

# Multi-Mission Modeling for Space-Based Exoplanet Imagers

[10400-54]

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August 10, 2017

# Motivation

**LUVOIR**  
Large Ultraviolet / Optical / Infrared Surveyor

LUVOIR is a cornerstone for a highly capable, multi-mission space observatory that will deliver game-changing results. This mission would realize great leaps forward in a broad range of astrophysics, from the epoch of reionization, through galaxy formation and evolution, star and planet formation, and individual objects of all kinds. The Solar System frontier will also be probed. LUVOIR will study a wide range of exoplanets, as do other, including those that might be habitable – or even uninhabited.

**Exo-S**  
Starshade Probe-Class  
Exoplanet Direct Imaging Mission Concept

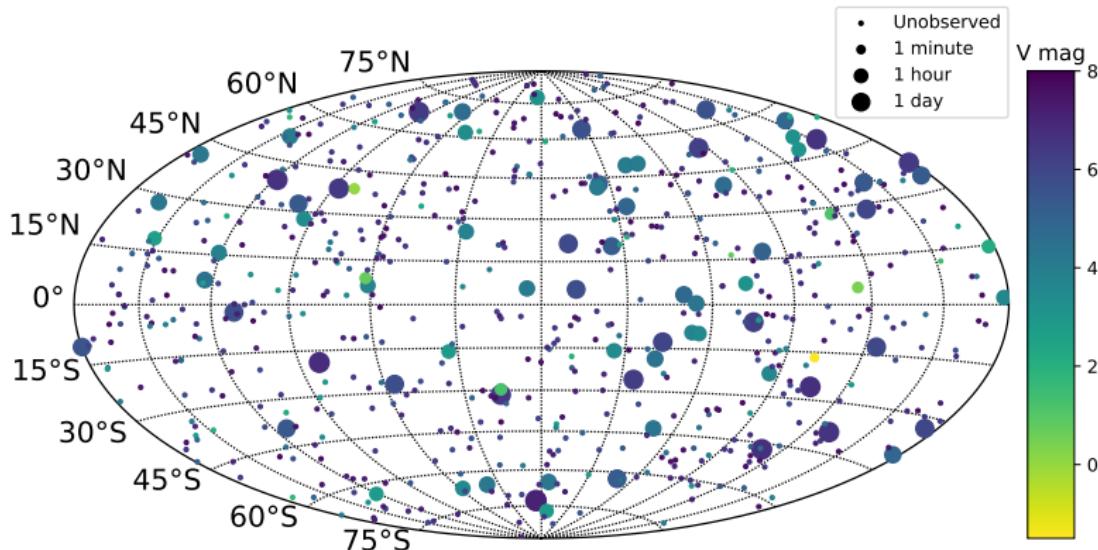
# Questions



What do we want to say about all these missions?

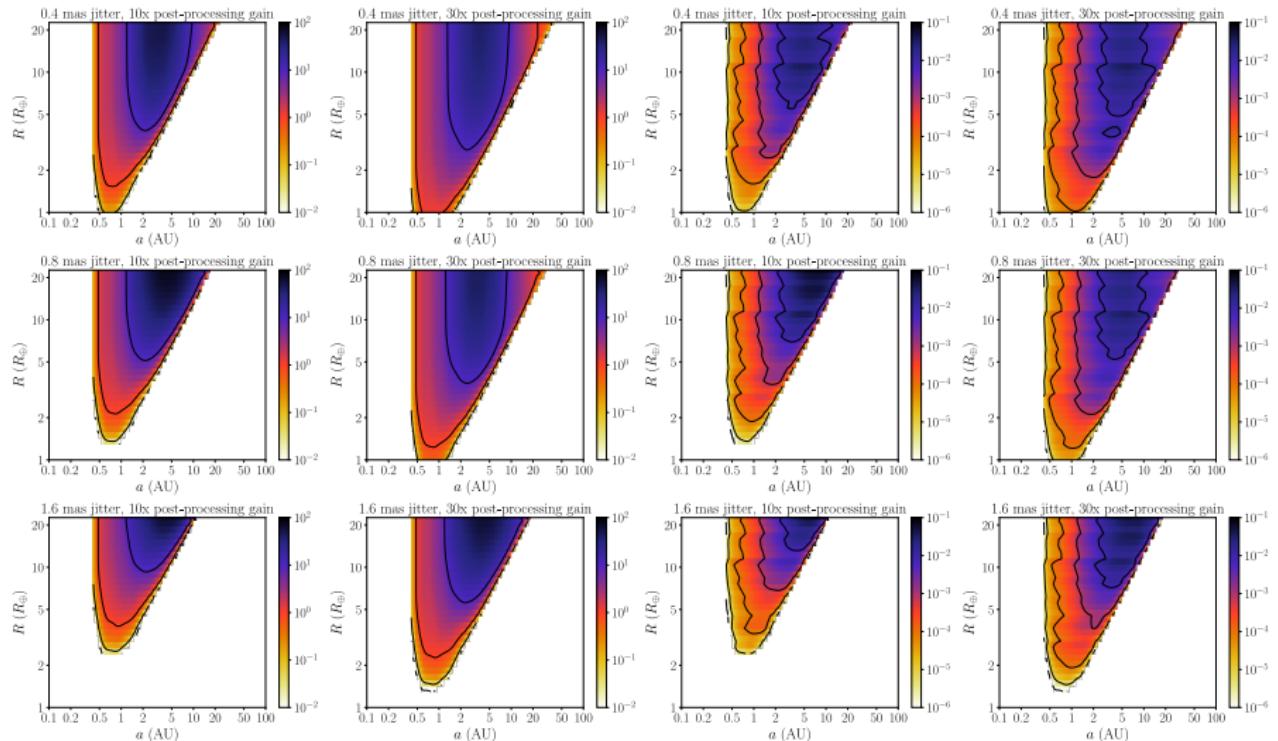
What tools do we have on hand to say it?

# Approach 1: Full Mission Simulations



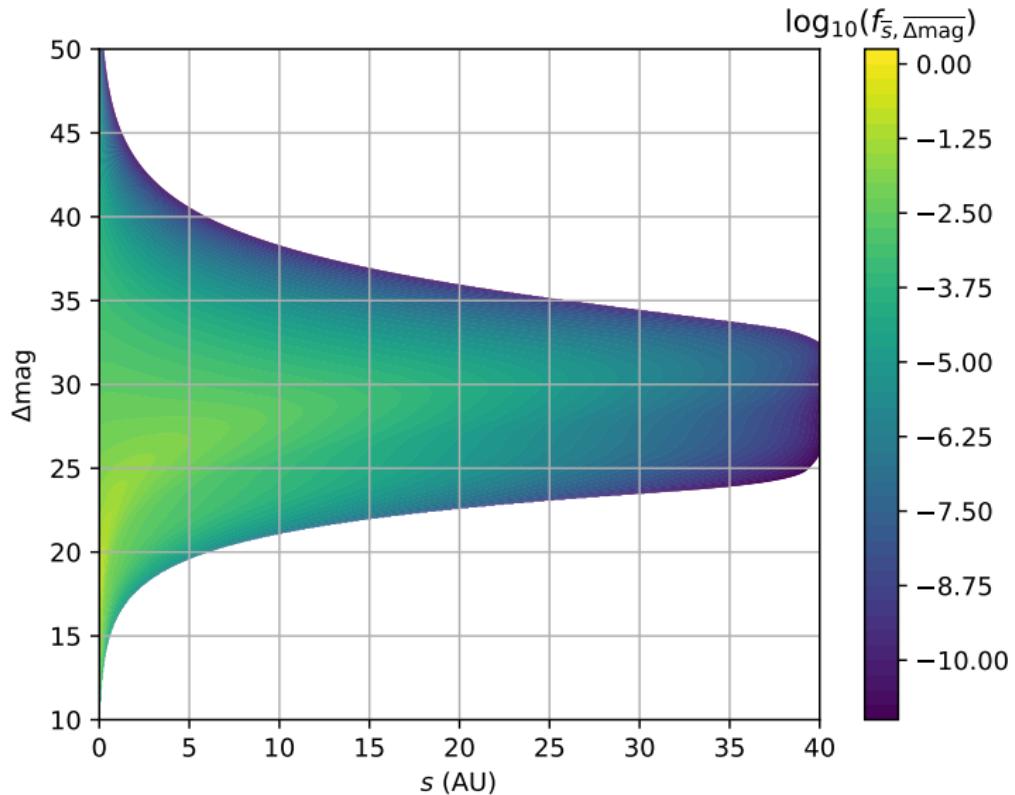
See: Savransky and Garrett (2015), Delacroix et al. (2016)  
EXOSIMS: <https://github.com/dsavransky/EXOSIMS>;  
<http://ascl.net/1706.010>

# Approach 2: Depth of Search



See: Lunine et al. (2008), Garrett et al. (2017)  
DoS: <https://github.com/dgarrett622/DoS>

# Completeness



See: Brown (2005), Garrett and Savransky (2016)

# Summed Completeness Maximization



$$\arg \min_{\{t_i\}} \left( - \sum_i^n c_i(t_i) \right)$$

subject to:

$$t_{\max} - \sum_i^n t_i - \left( \sum_i^n t_i^{\text{over}} (t_i > 0) \right) \geq 0$$

such that:

$$0 \leq t_i \leq t_{\max} \quad \forall i .$$

See: Hunyadi et al. (2007a,b); Stark et al. (2014)



# Tiny Bit of Math

$$c = \int_{\Delta\text{mag}_{\min}(s_{\min})}^{\Delta\text{mag}_u} \int_{s_{\min}}^{s_u(\Delta\text{mag})} f_{\bar{s}, \overline{\Delta\text{mag}}} (s, \Delta\text{mag}) \, ds \, d\Delta\text{mag}.$$

(See Garrett and Savransky (2016))

$$\Delta\text{mag}(t) = -m - 2.5 \log_{10} \left( \frac{\text{SNR}}{\mathcal{F}_0 T} \sqrt{\frac{C_b}{t} + C_{sp}^2} \right)$$

(See Nemati (2014) and Nemati (this conference))

$$\frac{d\Delta\text{mag}}{dt} = \frac{5C_b}{4 \ln(10)} \frac{1}{C_b t + (C_{sp} t)^2}$$

$$\left. \frac{dc}{dt} \right|_{t_{\text{int}}} = \left[ \int_{s_{\min}}^{s_u(\Delta\text{mag}(t_{\text{int}}))} f_{\bar{s}, \overline{\Delta\text{mag}}} (s, \Delta\text{mag}(t_{\text{int}})) \, ds \right] \left. \frac{d\Delta\text{mag}}{dt} \right|_{t_{\text{int}}}$$



# Where to Begin?

This is an unfriendly constraint:

$$t_{\min} - \sum_i^n t_i - \left( \sum_i^n t_i^{\text{over}} (t_i > 0) \right) \geq 0$$

Consider instead:

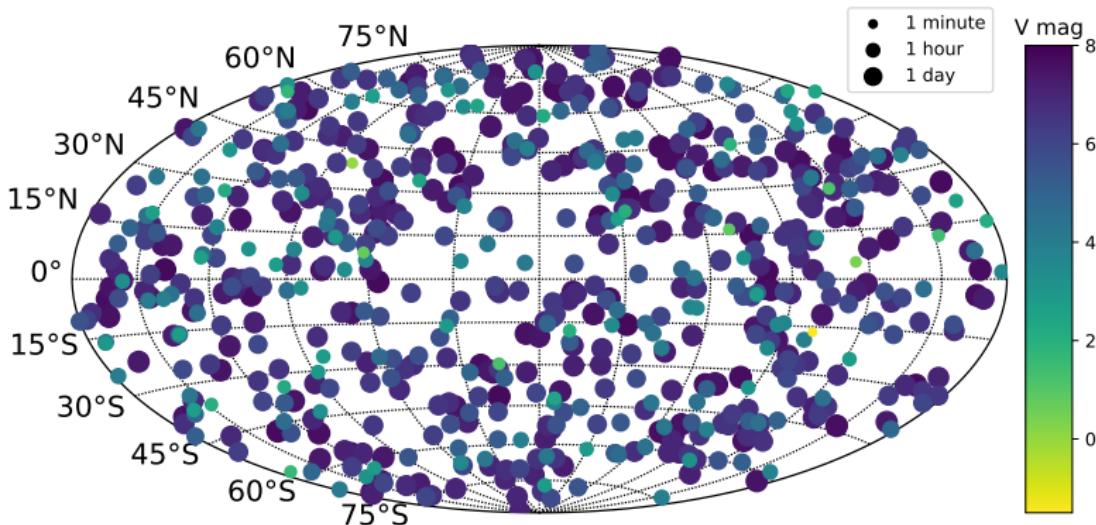
$$\left. \begin{array}{l} \arg \min_T \left( - \sum_{i \in T} c_i \right) \\ \text{subject to:} \\ \sum_{i \in T} (t_i + t_i^{\text{over}}) \leq t_{\max} \end{array} \right\} \begin{array}{ll} \arg \min_{\mathbf{x}} (-\mathbf{c}^T \mathbf{x}) & \mathbf{c}, \mathbf{t} \in \mathbb{R}^N \\ (\mathbf{t} + \mathbf{t}^{\text{over}})^T \mathbf{x} < t_{\max} & \\ \mathbf{x} \in \mathbb{Z}^N & \\ \mathbf{0} \leq \mathbf{x} \leq \mathbf{1} & \\ T = \{i : x_i = 1, \forall x_i \in \mathbf{x}\} & \end{array}$$

# Alphabet Soup



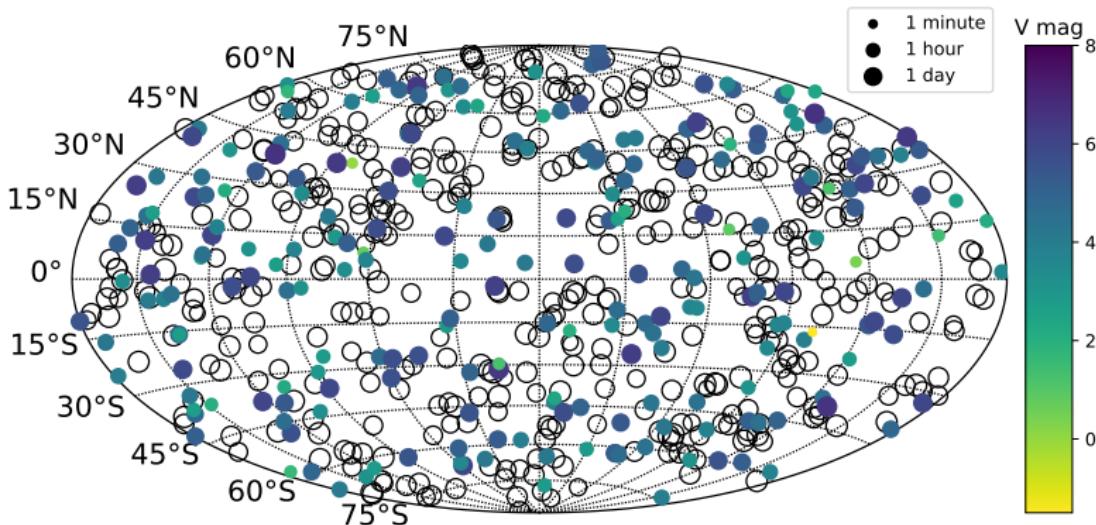
- The original problem is a nonlinear optimization. We would like to solve it using Sequential Least-Squares Quadratic Programming (SLSQP [Kraft, 1994])
  - This is hard and computationally expensive if you don't start near a local extremum
- The fixed integration time problem is a Binary Integer Linear Programming Problem (BILPP [Williams, 2009] )
  - Actually NP-complete, but computationally cheap for reasonably sized target lists using branch and cut

# Where to Begin (again)?



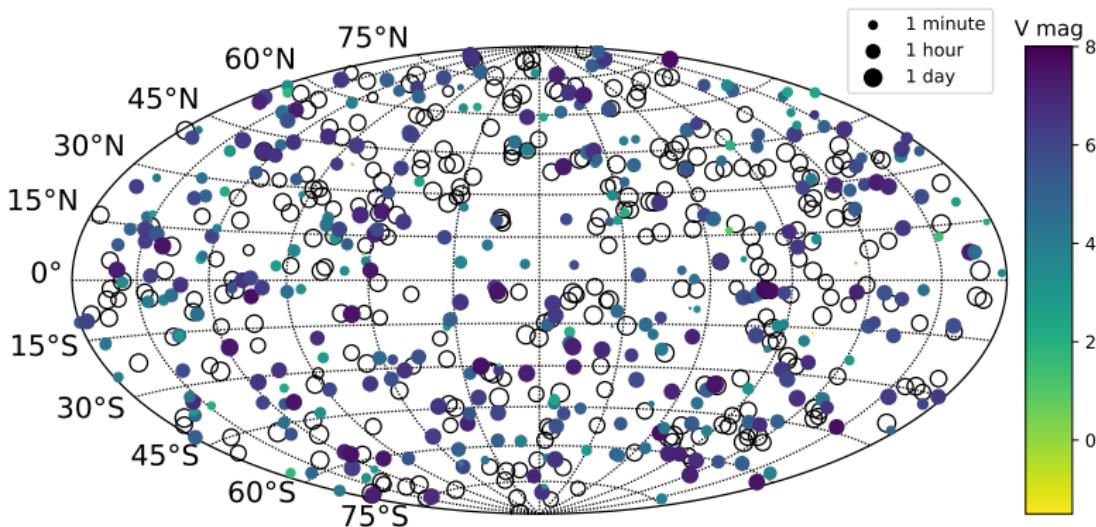
Option 1: Use an integration time for an assumed limiting planet  $\Delta\text{mag}$

# Where to Begin (again)?



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# Where to Begin (again)?

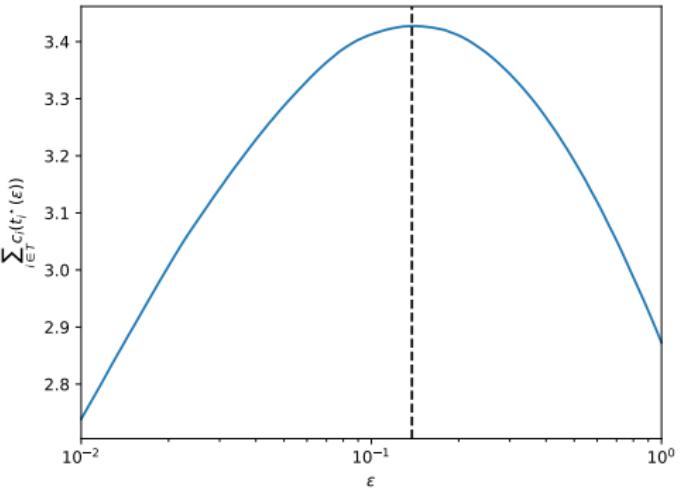


Option 2: Use an integration time to maximize  $c/t$

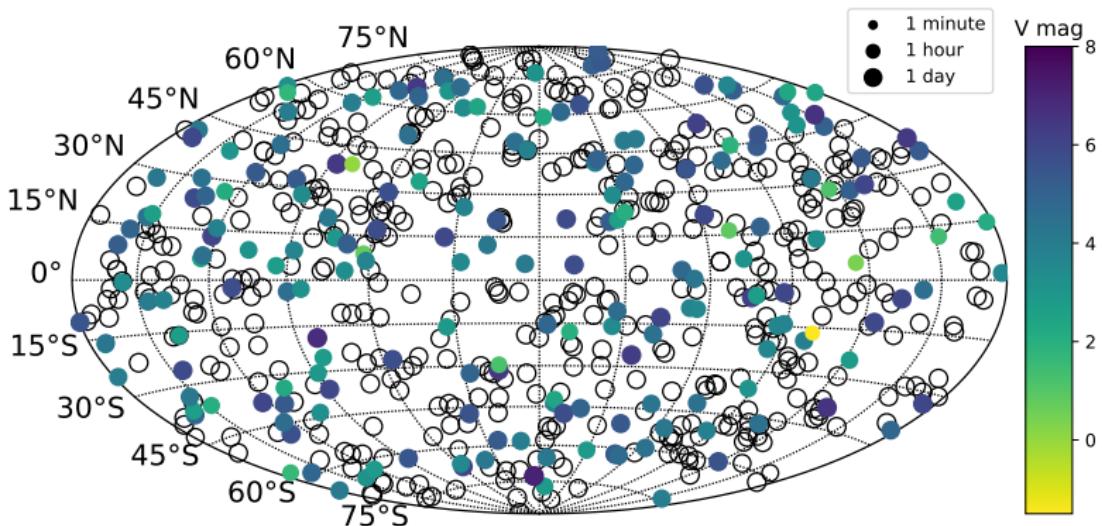
# Option 3

$$\frac{d\Delta \text{mag}}{dt} \Big|_{t^*} \leq \varepsilon$$

$$t^* = \frac{1}{2C_{sp}^2\varepsilon\sqrt{\log(10)}} \times \\ \left( -C_b\varepsilon\sqrt{\log(10)} + \sqrt{C_b\varepsilon(C_b\varepsilon\log(10) + 5C_{sp}^2)} \right)$$

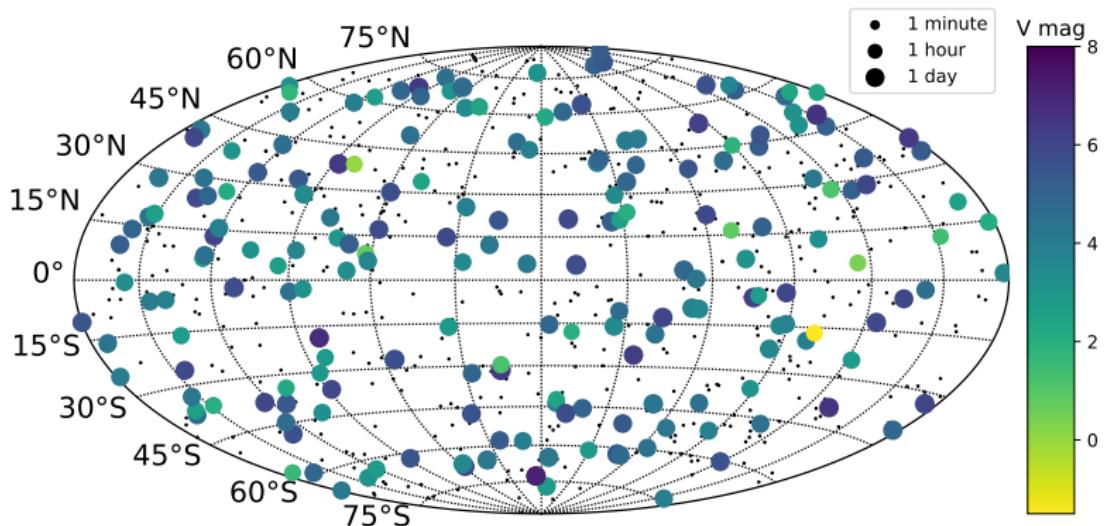


## Option 3



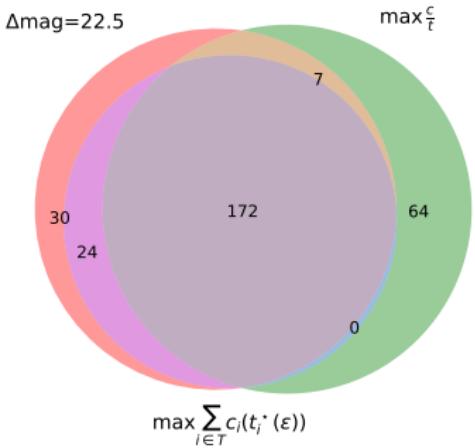
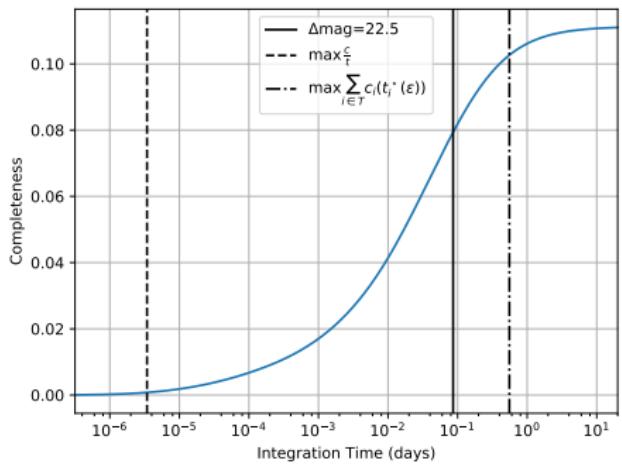
Option 3: BILPP step

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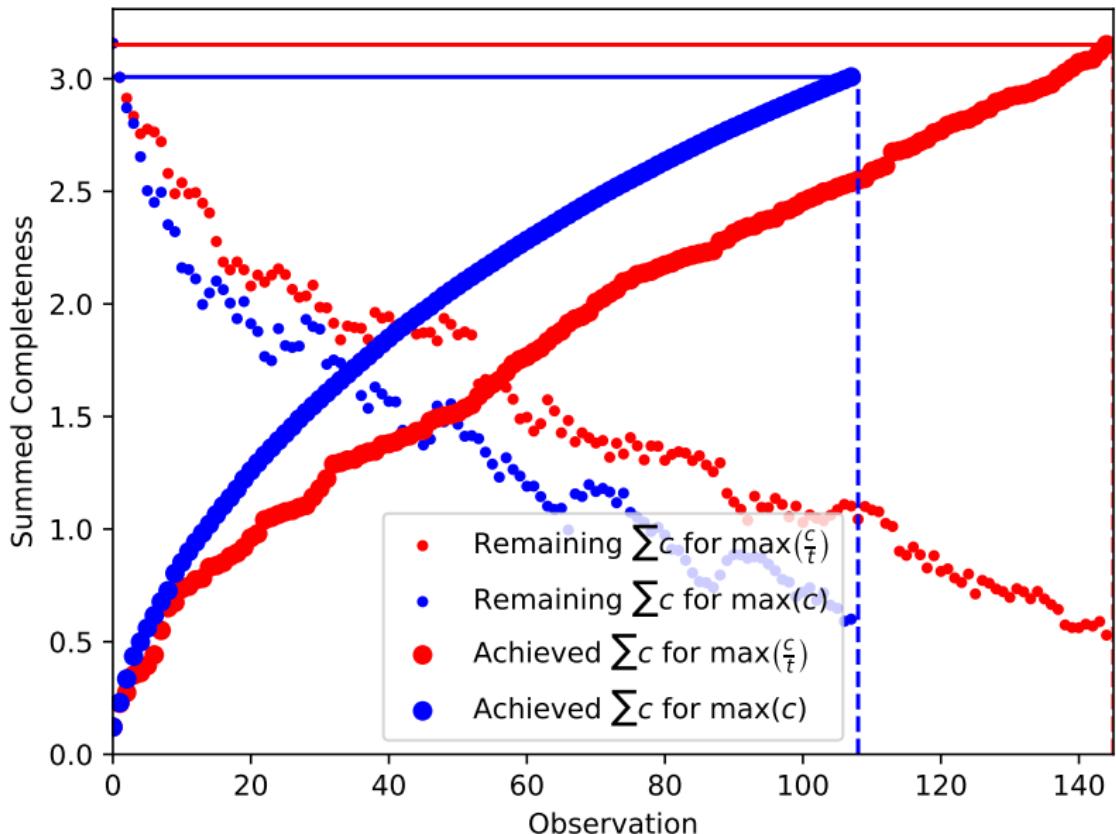
Option 3: SLSQP step

# Optimization Summary

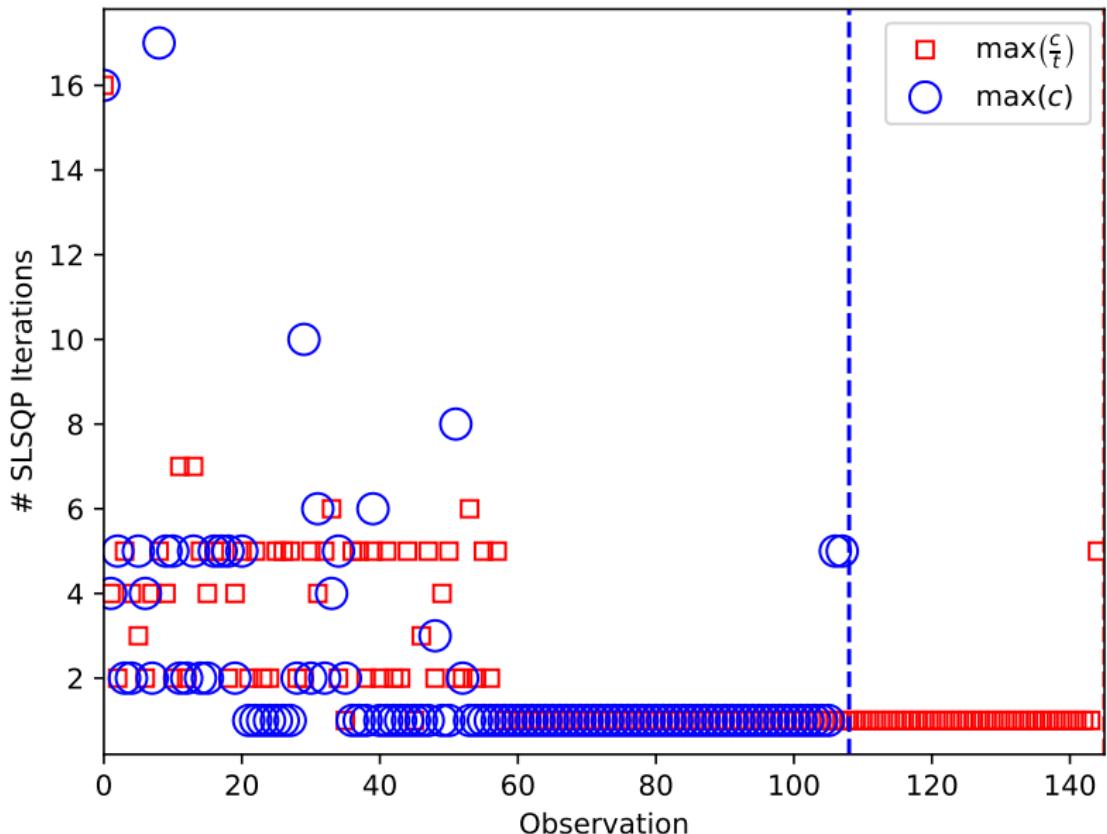


Case	BILPP		SLSQP			
	# Stars	$\sum c_i$	It	FC	# Stars	$\sum c_i$
$\Delta\text{mag}=22.5$	232	2.77	52	67	232	3.43
$\arg \max_t \frac{c}{t}$	345	0.26	97	305	193	3.45
$t^* (\arg \max_\varepsilon \sum c_i)$	196	3.42	28	30	196	3.47

# Completeness Maximization in Survey Simulations



# Turns Out to be Quite Feasible

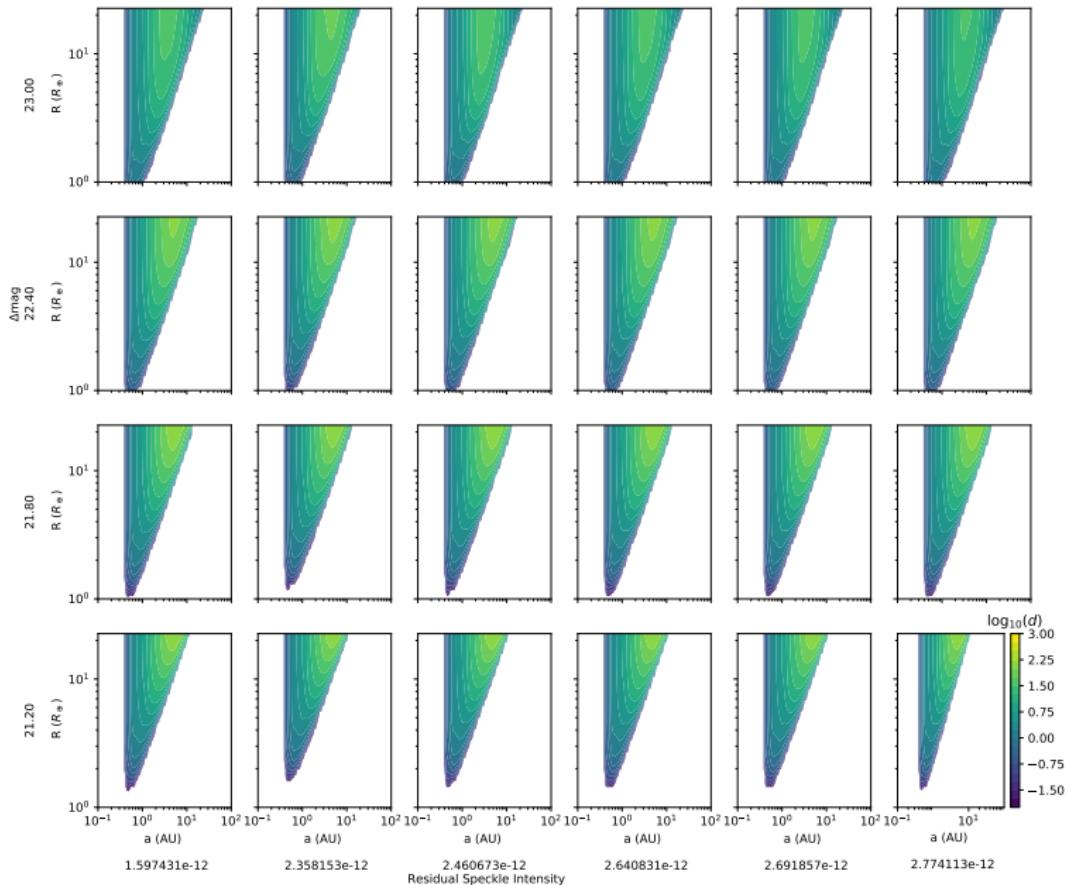


# So, What Can We Say?

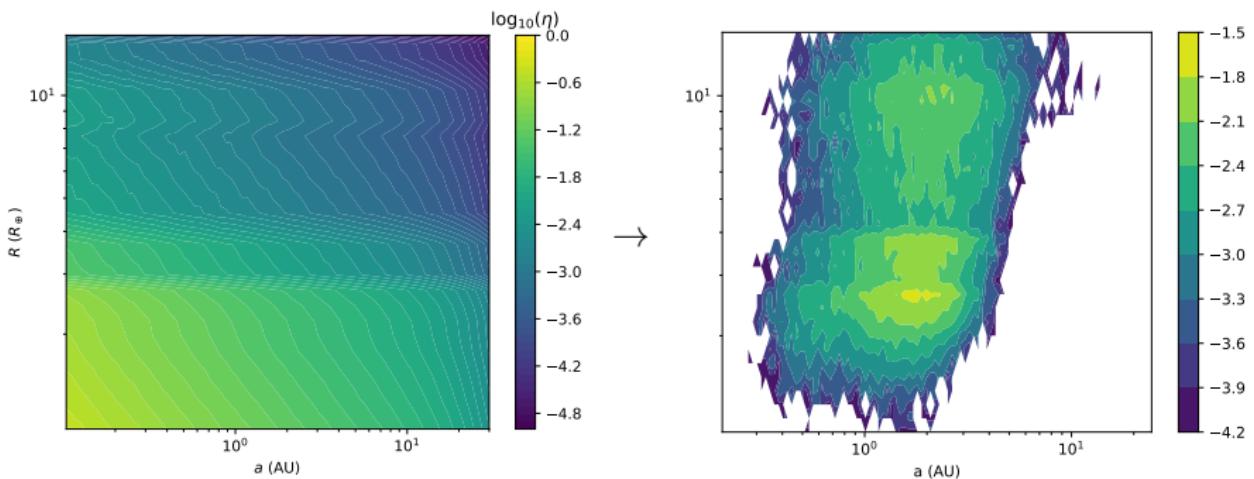


- Summed completeness gives you a number for how many planets from a given population you should expect to discover
- Mission simulations give you the distribution of this value (and many others)
- Depth of search tells you what kinds of planets your instrument is sensitive to

# Phase Space Exploration



We have the tools to evaluate how a mission interacts with the population of exoplanets



# References I



Kraft, D. (1994).

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*ACM Transactions on Mathematical Software (TOMS)*, 20(3):262–281.



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