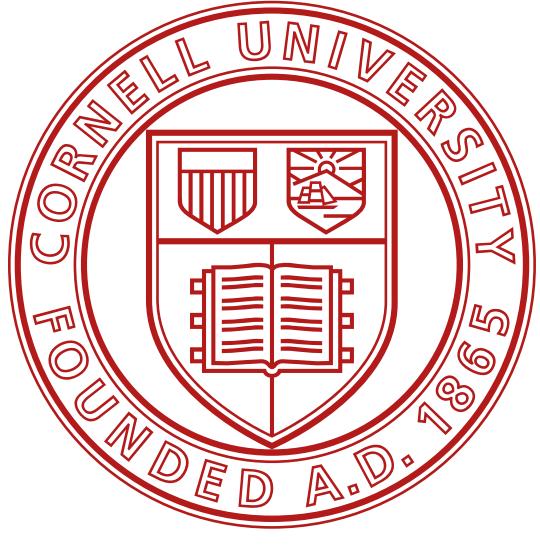


Detected Exoplanet Population Distributions Found Analytically



Daniel Garrett and Dmitry Savransky

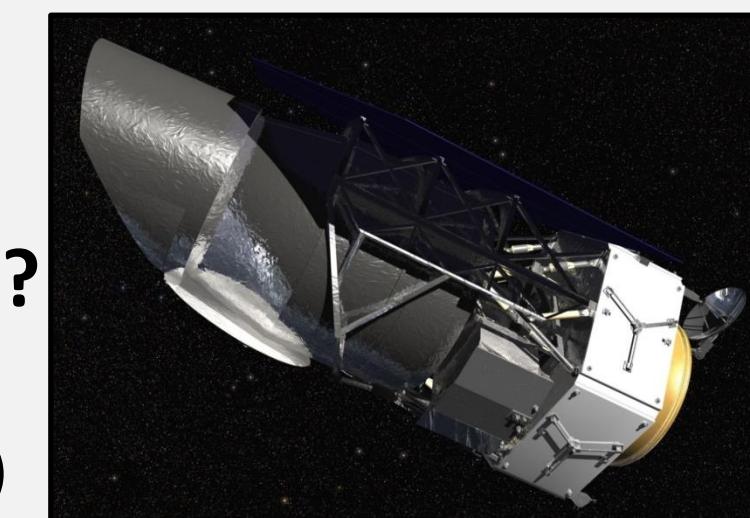
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Exoplanet Imaging Yield

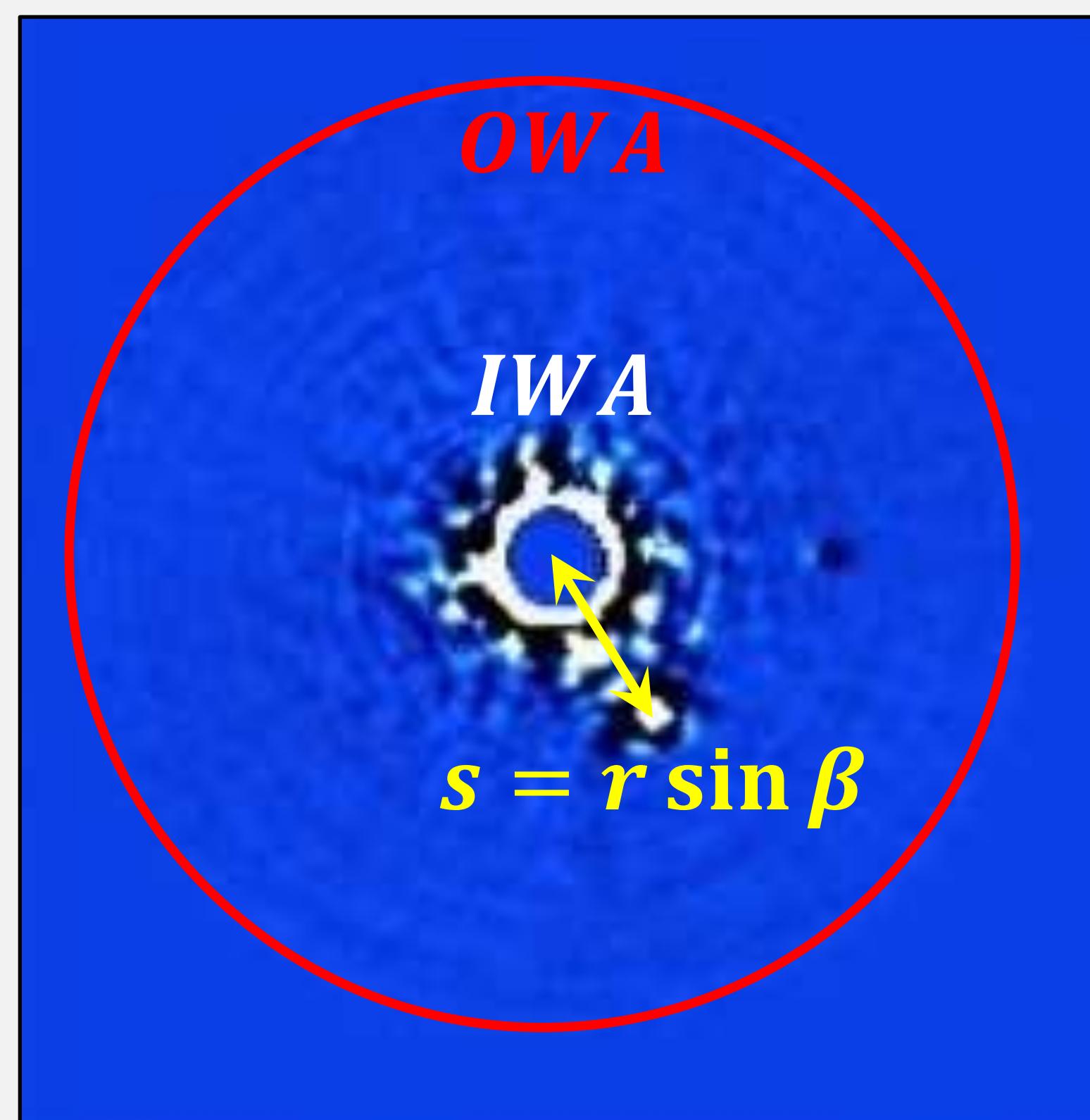
- How many planets will my instrument detect?
 - Completeness
- What kind of planets will my instrument detect?
 - Detected distributions

WFIRST (jpl.nasa.gov/spaceimages)



Detection Criteria

Geometric



Beta Pic image from [1]

Star-Planet Distance

$$r = \frac{a(1 - e^2)}{1 + e \cos \beta}$$

β – star-planet-observer (phase) angle

Photometric

Planet Population

$$F_R = p \left(\frac{R}{r} \right)^2 \Phi(\beta)$$

- $\Delta\text{mag} = -2.5 \log_{10} F_R$
- p – geometric albedo
- R – planetary radius
- $\Phi(\beta)$ - phase function

Survey Sensitivity

$$\Delta\text{mag}_0$$

Detection

$$IWA \times d < s < OWA \times d \text{ and } \Delta\text{mag} < \Delta\text{mag}_0$$

d – distance from star to observer (pc)

Completeness – How Many Planets?

- Fraction of detected planets from assumed population^[2]
- Probability of detecting planets from assumed population^[3]

$$\text{Comp} = \int_{IWA \times d}^{OWA \times d} \left[\int_0^{\Delta\text{mag}_0} f_{\bar{s}, \Delta\text{mag}}(s, \Delta\text{mag}) d\Delta\text{mag} \right] ds$$

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References

- [1] Marois, C., et al., "GPI PSF Subtraction with TLOCI: the Next Evolution in Exoplanet/Disk High-Contrast Imaging," in *[Adaptive Optics Systems IV]*, 91480U-91480U, International Society for Optics and Photonics (2014).
- [2] Brown, R. A., "Single-Visit Photometric and Obscurational Completeness," *The Astrophysical Journal* **624**(2), 1010 (2005).
- [3] Garrett, D. and Savransky, D., "Analytical Formulation of the Single-Visit Completeness Joint Probability Density Function," *The Astrophysical Journal* **828**(1), 20 (2016).

Detected Distributions – What Kind of Planets?

Derivation: Projected Separation and Δmag

$$f_{\bar{s}'}(s') = \frac{1}{C_{s'}} \times \begin{cases} \int_0^{\Delta\text{mag}_0} f_{\bar{s}, \Delta\text{mag}}(s, \Delta\text{mag}) d\Delta\text{mag}, & IWA \times d < s' < OWA \times d \\ 0, & \text{else} \end{cases}$$

$$f_{\Delta\text{mag}'}(\Delta\text{mag}') = \frac{1}{C_{\Delta\text{mag}'}} \times \begin{cases} \int_{IWA \times d}^{OWA \times d} f_{\bar{s}, \Delta\text{mag}}(s, \Delta\text{mag}) ds, & \Delta\text{mag}_{\min}(IWA \times d) < \Delta\text{mag}' < \Delta\text{mag}_0 \\ 0, & \text{else} \end{cases}$$

- $\Delta\text{mag}_{\min}(s)$ from [3]
- C 's normalize integral over s' or $\Delta\text{mag}'$ to one

Derivation: Planetary Parameters

- Assume parameters are all independent and form joint probability density function

$$f_{\bar{a}, \bar{e}, \bar{E}, \bar{\beta}, \bar{p}, \bar{R}}(a, e, E, \beta, p, R) = \frac{\sin \beta}{4\pi} (1 - e \cos E) f_{\bar{a}}(a) f_{\bar{e}}(e) f_{\bar{p}}(p) f_{\bar{R}}(R)$$

- E - eccentric anomaly

- Marginalize all but desired parameter subject to detection criteria:

$$f_{\bar{a}'}(a') = \frac{1}{C_{a'}} \int_e \int_E \int_{\beta} \int_p \int_R f_{\bar{a}, \bar{e}, \bar{E}, \bar{\beta}, \bar{p}, \bar{R}}(a, e, E, \beta, p, R) dR dp d\beta dE de$$

- $C_{a'}$ normalizes integral over a' to one

Survey Sensitivity

Quantity	Value
IWA	0.1"
OWA	0.5"
Δmag_0	25.0

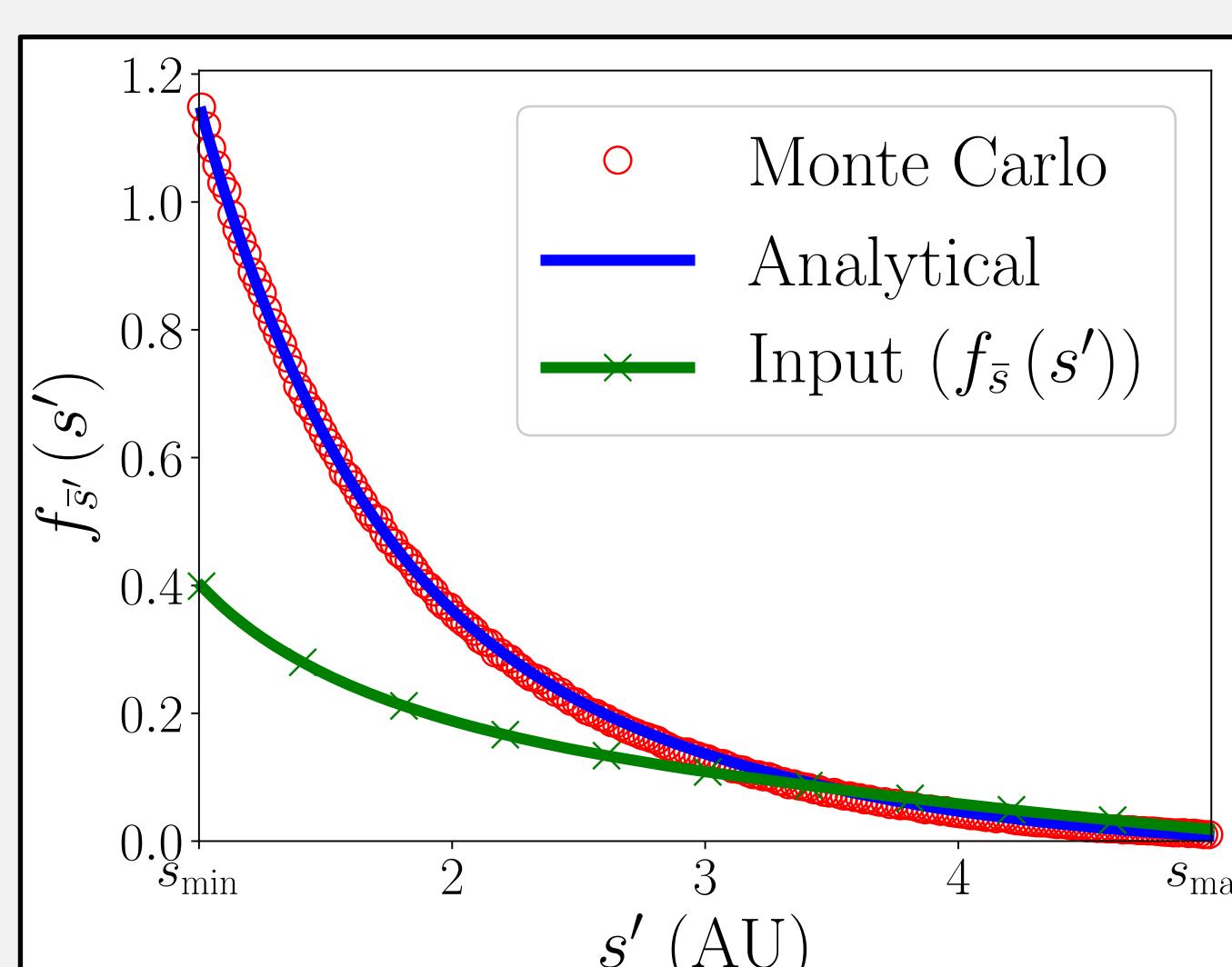
Target Star
 $d = 10$ pc

Assumed Population

Quantity	Min	Max	Distribution
a (in AU)	0.5	5.0	Log-uniform
e	0.0	0.35	Uniform
p	0.2	0.4	Uniform
R (in km)	6,000	30,000	Log-uniform

Phase Function

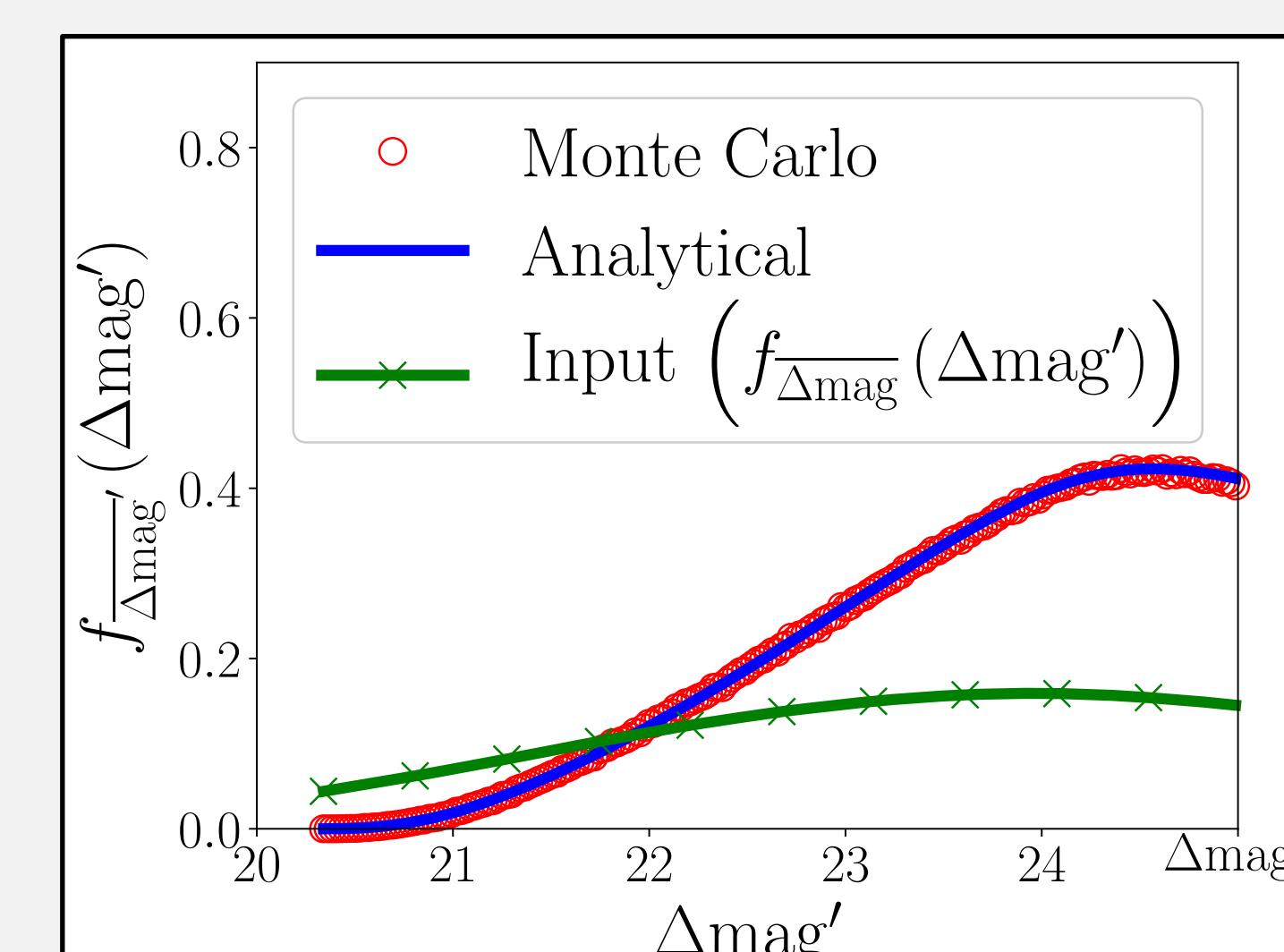
$$\Phi(\beta) = \frac{1}{\pi} [\sin \beta + (\pi - \beta) \cos \beta]$$



- Input separation distribution:

$$s \in [0, a_{\max}(1 + e_{\max})]$$

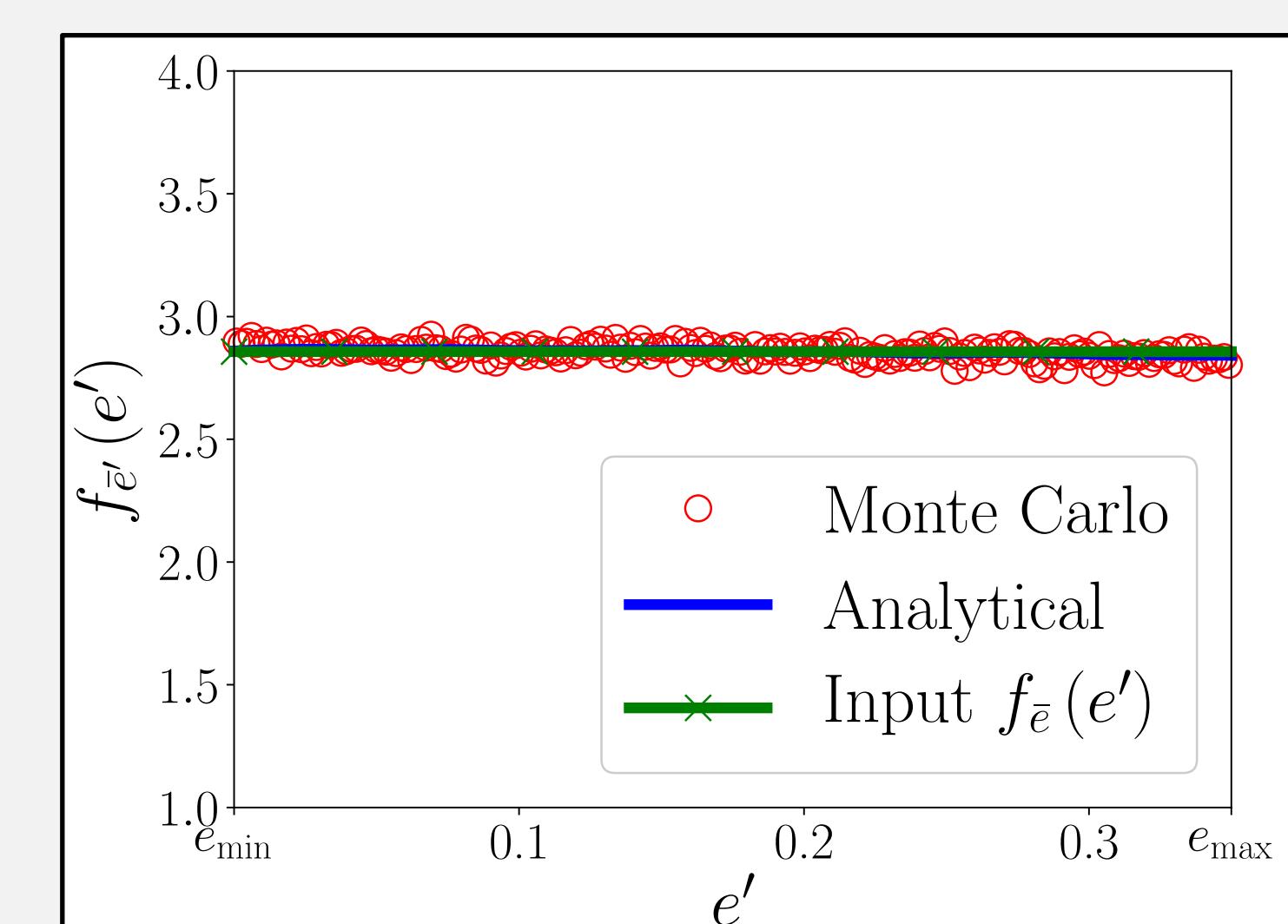
- $s \downarrow, \Delta\text{mag} \downarrow$, brighter, more detectable



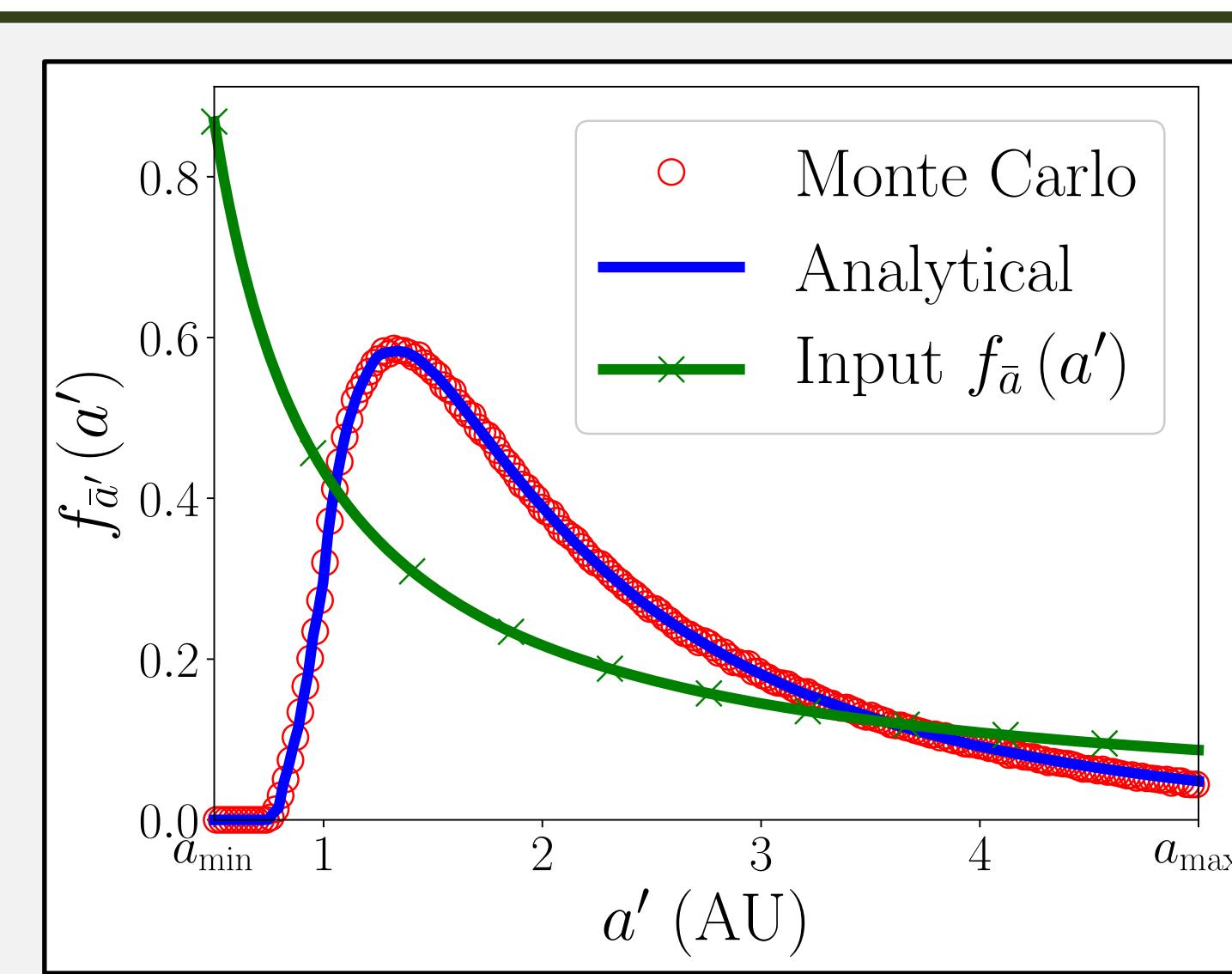
- Input Δmag distribution:

$$\Delta\text{mag} \in [\Delta\text{mag}_{\min}(0), \infty)$$

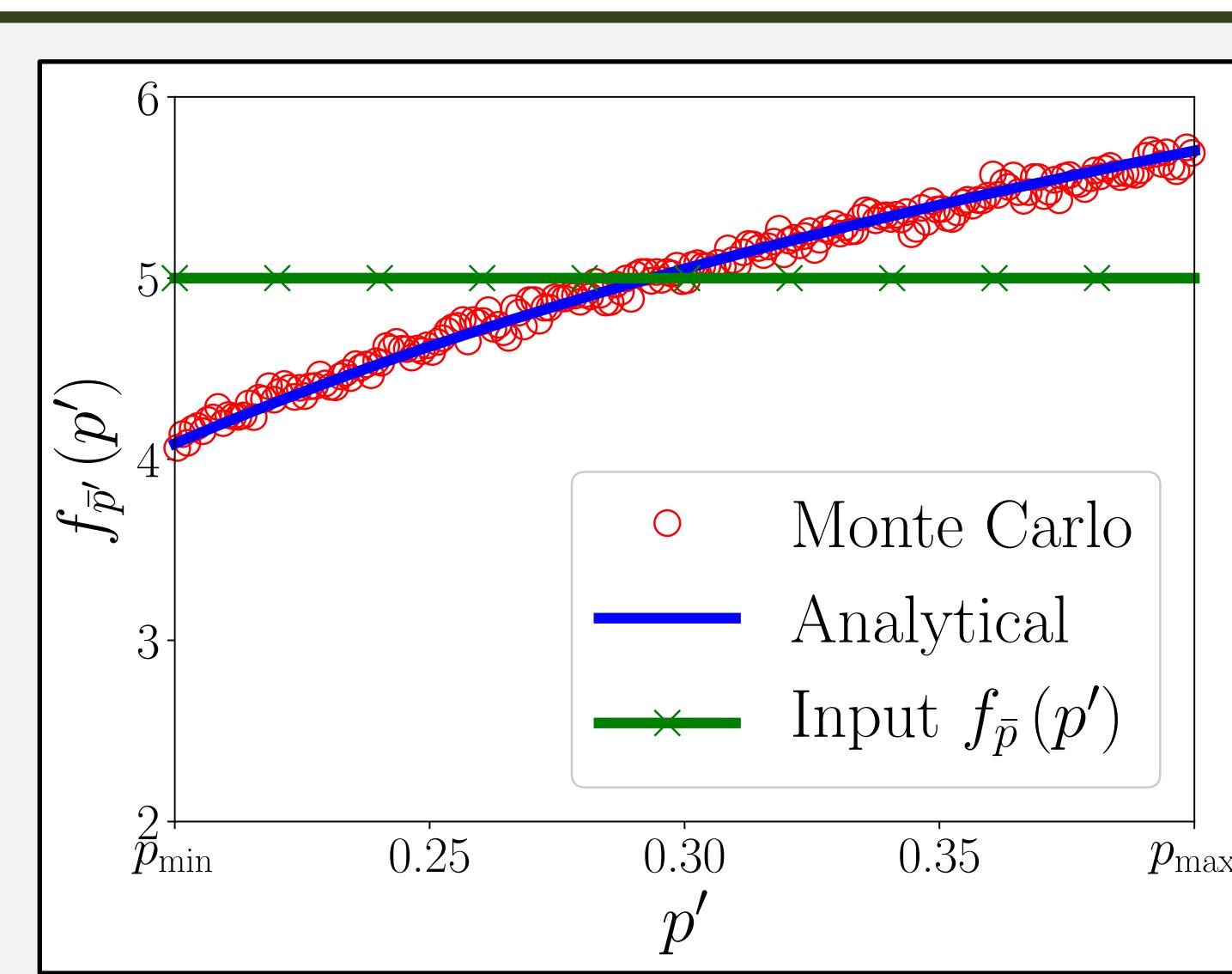
- Truncated range causes steeper slope



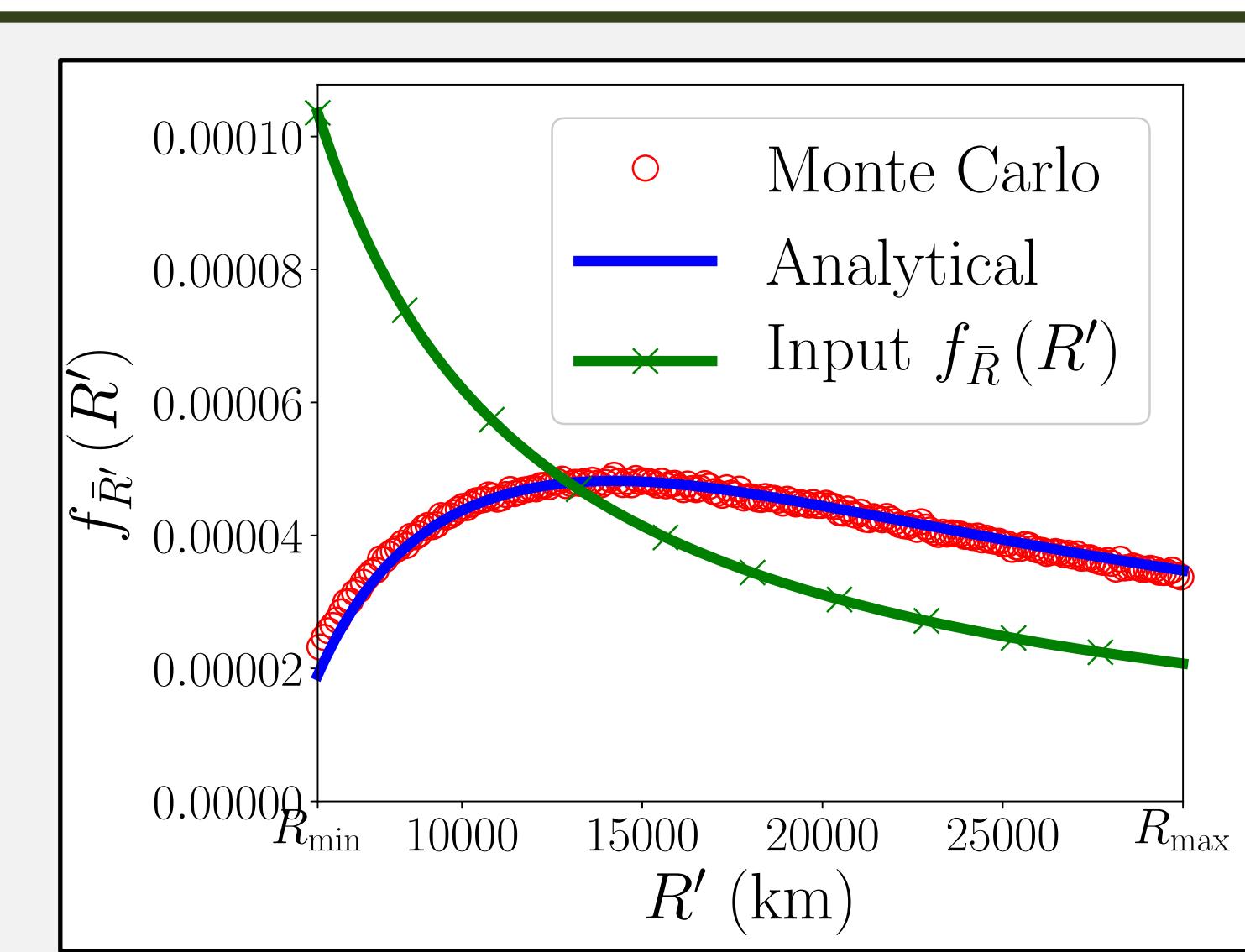
- Unchanged from input distribution



- Zero below $a_l = (IWA \times d)(1 + e_{\max})^{-1}$
- $a \uparrow, s \uparrow$, more likely inside IWA/OWA
- $a \uparrow, \Delta\text{mag} \uparrow$, dimmer, less detectable



- Indirectly affected by IWA/OWA
- $p \uparrow, \Delta\text{mag} \downarrow$, brighter, more detectable



- Indirectly affected by IWA/OWA
- $R \uparrow, \Delta\text{mag} \downarrow$, brighter, more detectable