Misalignment Retrieval of an Off-axis Parabolic Mirror using Kalman Filtering

JAN 20, 2017

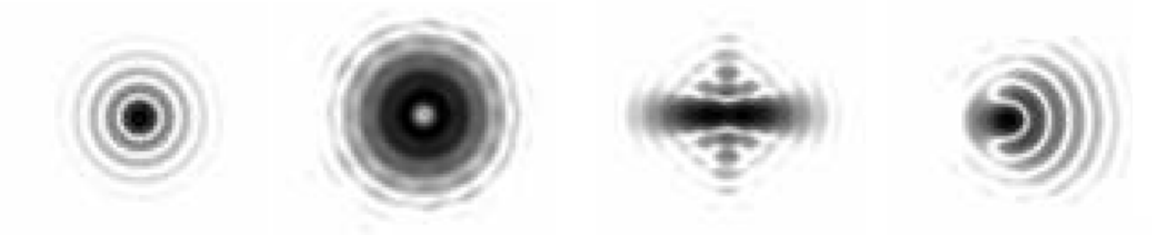
Outline

- Introduction
- Model
- Image processing and reconstruction
- State estimation and control
- Simulation result
- Conclusion

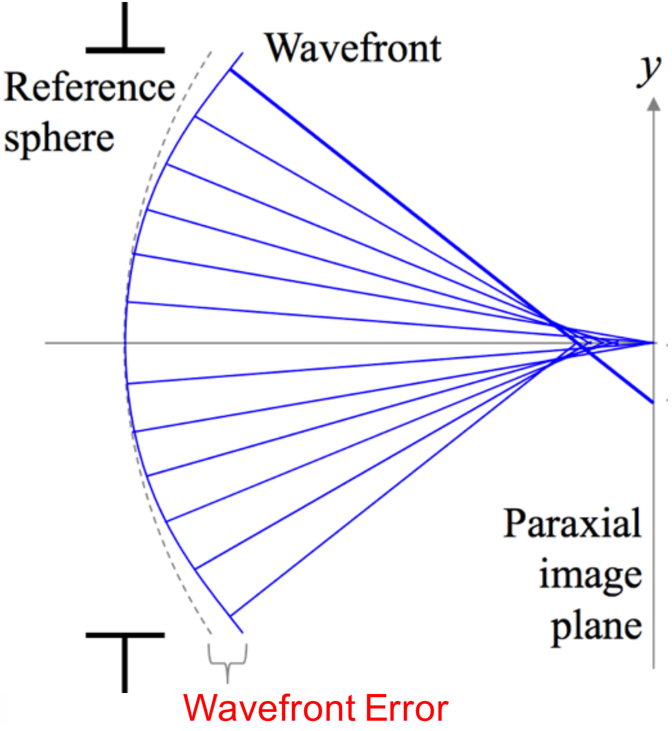
Optical misalignments introduce wavefront aberration

- Wavefront aberration - *Any deviation of the wavefront formed by an optical system from perfect spherical*

- Main aberrations

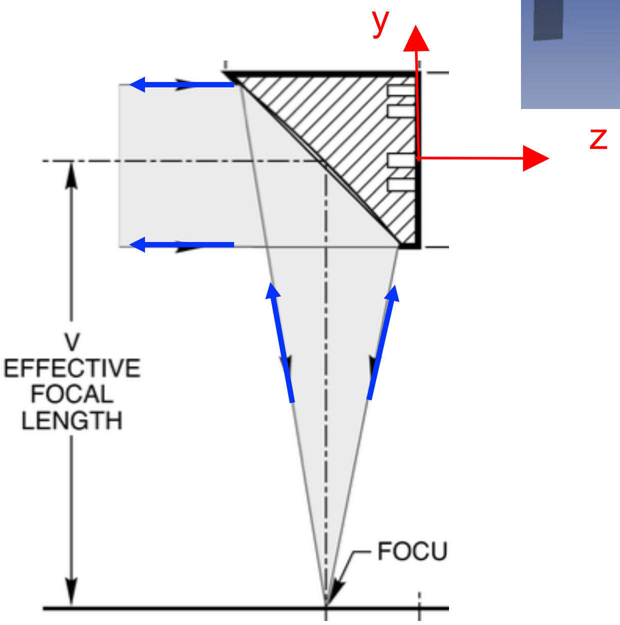


Ideal Defocus Astigmatism Coma

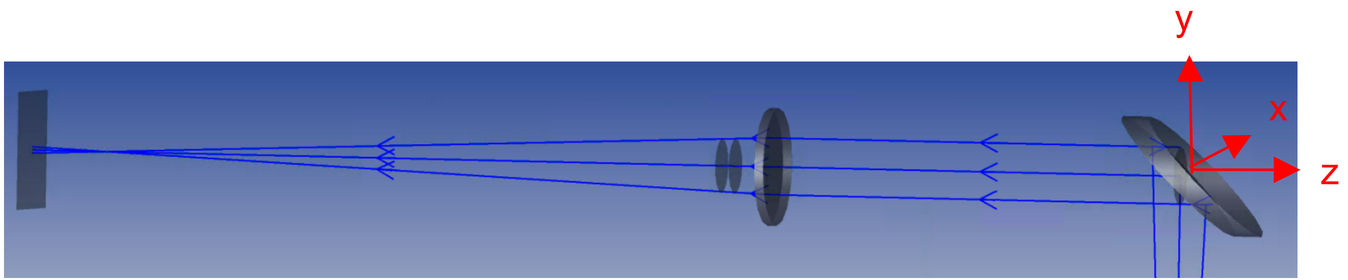


Robert McLeod, 2015

Model



http://assets.newport.com/

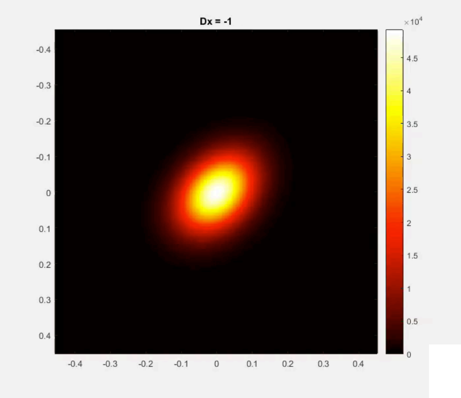


DOF:

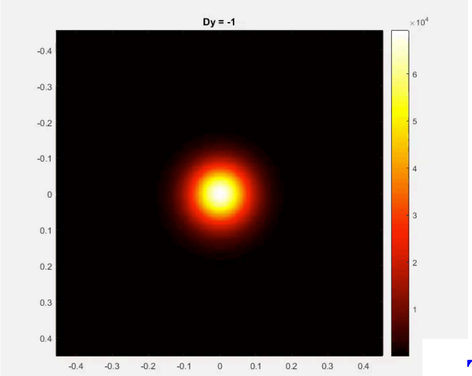
- Decenter **Dx**
- Decenter **Dy**
- Decenter **Dz**
- Tilt **Tx**
- Tilt **Ty**

Misalignment Effect

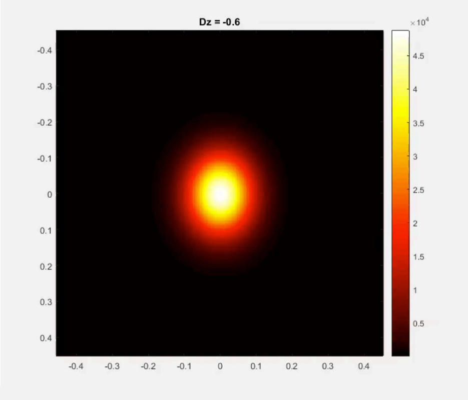
Decenter **Dx**



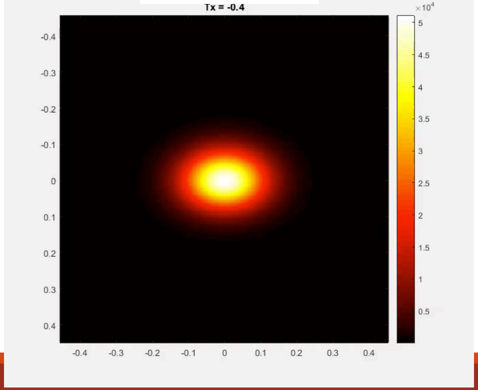
Decenter **Dy**



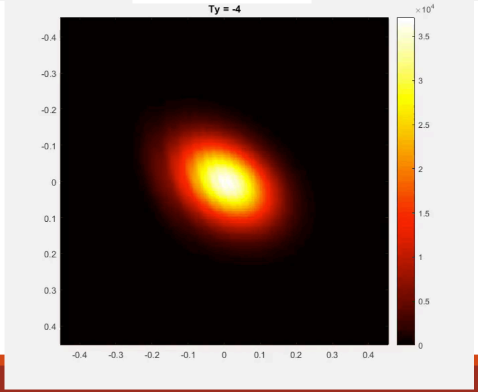
Decenter **Dz**



Tilt **Tx**



Tilt **Ty**



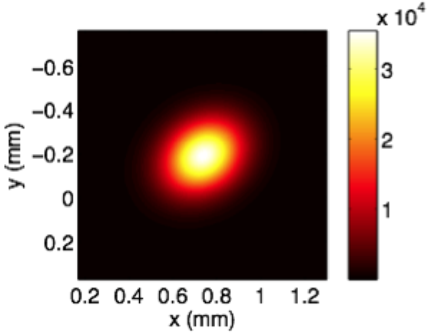
Gaussian Fitting and Image Decomposition

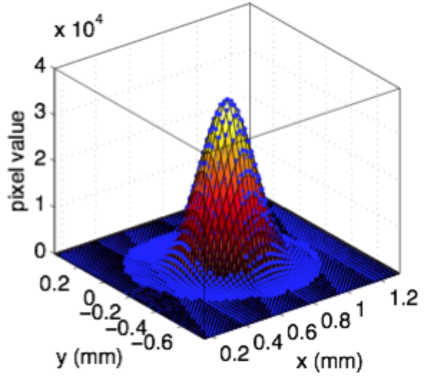
- Gaussian fitting to get center position **Cx** and **Cy**

$$F(x,y) = G_1 + G_2 \exp\left(\frac{-(\frac{x'}{a})^2 - (\frac{y'}{b})^2}{2}\right)$$

$$x' = (x - C_x)\cos\phi - (y - C_y)\sin\phi$$

$$y' = (x - C_x)\sin\phi + (y - C_y)\cos\phi$$



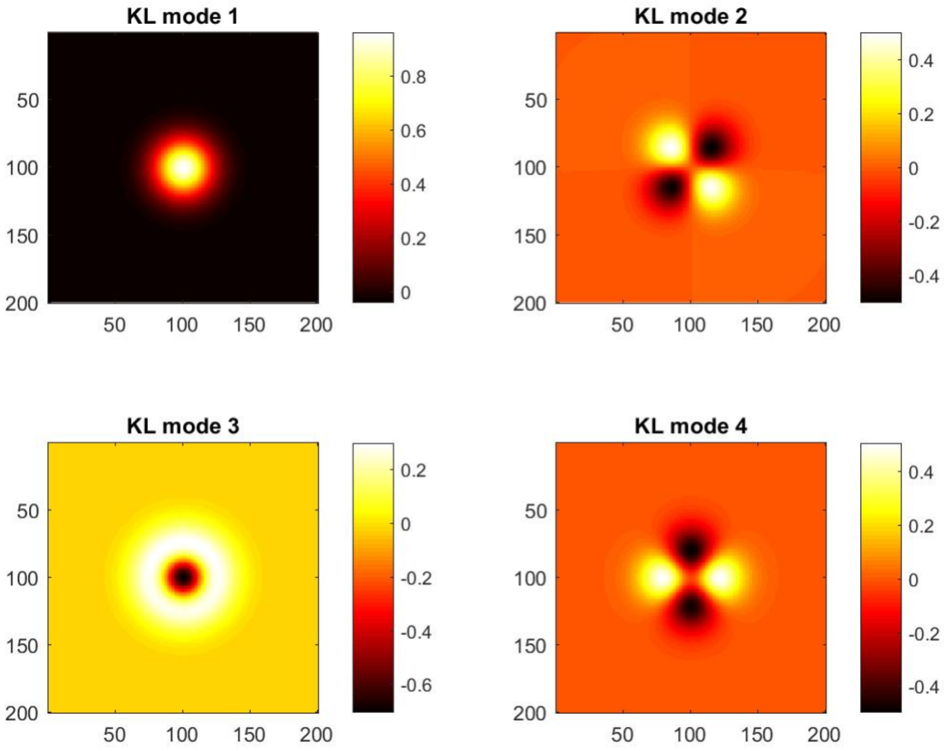


- Vector-mean-subtracted image vector

$$\bar{\mathbf{v}}_i = \mathbf{v}_i - \mu(\mathbf{v}_i) \xrightarrow{collect\ image\ dataset} \bar{\mathbf{V}} = [\bar{\mathbf{v}}_1, \bar{\mathbf{v}}_2, \dots, \bar{\mathbf{v}}_n]$$

- Image-to-image covariance $\mathbf{S} = \frac{1}{p-1}\bar{\mathbf{V}}^T\bar{\mathbf{V}}$
- Eigen decomposition $\mathbf{S\Phi} = \mathbf{\Phi\Lambda}$
- Karhunen-Loève modes (eigen images) $\mathbf{Z} = \bar{\mathbf{V}}\mathbf{\Phi}$

Karhunen-Loève Modes



Weights:
w1, w2, w3, w4

Measurements:
w2/w1, w3/w1, w4/w1

Measurement Function

- Measurements in terms of states (Dx, Dy, Dz, Tx, Ty).

$$C_x = a_1 x_1 + a_2 x_2$$

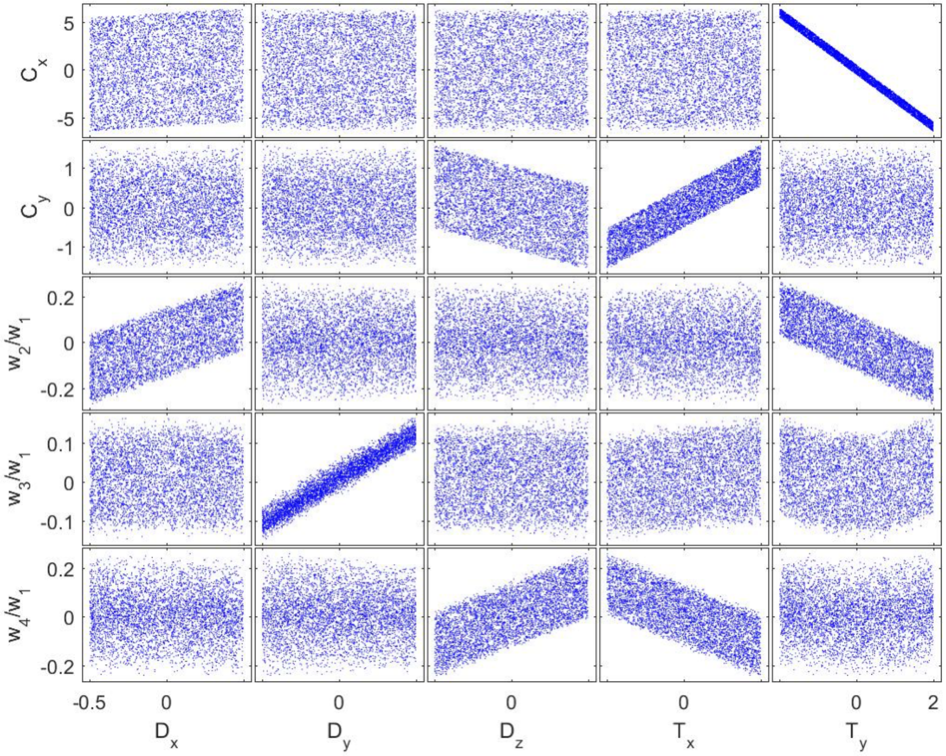
$$C_y = b_1 x_2 + b_2 x_3 + b_3 x_4 + b_4 x_1^2 + b_5 x_5^2$$

$$\frac{w_2}{w_1} = c_1 x_1 + c_2 x_5$$

$$\frac{w_3}{w_1} = d_1 x_2 + d_2 x_4 + d_3 x_1^2 + d_4 x_3^2 + d_5 x_5^2$$

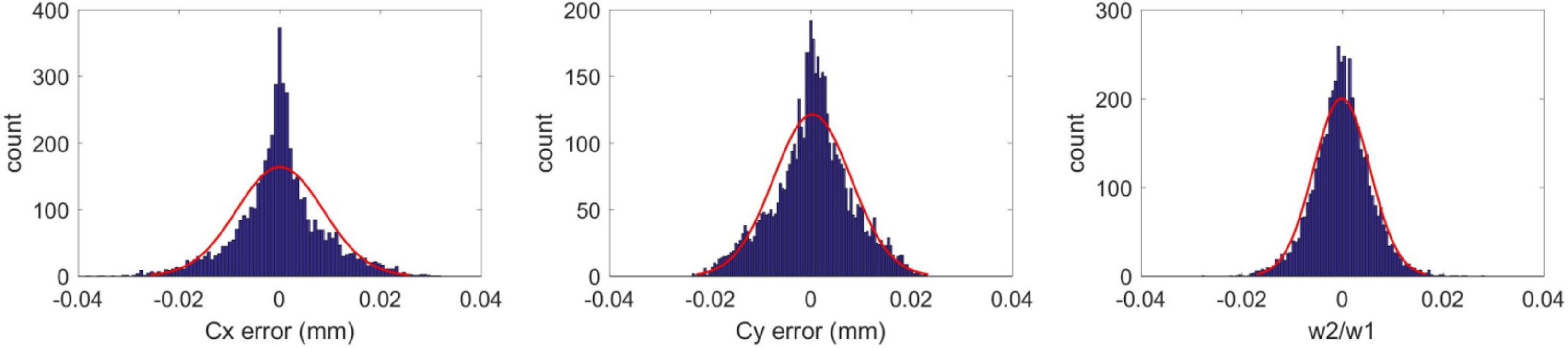
$$\frac{w_4}{w_1} = e_1 x_3 + e_2 x_4 + e_3 x_2^2 + e_4 x_5^2$$

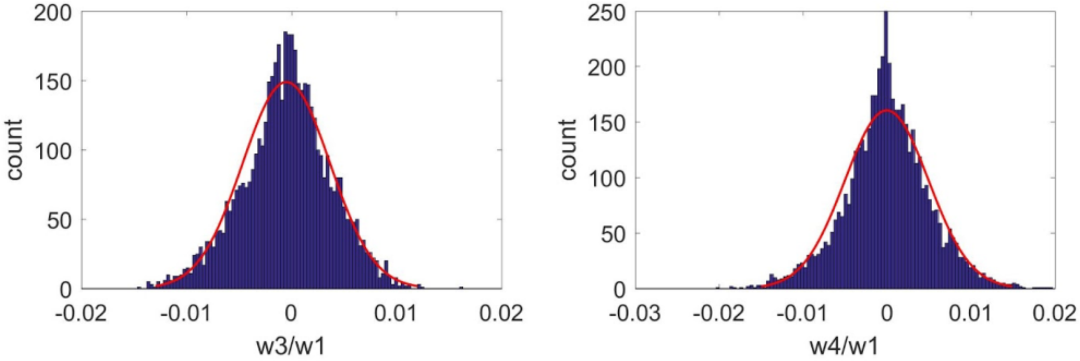
Training set 10000 images



Measurement Function Error

Test set 5000 images





$$\sigma = \begin{bmatrix} 8.6\mu m \\ 7.7\mu m \\ 4.4\times10^{-3} \\ 3.9\times10^{-3} \\ 4.3\times10^{-3} \end{bmatrix}$$

Image noise

- Shot noise – The random arrival of photons → **Poisson distribution**
- CCD read noise – Noise generated by electronics as the charge present in the pixels is transferred to the camera → **Gaussian distribution**

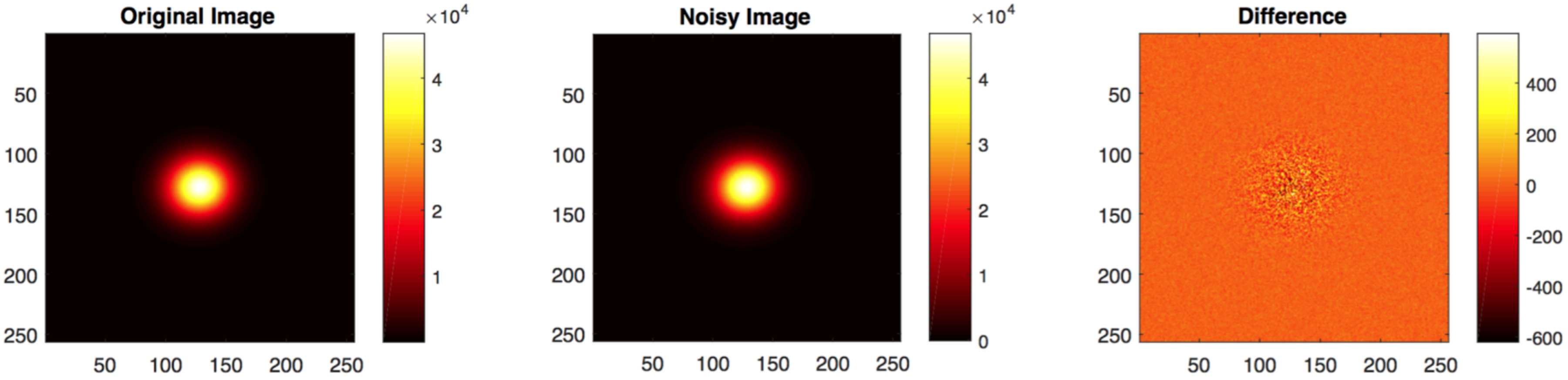
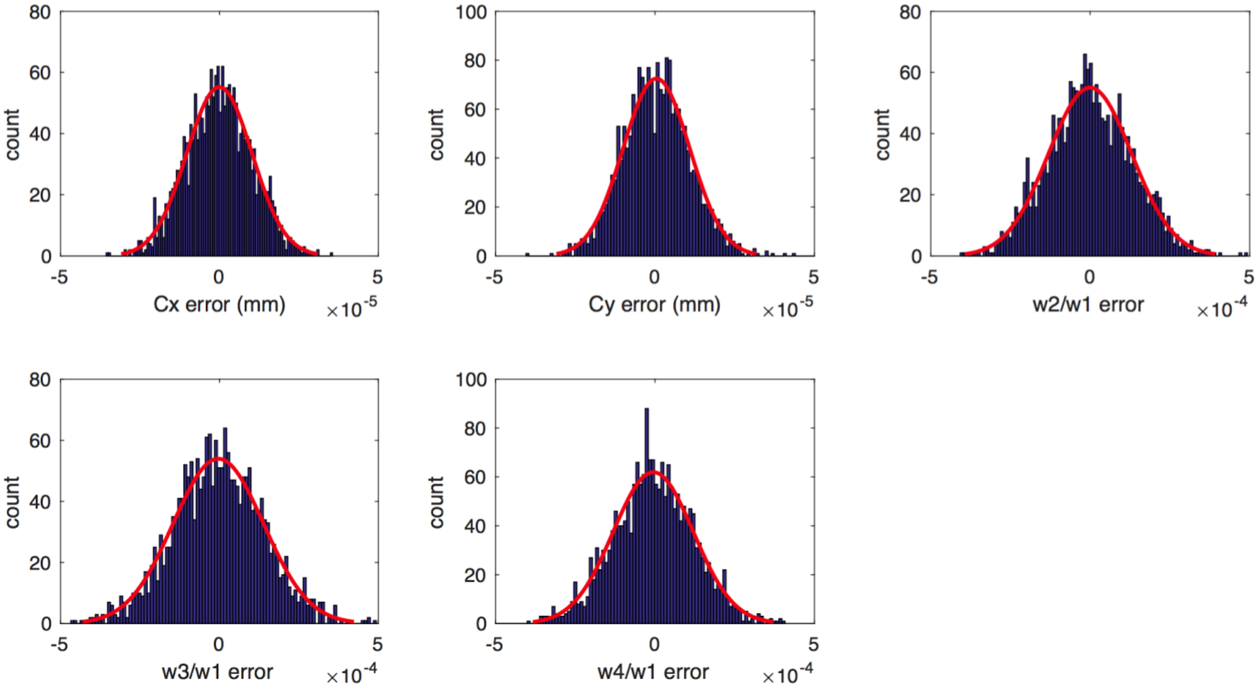
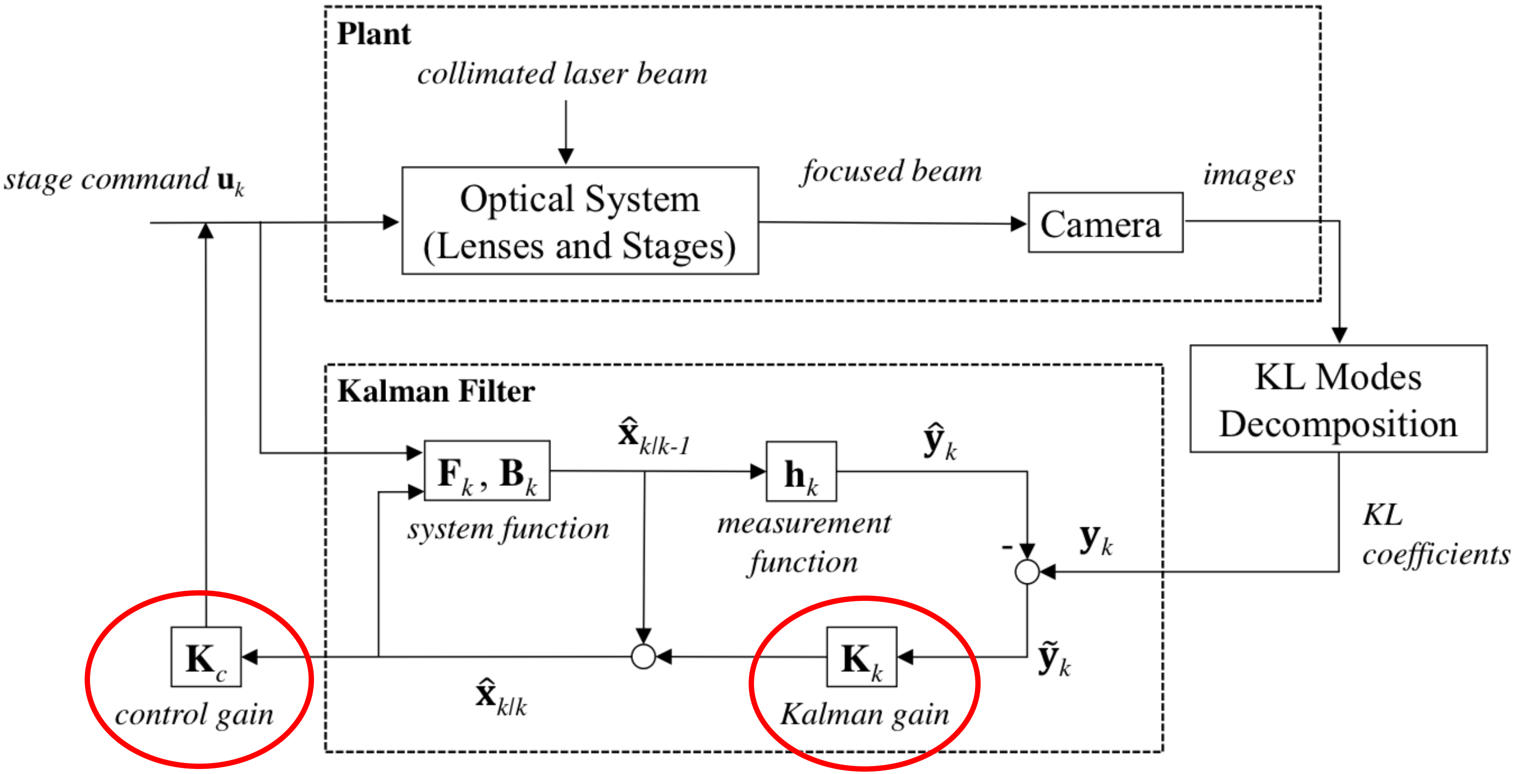


Image Noise Effects on Center Position and KL weight deviation

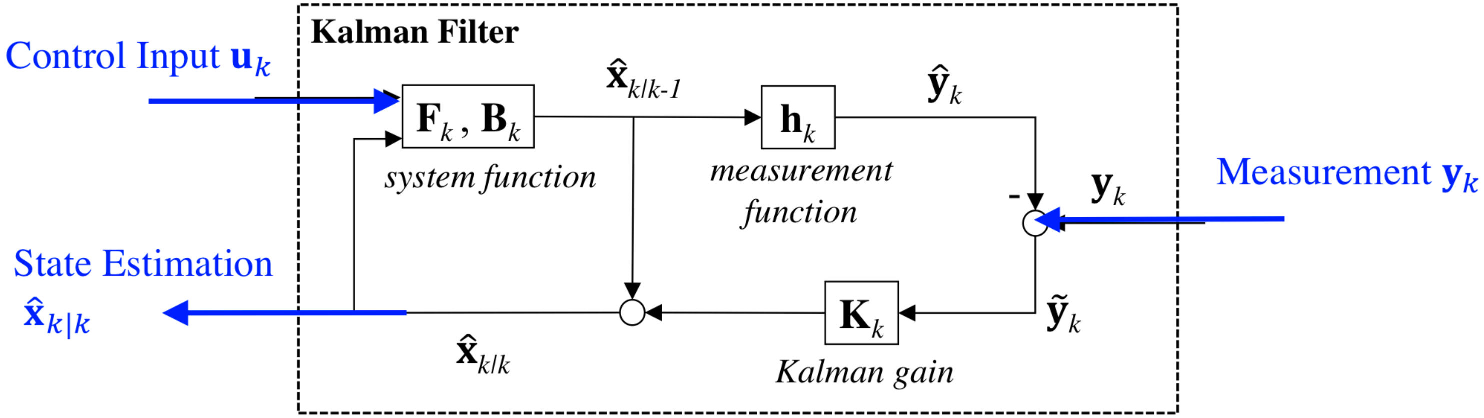
- 2000 images with random misalignment and noise
- The effect of random noise (both Poisson and Gaussian noise) on the measurements are normal distribution



Estimation and Control



Kalman Filtering

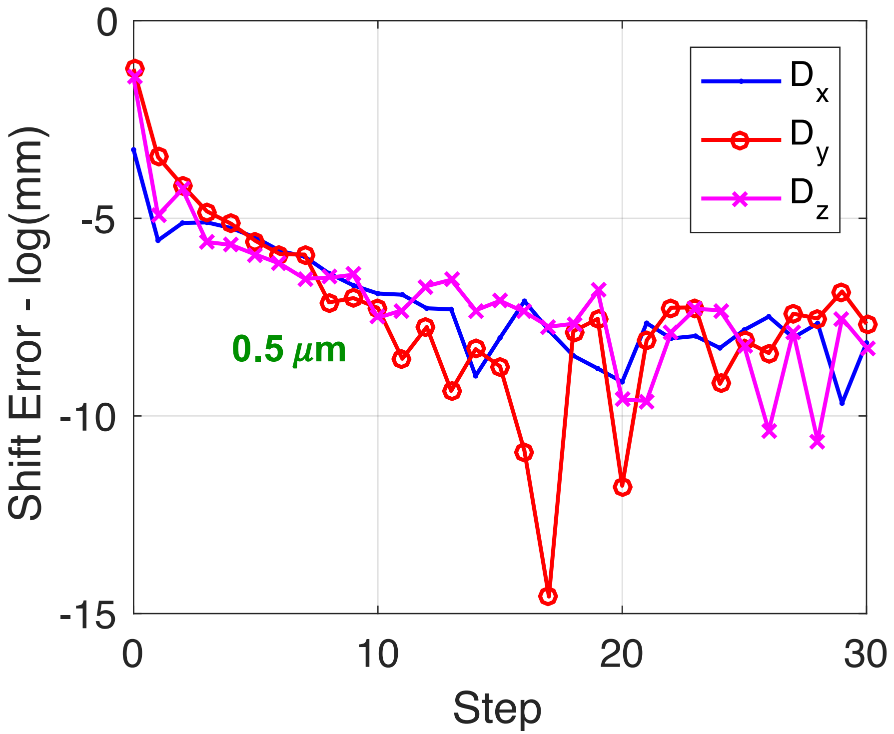


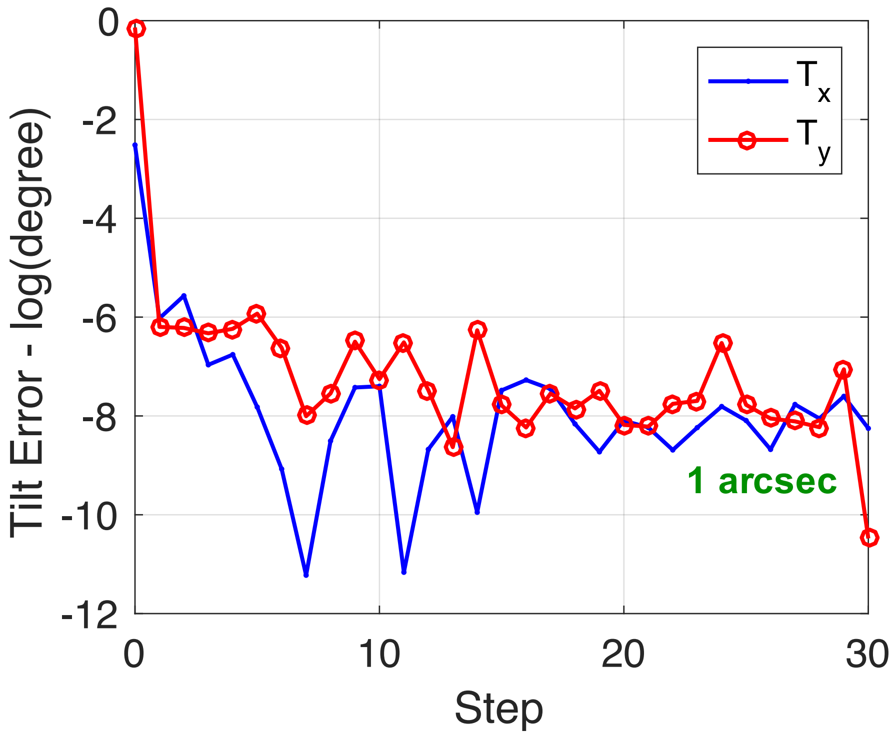
- Given variables: control input $\mathbf{u}_k$, measurements $\mathbf{y}_k$
- State space representation:

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \qquad \mathbf{q}_k \sim N(0, \mathbf{Q}_k)$$
$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{r}_k \qquad \mathbf{r}_k \sim N(0, \mathbf{R}_k)$$

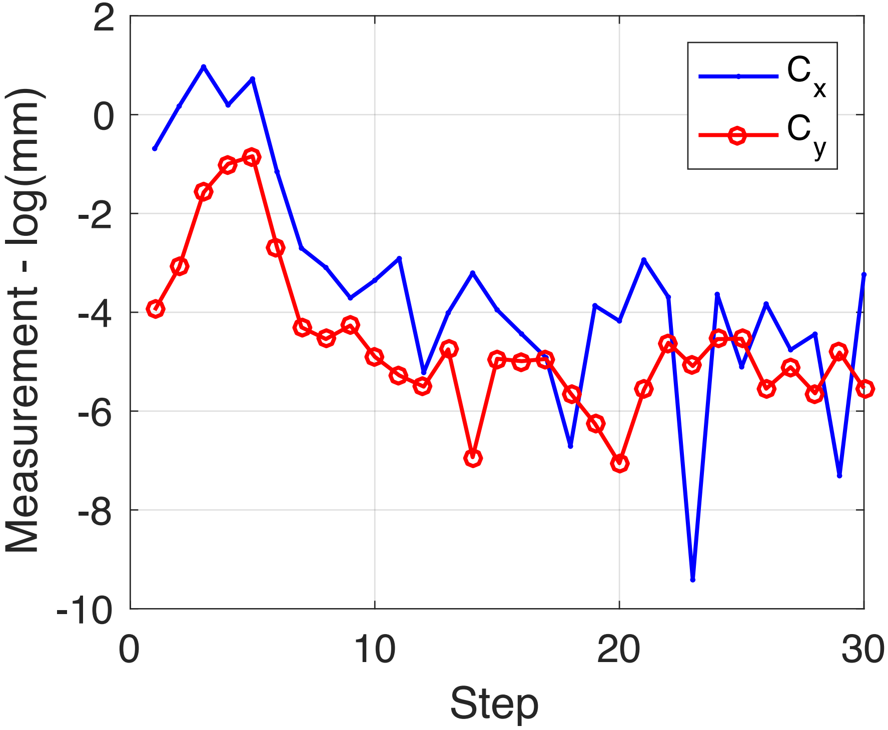
- $\mathbf{R}_k = \mathbf{R}_{model} + \mathbf{R}_{meas}$

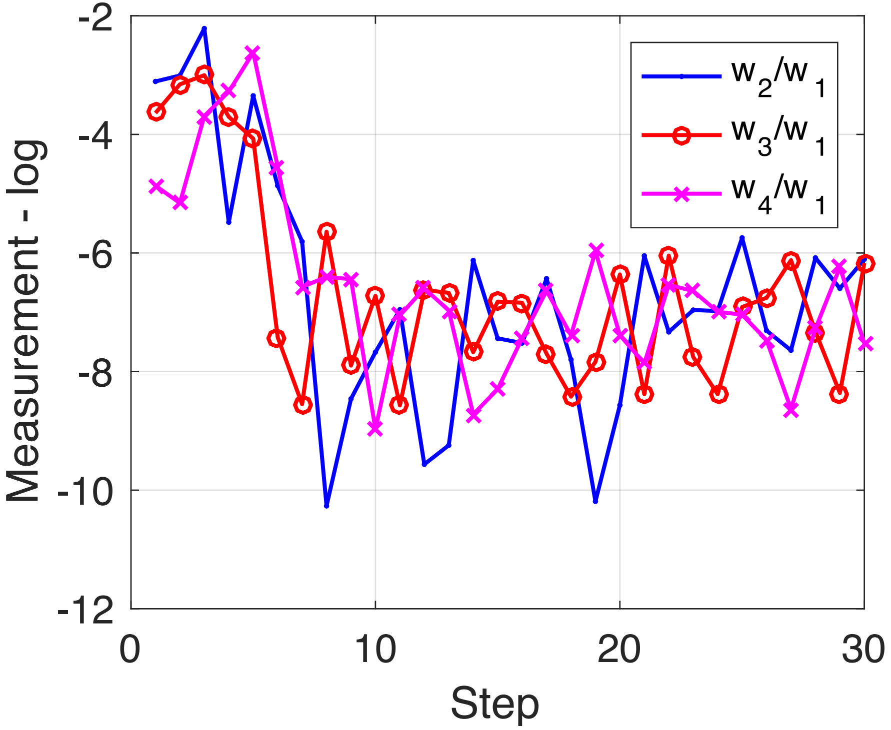
Simulation Result - State (IEKF)



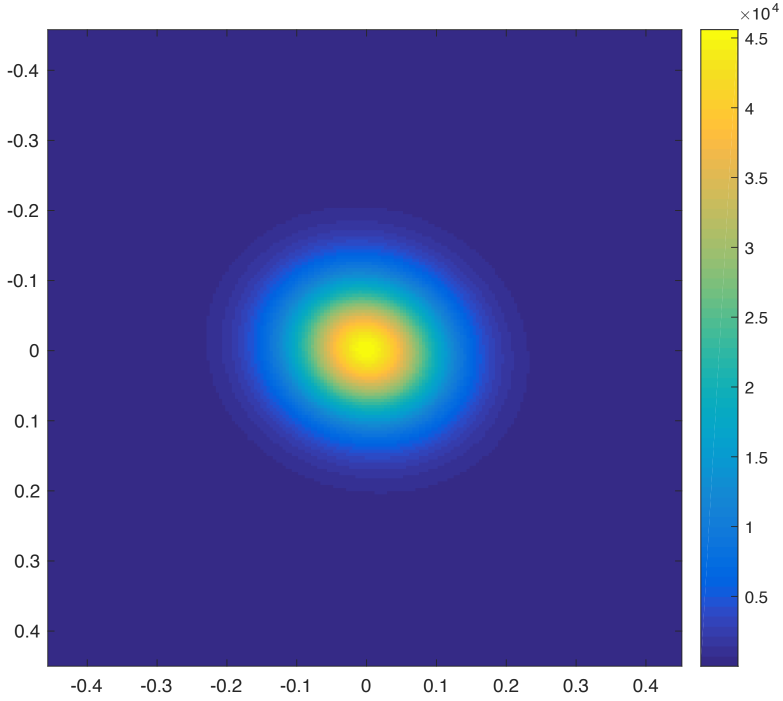


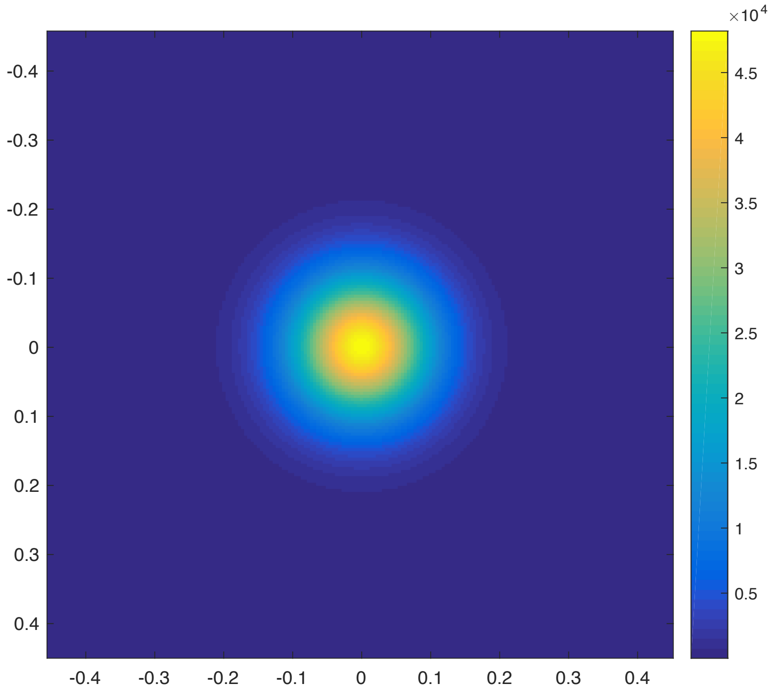
Simulation Result - Measurement (IEKF)





Before and After





Conclusion and Future Work

- Image processing and optimal estimation achieve automated alignment of an off-axis parabolic mirror without using a wavefront sensor.
- Other estimation and control technique can be applied (such as particle filter, robust control).
- An experiment will be conducted on an optical bench.

https://github.com/JFgithubJF/oap_model

Thank You