Common Spatial Pattern Filtering for Imaging of Circumstellar Discs

Jacob Shapiro[1], Dmitry Savransky[1,2], Jean-Baptiste Ruffio[3], and Bruce Macintosh[3]

1. Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY, United States

2. Carl Sagan Institute, Cornell University, Ithaca, NY, United States

3. Kavli Institute for Particle Astrophysics and Cosmology, Stanford University, Stanford CA, United States

Introduction

Direct Imaging is the search for exoplanets and circumstellar discs by observing their star with a coronagraph. Unfortunately, the astrophysical target signal is a similar order of magnitude to that of the noise sources.

Recently, work has been done to show that a new method, Common Spatial Pattern Filtering, can be used to extract exoplanets from the data. The research presented here is an early attempt at using the same method to gather information about circumstellar discs and dust clouds.

Common Spatial Pattern Filtering[1] Overview

Objective: $\arg\max_{\mathbf{w}} \frac{\|\mathbf{X}_1^T\mathbf{w}\|^2}{\|\mathbf{X}_2^T\mathbf{w}\|^2} \quad \text{s.t.} \quad \|\mathbf{w}\| = 1$

Solution:

Find the each covariance matrix:

$$\mathbf{C} = \frac{\mathbf{X}\mathbf{X}^T}{P-1}$$

Whiten the matrices:

$$\mathbf{C}_+ = \mathbf{C}_1 + \mathbf{C}_2 = \mathbf{U}\mathbf{\Gamma}\mathbf{U}^\mathbf{T}$$

$$\mathbf{P} = \mathbf{\Gamma}^{-1/2}\mathbf{U}^\mathbf{T}$$

$$\bar{\mathbf{C}}_n = \mathbf{P}\mathbf{C}_n\mathbf{P}^T$$

so that:

$$\bar{\mathbf{C}}_1 + \bar{\mathbf{C}}_2 = \mathbf{I}$$

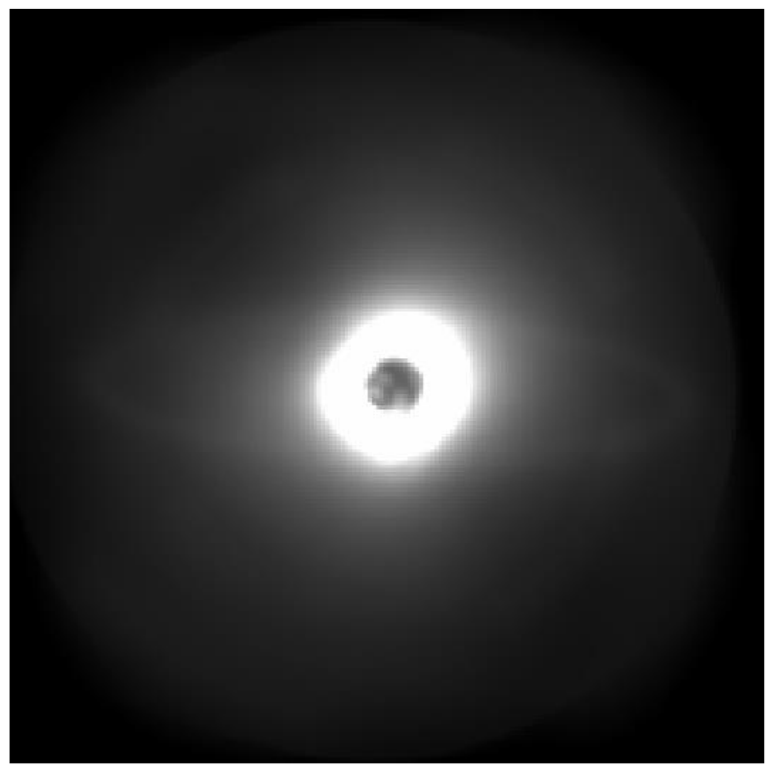
As $\|\bar{\mathbf{X}}_1^T\mathbf{w}\|^2$ increases, then $\|\bar{\mathbf{X}}_2^T\mathbf{w}\|^2$ must decrease. Thus ***w*** also satisfies:

$$\arg\max_{\mathbf{w}} \|\bar{\mathbf{X}}_1^T\mathbf{w}\|^2 \quad \text{s.t.} \quad \mathbf{w}^T(\bar{\mathbf{C}}_1 + \bar{\mathbf{C}}_2)\mathbf{w} = \mathbf{I}$$

Reduce to an eigenvalue problem:

$$\bar{\mathbf{C}}_1\mathbf{w} = \phi\mathbf{w}$$

Data: HR 4796a[2]



CSP in Direct Imaging

Design datasets such that the target signal is a component of the difference between them:

Objective: After developing the eigenvalue problem, project the data into the difference modes that highlight the circumstellar disk. Build a projection matrix $\mathbf{Z}$:

$$\mathbf{Z} = \mathbf{\Phi}^{-1/2}\mathbf{W}^T\mathbf{P}\mathbf{X}_1$$

$\mathbf{\Phi}$ = pure diagonal matrix of eigenvalues of $\bar{\mathbf{C}}_1$

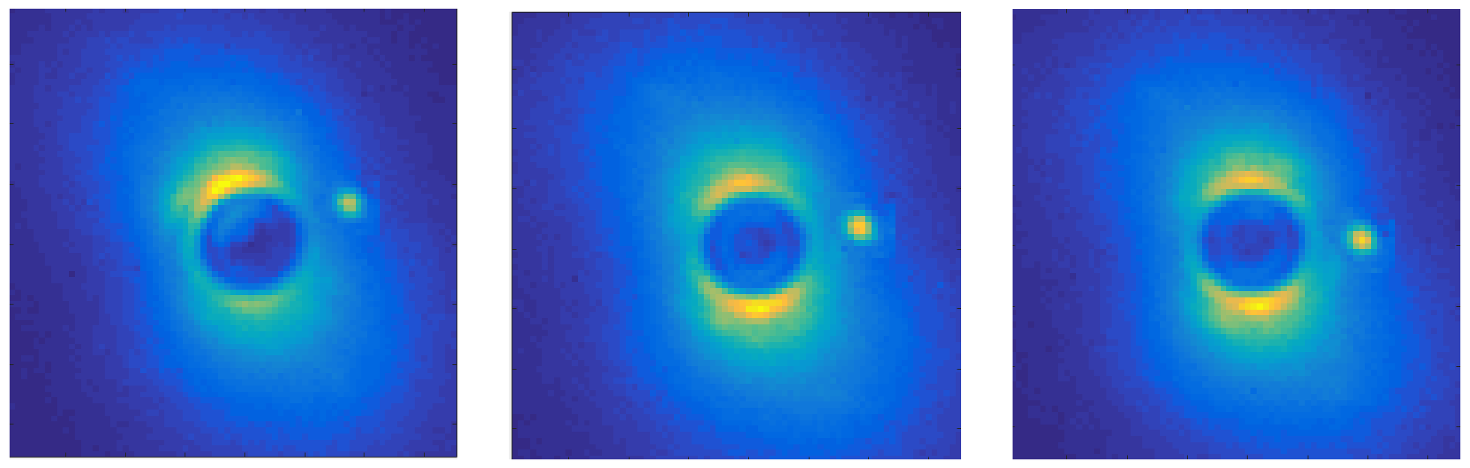
$\mathbf{W}$ = eigenvector matrix of $\bar{\mathbf{C}}_1$

$\mathbf{P}$ = whitening matrix of $\mathbf{C}_+$

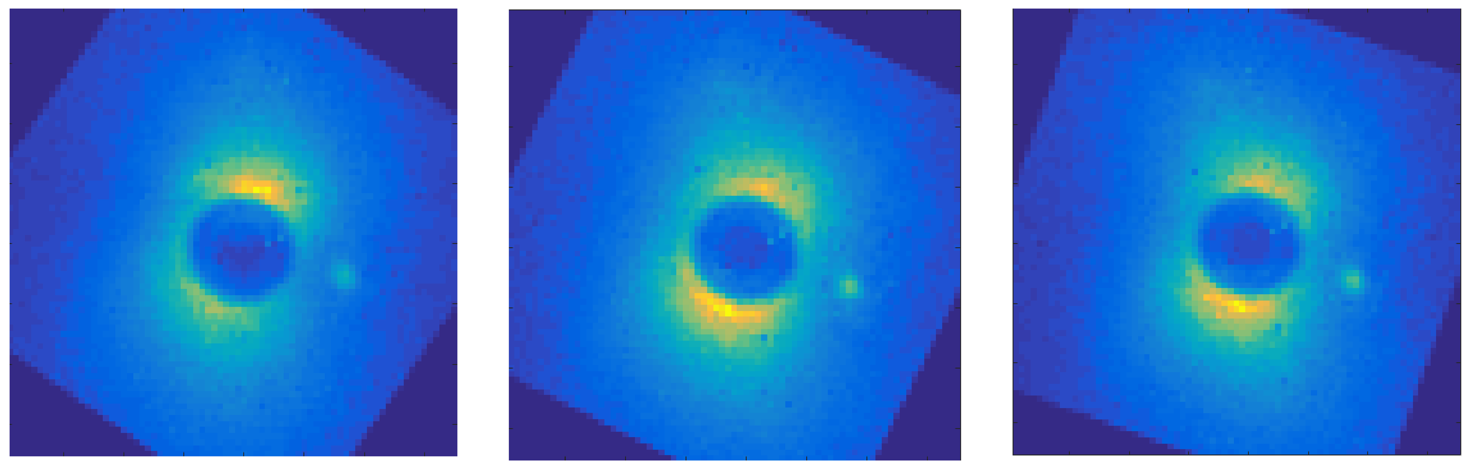
$\mathbf{X}_1$ = dataset listed right

The modes from projecting original images into $\mathbf{Z}$ that contain the target signal are final K rows of $\mathbf{Z}$; K is typically less than 10.

$\mathbf{X}_1$ Dataset: semi-static speckles, rotating signal

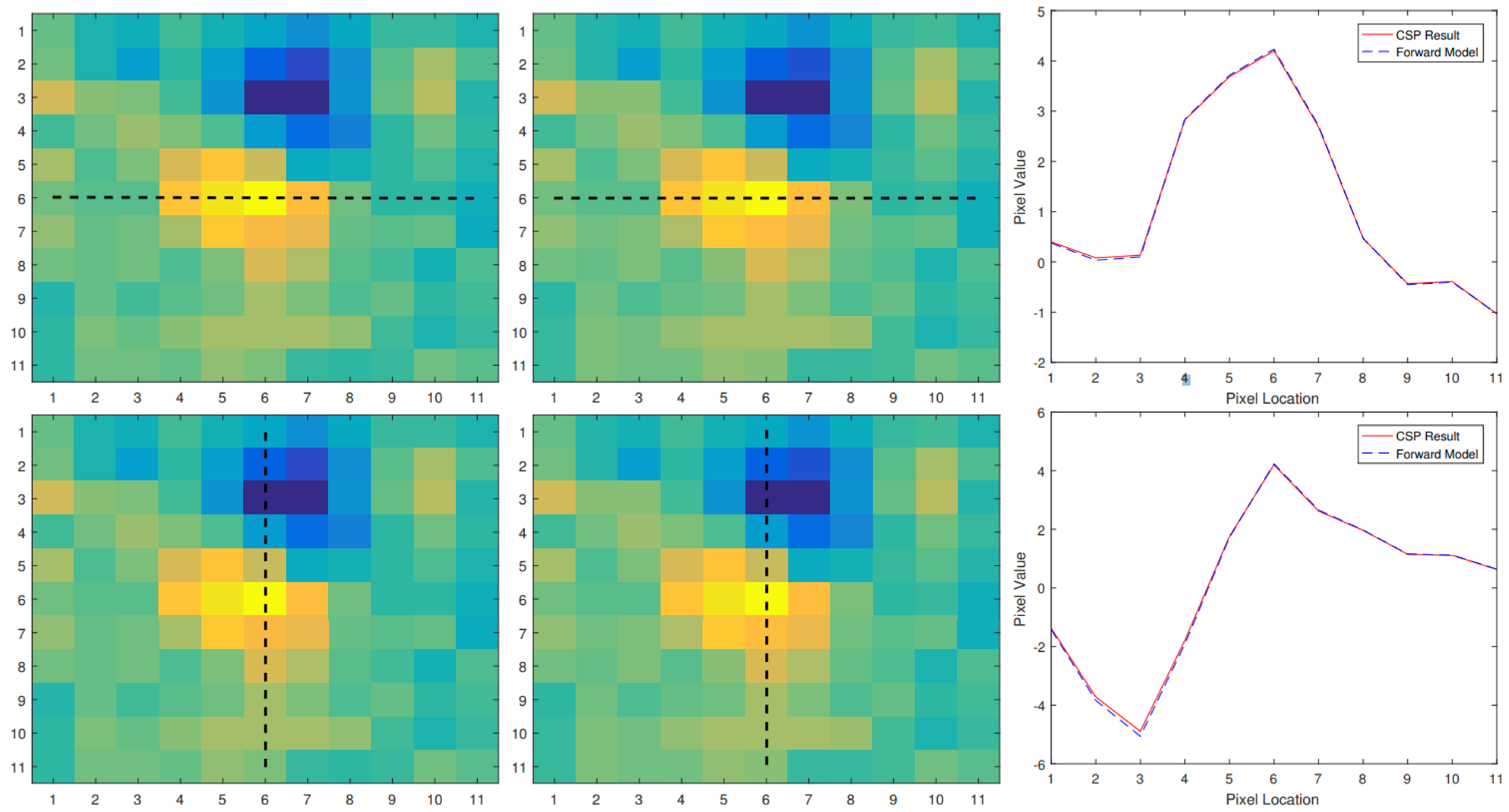


$\mathbf{X}_2$ Dataset: derotated speckles, stationary signal



CSP Exoplanet Detection[3]

Forward Model Matched Filter: Assume the projection matrix is a sum of speckle effects ($\mathbf{Z}_S$) and target signal perturbations ($\epsilon\delta\mathbf{Z}$).

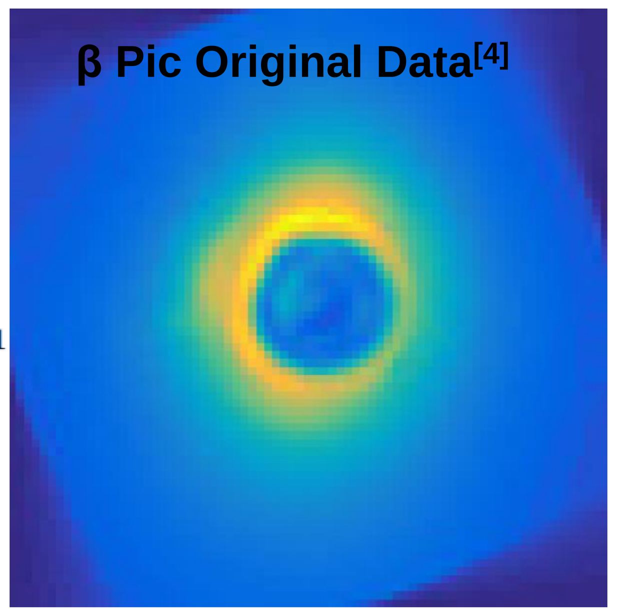


The forward model results compared to the CSP results in one example mode. Top row: horizontal cut. Bottom row: vertical cut. Left column: CSP result. Middle column: forward model result. Right column: pixel comparison.

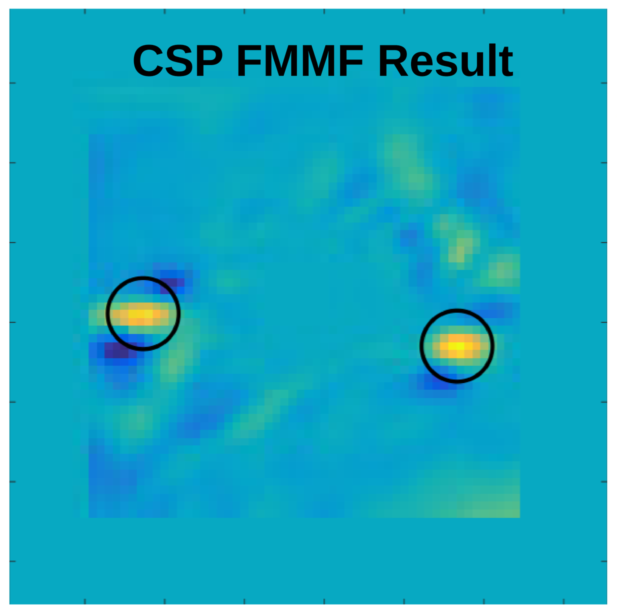
$$\mathbf{Z} = \mathbf{Z}_s + \epsilon\delta\mathbf{Z}$$

$$\mathbf{Z}_s = \mathbf{\Omega}^{-1/2}\mathbf{W}^T\mathbf{\Lambda}^{-1/2}\mathbf{V}^T\mathbf{S}_1$$

β Pic Original Data[4]



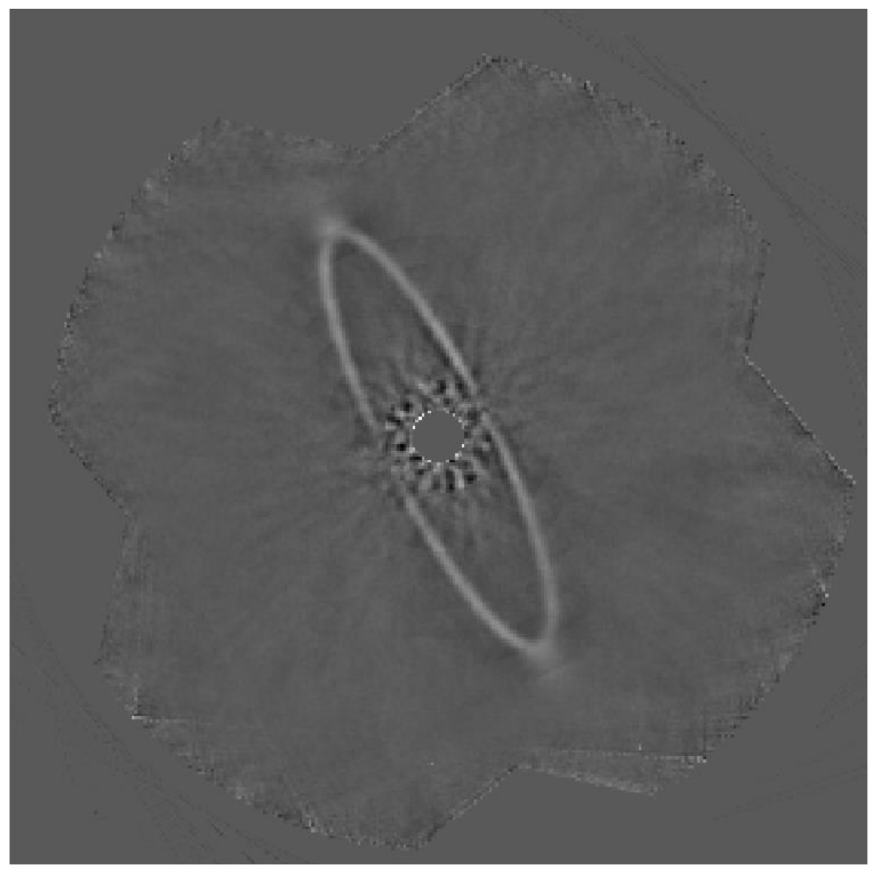
CSP FMMF Result



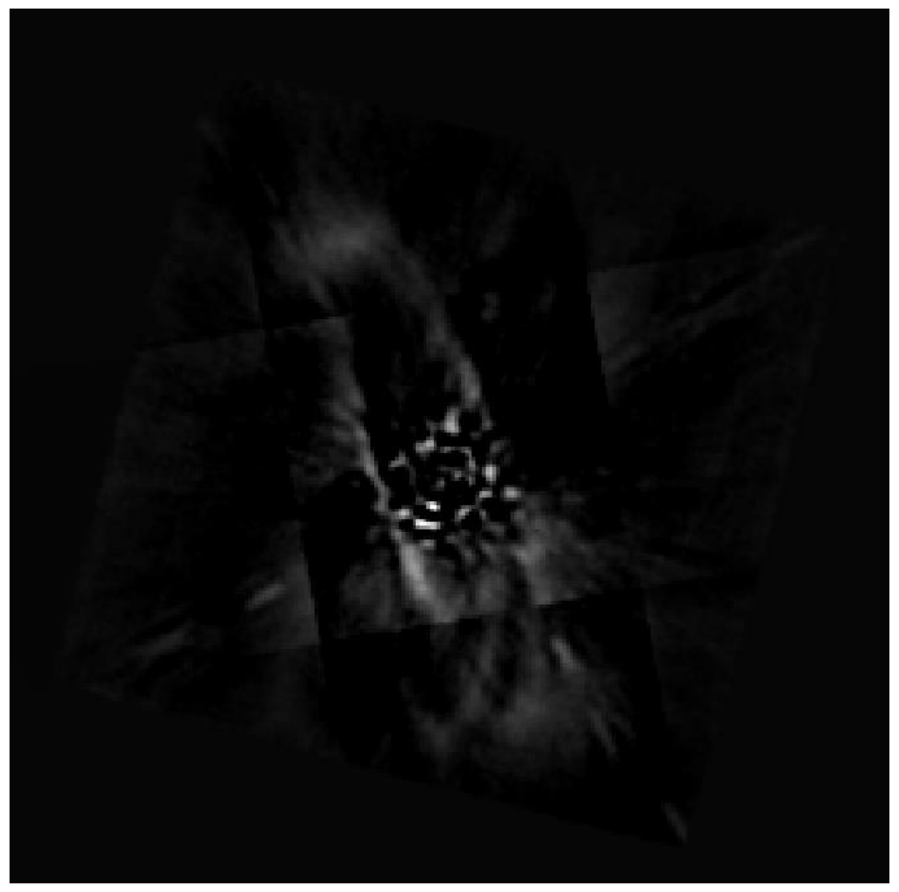
$$\delta\mathbf{Z} = \mathbf{\Omega}^{-1/2}\mathbf{Y}^T\mathbf{\Lambda}^{-1/2}\mathbf{V}^T\mathbf{A}_1 + \mathbf{\Omega}^{-1/2}\mathbf{Y}^T\mathbf{\Lambda}^{-1/2}\delta\mathbf{V}^T\mathbf{S}_1 + \mathbf{\Omega}^{-1/2}\delta\mathbf{Y}^T\mathbf{\Lambda}^{-1/2}\mathbf{V}^T\mathbf{S}_1 - \frac{1}{2}\mathbf{\Omega}^{-1/2}\mathbf{Y}^T\mathbf{\Lambda}^{-3/2}\delta\mathbf{\Lambda}\mathbf{V}^T\mathbf{S}_1 - \frac{1}{2}\mathbf{\Omega}^{-3/2}\delta\mathbf{\Omega}\mathbf{Y}^T\mathbf{\Lambda}^{-1/2}\mathbf{V}^T\mathbf{S}_1$$

Results

pyKLIP[5] Result



CSP Result:



Conclusions

- CSP filtering has been developed for direct imaging reduction, specifically for discs.
- No forward model can be created for circumstellar discs
- CSP can detect discs, but at a currently lower SNR than KLIP
- CSP is better suited for point source detection

Acknowledgements

This work is supported by NASA Grant No. NNX16AI13G.

Based on observations obtained at the Gemini Observatory, which is operated by the Association of Universities for Research in Astronomy, Inc., under a cooperative agreement with the National Science Foundation (NSF) on behalf of the Gemini partnership: the NSF (United States), the National Research Council (Canada), CONICYT (Chile), the australian Research Council (Australia), Ministério da Ciência, Tecnologia e Inovação (Brazil), and Ministerio de Ciencia, Tecnología e Innovación Productiva (Argentina).

References

[1] Y. Wang, S. Gao and X. Gao, "Common Spatial Pattern Method for Channel Selection in Motor Imagery Based Brain-Computer Interface", in *Engineering in Medicine and Biology 27th Annual Conference*, Shanghai, China, 2005.

[2] Gemini Planet Imager, GPIES team. Dec. 2013

[3] Shapiro, J, et al. "Detecting Planets from Direct Imaging Observations using Common Spatial Pattern Filtering," (in preparation)

[4] Gemini Planet Imager, GPIES team. Nov, 2015

[5] Wang, J. J., Ruffio, J.-B., De Rosa, R. J., et al. 2015, pyKLIP: PSF Subtraction for Exoplanets and Disks, Astrophysics Source Code Library, ascl:1506.001