



State Estimation in Optical System Alignment Using Monochromatic Beam Imaging

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August, 2015

Outline



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4 Results

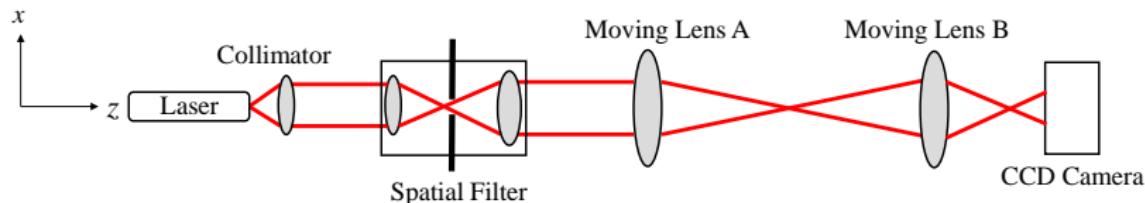
5 Conclusion

Motivation



- Advantages:
 - 1 Make static components in optical systems more flexible, use the same optical components in different beam bath (e.g. satellites and space telescopes).
 - 2 Distributed optical systems that has limited setup time and space (e.g. communication system).
 - 3 Save time and energy spent for aligning manually.
- Some optical assembling stations use rotational stages.
- Most of self-aligning systems use wavefront sensors.
- Align the system using camera without wavefront sensor:
 - 1 More compact, cheaper
 - 2 Easily redesign an optical system: changing fixed mounts to kinematic ones without splitting the beam.
 - 3 No throughput loss or non-common path error.

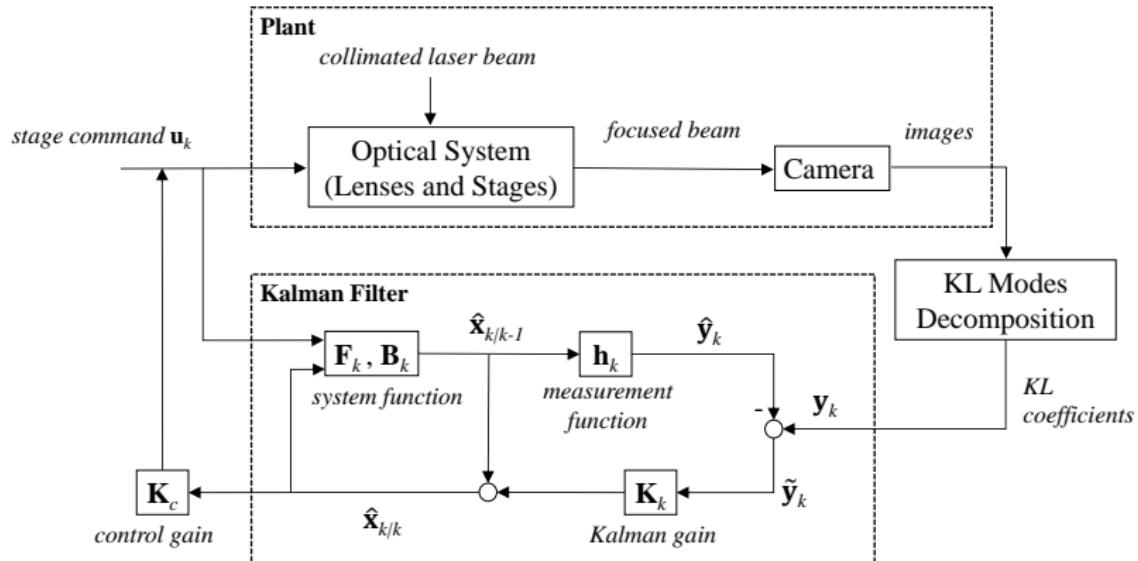
Optical Model



- State vector: $\mathbf{x} = [T_{xA}, T_{yA}, T_{xB}, T_{yB}]^T$
- Goal: Using only image data from CCD to correct tip-tilt misalignments of moving lenses A and B with respect to the beam.

Control Scheme

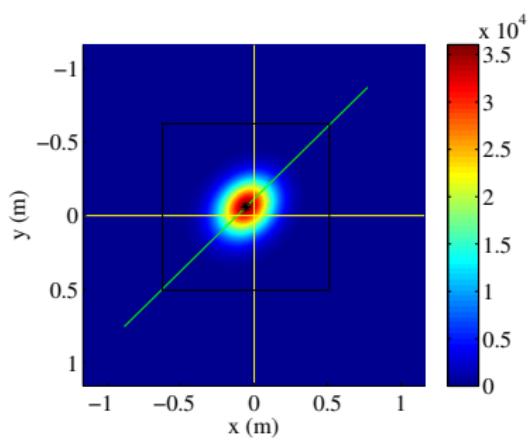
Plant & Kalman filter



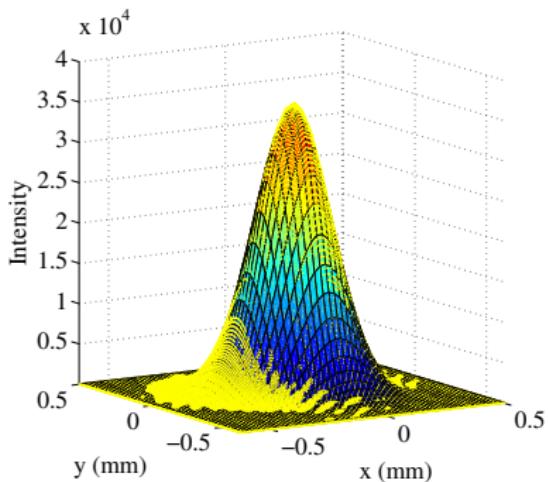
Simulated Image in ZEMAX



Simulated image



Gaussian fitting



Vector-mean-subtracted image vector

$$\bar{\mathbf{r}}_i = \mathbf{r}_i - \mu(\mathbf{r}_i) \xrightarrow{\text{scan through } n \text{ images}} \bar{\mathbf{R}} = [\bar{\mathbf{r}}_1, \bar{\mathbf{r}}_2, \dots, \bar{\mathbf{r}}_n]$$

Principal Component Analysis

Karhunen-Loève

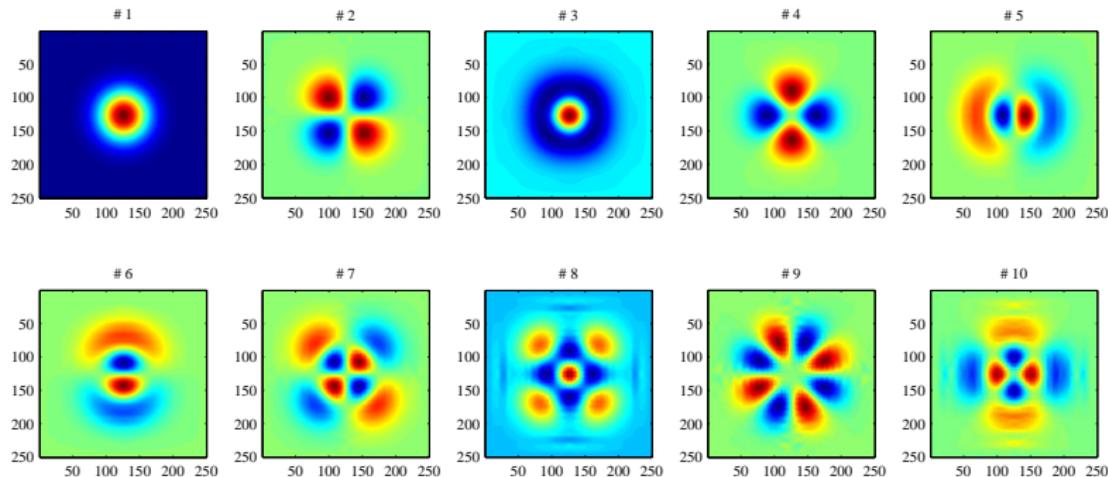


KL decomposition

Image-to-image covariance: $\mathbf{S} = \frac{1}{p-1} \bar{\mathbf{R}}^T \bar{\mathbf{R}}$

Eigendecomposition: $\mathbf{S}\Phi = \Phi\Lambda$

KL modes (eigen images): $\mathbf{Z} = \bar{\mathbf{R}}\Phi$



Karhunen-Loève Modal Reconstruction

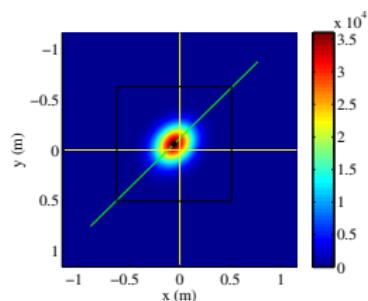


KL coefficients and reconstruction

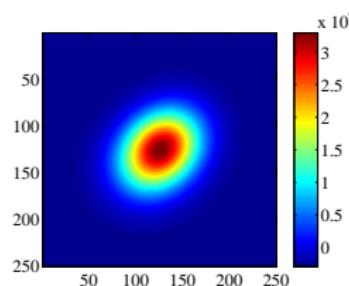
KL coefficients: $\mathbf{w}_i = \mathbf{Z}_m^\dagger \bar{\mathbf{r}}_i$

Reconstructed image: $\bar{\mathbf{c}}_i = \mathbf{Z}_m \mathbf{w}_i$

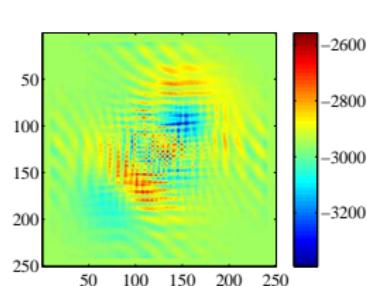
Simulated image



Reconstructed image



Subtracted image



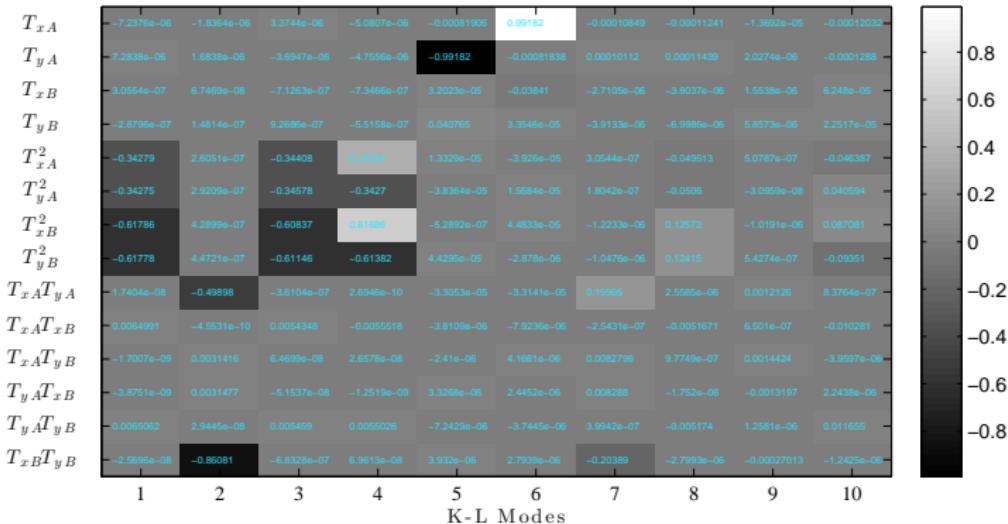
Correlation Analysis



Correlation matrix

State feature extension: $\mathbf{x} \xrightarrow{4\text{-elements to } 14\text{-elements}} \mathbf{t}$

$$\text{corr}(\mathbf{T}, \mathbf{W}) = \frac{[\mathbf{T} - \mu(\mathbf{T})][\mathbf{W} - \mu(\mathbf{W})]^T}{\sigma(\mathbf{T})\sigma(\mathbf{W})^T}$$



Model Fitting

Ridge linear regression



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Normalized measurements

$$\mathbf{y}_i = \frac{1}{w_1} [w_2, w_4, w_5, w_6]^T$$

Ridge regression (10-fold cross-validation)

Design matrix: \mathbf{T}_{train} ($14 \times n$), Output matrix: \mathbf{Y}_{train} ($4 \times n$)

Model function: $\hat{\mathbf{y}}_i = \mathbf{Gt}_i \xrightarrow{\text{nonlinear}} \hat{\mathbf{y}}_i = \mathbf{h}(\mathbf{x}_i)$

Normalized root mean square error (NRMSE)

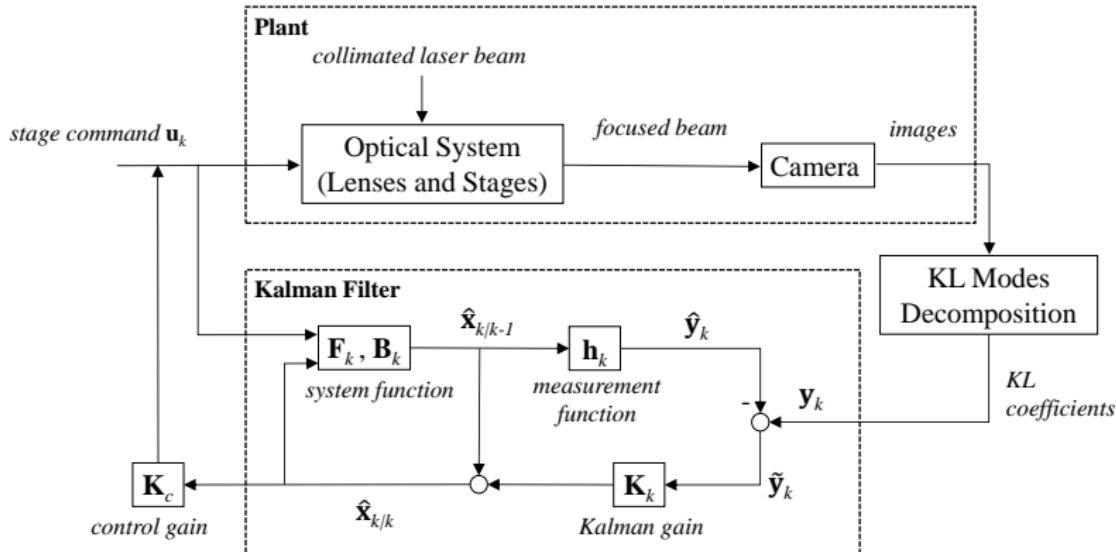
$$\mathbf{e}_i = \mathbf{y}_i - \hat{\mathbf{y}}_i$$

$$\text{NRMSE}(e_j) = \frac{\sqrt{(\sum_{i=1}^n e_{ji})/n}}{y_{max} - y_{min}}$$

<i>NRMSE</i>	e_1	e_2	e_3	e_4
Training Error	5.913e-3	5.916e-3	1.943e-2	1.961e-2
Test Error	5.966e-3	5.602e-3	1.448e-2	1.446e-2

State Estimation

Extended Kalman filter



State space representation

$$\begin{aligned}\mathbf{x}_k &= \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\ \mathbf{y}_k &= \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v}_k\end{aligned}$$

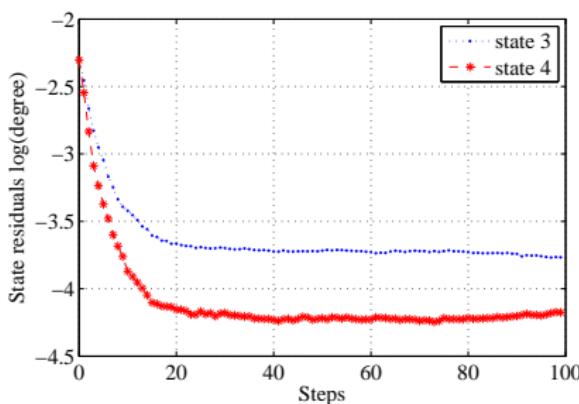
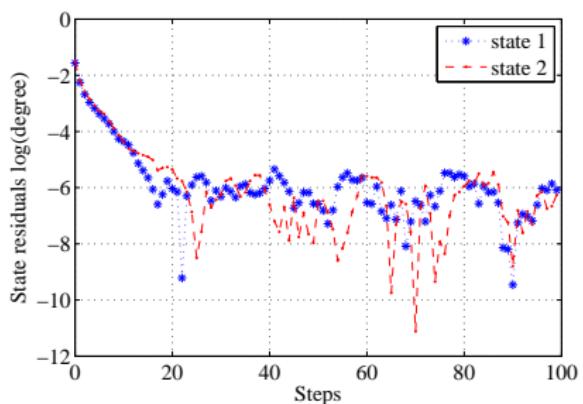
Jacobian: $\mathbf{H}_k = \frac{\partial \mathbf{h}}{\partial \mathbf{x}}|_{\mathbf{x}_k|k-1}$

Kalman Filter Test in Simulation

State residuals



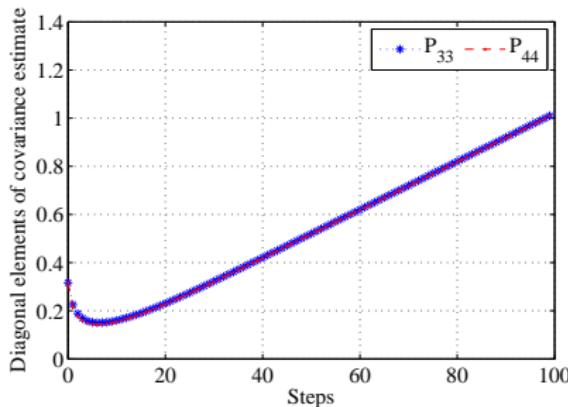
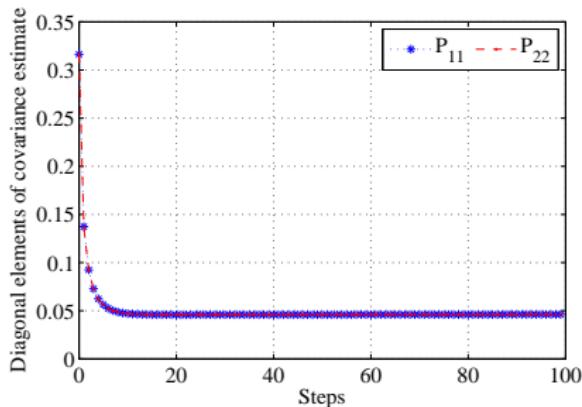
$$\mathbf{q}_k \sim N(0, \mathbf{Q}_k) \text{ and } \mathbf{v}_k \sim N(0, \mathbf{V}_k)$$



- State 1 and 2 converge to ~ 9 arcsec (0.0025 degree).
- State 3 and 4 have similar precision but much lower accuracy ~ 65 arcsec (0.018 degree).

Kalman Filter Test in Simulation

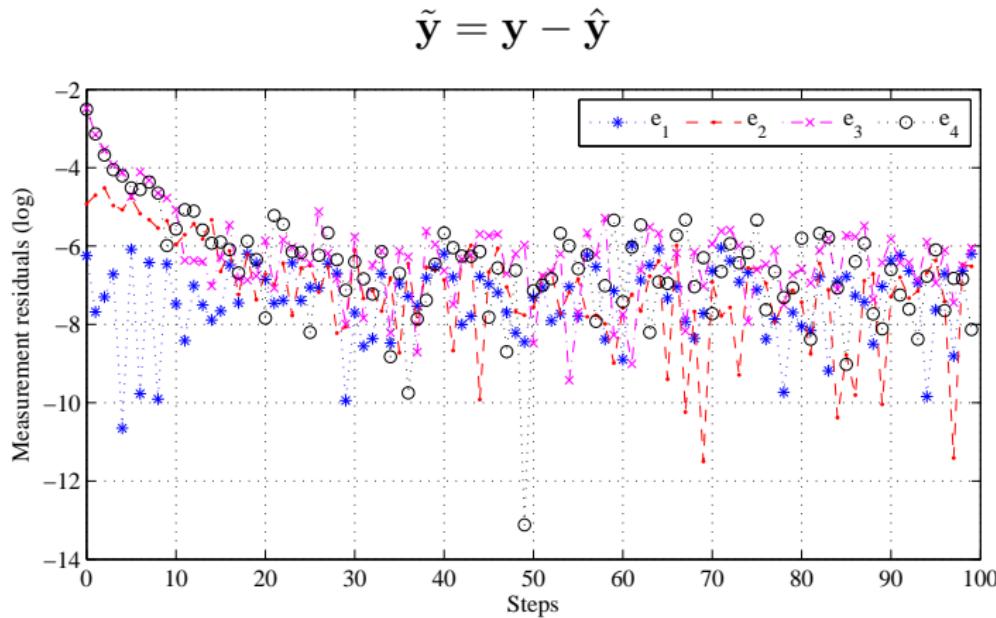
Diagonal elements of covariance matrix P



- P_{11} and P_{22} decrease monotonically to a stable value.
- P_{33} and P_{44} decrease in the beginning, but start increasing after few steps.

Kalman Filter Test in Simulation

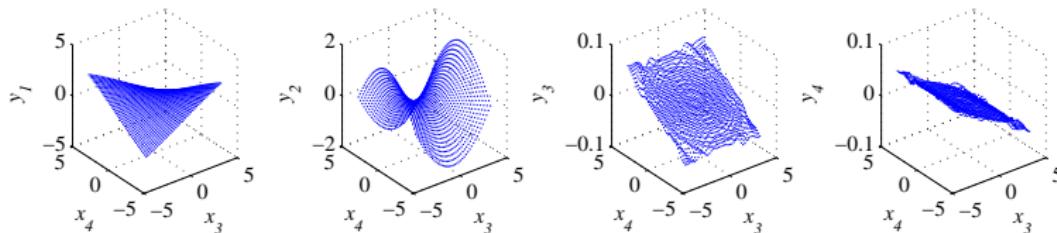
Measurement residuals



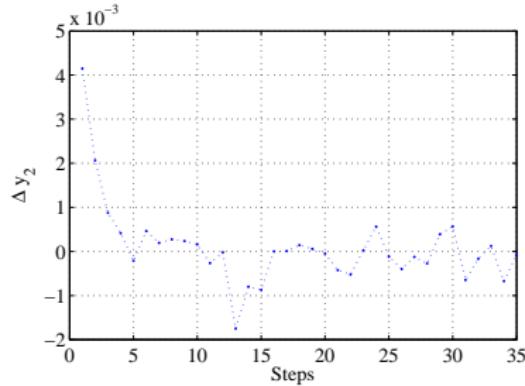
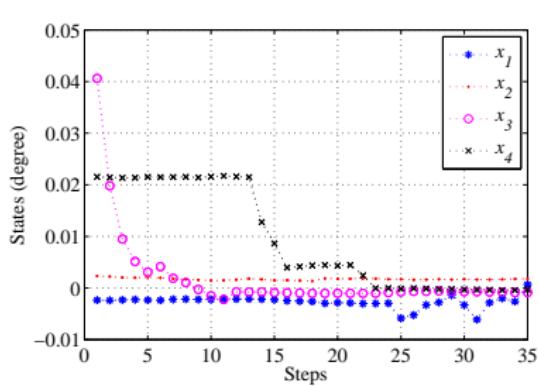
- Measurements converge to near zero → Kalman filter converges.
- Lens B is not fully observable using EKF with our 4 measurements y_1 to y_4 .

Alignment of Lens B

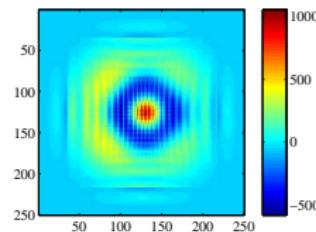
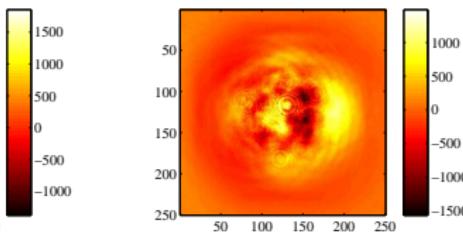
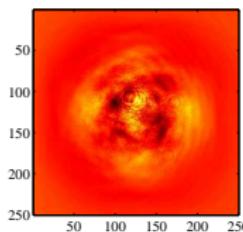
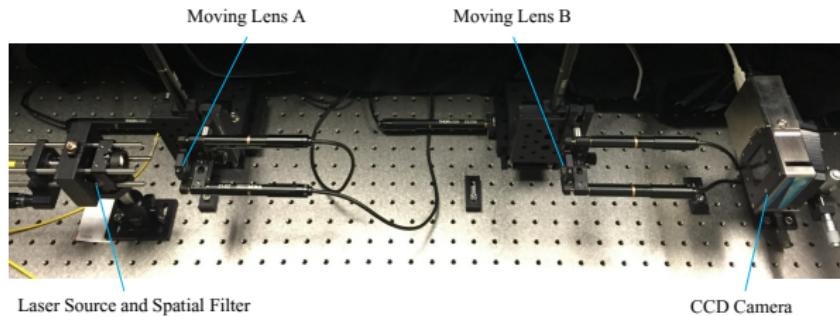
Simulation



- Measurement y_2 is symmetric with respect to both x_3 and x_4 .
- Increase and decrease x_3 by $\Delta x_3 \rightarrow$ measurements y_2^+ and y_2^- .
- Calculate difference $\Delta y_2 = y_2^+ - y_2^-$ for correction.



Experimental Setup



- Images collected in the experiment show large bias compared to our model → lens shift.
- Test lens shift in ZEMAX: reconstructed image with mode w_1 to w_4
- Projecting the remaining image onto mode coma-related modes will result in non trivial coefficients.

Conclusion

Future work - more complicated reconfigurable systems



- 1** Correct shift and tilt together in the experiment:
 - Aligning the lenses separately
 - Remove one lens and translate the other one along z – axis.
 - Might introduce stage error.
 - Estimating with a 8 DOF model
 - More measurements (e.g. spot center and coefficients).
 - Require more image data and computation time.
- 2** Improve the model describing optical system:
 - Higher order model functions
 - Learning methods other than linear regression
 - Different methods for modal decomposition (e.g. ICA)
- 3** Estimation and control:
 - Apply other estimation methods (e.g. Particle filter, Unscented Kalman filter)
 - A systematic control process will be built



Thank you