2 & N(j) = 0 
\end{cases} \]

\[ f_{rev}(j) = \begin{cases} 
0 & j \notin E \\
1 & j \in E 
\end{cases} \]

\[ J = \arg \min_{j} (c) \]

Number of Observations \( N(j) \)

Revisit List \( E \subseteq T \)

Observation Schedule

Conclusions

• Re-parameterized fuel cost calculation

• Constraints placed on these calculations due to keepout

• Enables faster end-to-end mission simulations
Part III: Future Work
Contributions

Extracurricular Activities
• SGRS Marketing Director 2017
• SiGMA Social Co-Chair 2017-2018

Conference Presentations
3. 229th AAS – “Starshade Orbital Maneuver Study for WFIRST” (2017 – Poster in Grapevine, TX)
5. SPIE Techniques and Instrumentation for Detection of Exoplanets VIII – “Starshade Orbital Maneuver Study for WFIRST” (2017 – Poster in San Diego, CA)
6. SiGMA Seminar Series “Starshade Orbital Maneuvers and Observation Scheduling” (2017 - Speaker in Ithaca, NY)
7. 231st AAS – “Starshade Observation Scheduling for WFIRST” (2018 - Poster in Washington, DC)
8. SPIE Astronomical Telescopes + Instrumentation – “Optimal Starshade Observation Scheduling” (2018 - Poster in Austin, TX)

Conference Papers

Journal Publications

Teaching Assistantships
• MAE 2030 – Dynamics (Spring 2016)
• MAE 2030 – Dynamics (Head TA – Spring 2018)
Coursework

- MAE 5780 – Feedback Control Systems
- MAE 5730 – Intermediate Dynamics & Vibrations
- MAE 5950 – Systems Architecture
- MAE 6060 – Spacecraft Dynamics, Estimation & Control
- MAE 6700 – Advanced Dynamics
- MAE 6780 – Multivariable Control Theory
- ASTRO 6579 – Celestial Mechanics
- ASTRO 7671 – Topics in Planetary Science (Comets)

Future Courses

- ASTRO 7690 – Computational Physics
- ASTRO 6560 – Stellar Structure
- MATH 5790 – Nonlinear Dynamics and Chaos
- MATH 7170 – Applied Dynamical Systems
Starshade With Continuous Thrust

• Optimal control problem
  – Develop heuristics for retargeting case
• Extend to starshade station-keeping during observation
Starshade Attitude

- Starshade attitude dynamics during retargeting
- Configurations due to SRP perturbations
Analytical Approximations

• Richardson Third Order Expansion
  – Nonlinear terms written as Legendre Polynomials

• Study dynamics in Floquet modes
  – Flow equation used to find halo orbits in CR3BP
  – Project motion onto eigenvectors
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Keith Grogan (JPL)

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Daniel Garrett
Christian Delacroix
Jacob Shapiro
Dean Keithly
Amlan Sinha
Backup Slides
Flyby State Rotation

\[ P \mathbf{h}_E = P \mathbf{h} = P \mathbf{r}_E \times P \mathbf{v}_E \]

**Velocity Turning Angle**
\[ \sin \frac{\delta_v}{2} = \frac{\mu_P}{\mu_P + r_{min}v_\infty^2} \]

**Position Turning Angle**
\[ \delta_r = \delta_v - 2\gamma - \pi \]

**Flight Path Angle**
\[ \sin \gamma = \frac{B}{R_{SOI}} = \frac{||P \mathbf{h}||}{R_{SOI}v_\infty} \]

\[ B = \frac{||\mathbf{r}_E \times \mathbf{v}_E||}{v_E} \]

\[ P \mathbf{v}_X = Q_v P \mathbf{v}_E \]
\[ P \mathbf{r}_X = Q_r P \mathbf{r}_E \]

\[ Q_v = \exp \left( \left[ P \hat{\mathbf{h}} \right] \times \delta_v \right) \]
\[ Q_r = \exp \left( \left[ P \hat{\mathbf{h}} \right] \times \delta_r \right) \]
## Second Earth Flyby Results

<table>
<thead>
<tr>
<th>Primary Spacecraft</th>
<th>Europa Clipper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flyby Date</td>
<td>05/29/2023</td>
</tr>
<tr>
<td>Max. Orbital Height</td>
<td>0.1013 AU</td>
</tr>
<tr>
<td>Increase in Orbital Height</td>
<td>0.0081 AU</td>
</tr>
<tr>
<td>Δv Burn at Detachment</td>
<td>35.49 m/s</td>
</tr>
<tr>
<td>Detachment Point D</td>
<td>94.7%</td>
</tr>
<tr>
<td>Altitude of Closest Approach</td>
<td>25,142 km</td>
</tr>
<tr>
<td>Orbital Period Ratio w.r.t. Earth</td>
<td>1.000</td>
</tr>
<tr>
<td>Date of Next Earth Intercept</td>
<td>05/29/2024</td>
</tr>
<tr>
<td>Next Closest Earth Approach</td>
<td>1.25 ×10^6 km</td>
</tr>
</tbody>
</table>
Microlensing

– Mass & Parallax
Radial Velocity/Doppler Spectroscopy

- Doppler shift in radial velocity of the star
  - In the observer line of sight
- Semi-major Axis & Mass
- Minimum mass = $M \sin(i)$
- Need high SNR to do doppler spectroscopy as opposed to astrometry

http://earthsky.org/space/how-do-astronomers-discover-exoplanets
Transit

- Dip in light curve
- **Planet Radius, Atmosphere and Temperature**
  - Length of the dip
  - Stellar spectroscopy
  - Second eclipse – subtract out full planet signal

http://www.planetary.org/explore/space-topics/exoplanets/transit-photometry.html
Retargeting Trajectories

Collocation:
- Cubic polynomial
- Equal at endpoints
- Creates mesh and minimizes residual error
Retargeting Trajectories
Error Analysis

- Test 1, $\mu_{abs} = 2.53$, $med_{abs} = 1.36$
- Test 2, $\mu_{abs} = 8.97$, $med_{abs} = 3.90$
- Test 3, $\mu_{abs} = 6.83$, $med_{abs} = 3.61$
Parameterizing Fuel Costs - Errors

\[ \Delta v_{BVP} = f(\Delta t)|_{(\psi_0, t_0)} \]

\[ \Delta v_{INT} = f(\Delta t, t_0)|_{\psi_0} \]
Parameterizing Fuel Costs - Errors

\[ \Delta v_{BVP} = f(\psi, \Delta t, t_0) \]
\[ \Delta v_{INT} = f(\psi, \Delta t, t_0) \]
Scheduler

- Star Catalog
- Planet Population
- Planet Physical Model
- Optical System

Target List

- Generate star completeness values

- Apply completeness, binary star, other filters

Keepout Binary Map

- Keepout Parameters
- Mission Start and End Times

Fake Star Catalog

Fuel Cost Interpolant

Mission Simulation Start
Scheduler

- Old Star
- Target List
- Current Time

Find Next Best Target Star

- Calculate Observable Time Windows

- Estimate Fuel Costs

Filter Target List

Select Star with Lowest Total Cost

- Stars visited more than a maximum amount of times
- Long integration times
- Retargeting fuel costs too expensive
- Observable times too far in the future

Determine Detections and Populate Schedule

Update

While Fuel Left on Board
Completeness

- Joint Probability Density function
  - Star-planet brightness difference
  - Star-planet projected separation
- Based on instrument parameters, integrate over region
- Probability that a planet with assumed parameters is observable near a star

Uniqueness of BVP Solutions Near L2

\[ \Delta t = 20d \]
Uniqueness of BVP Solutions Near L2

$\Delta t = 270d$