Exoplanet Direct Imaging Detection Metrics and Exoplanet Populations

Daniel Garrett

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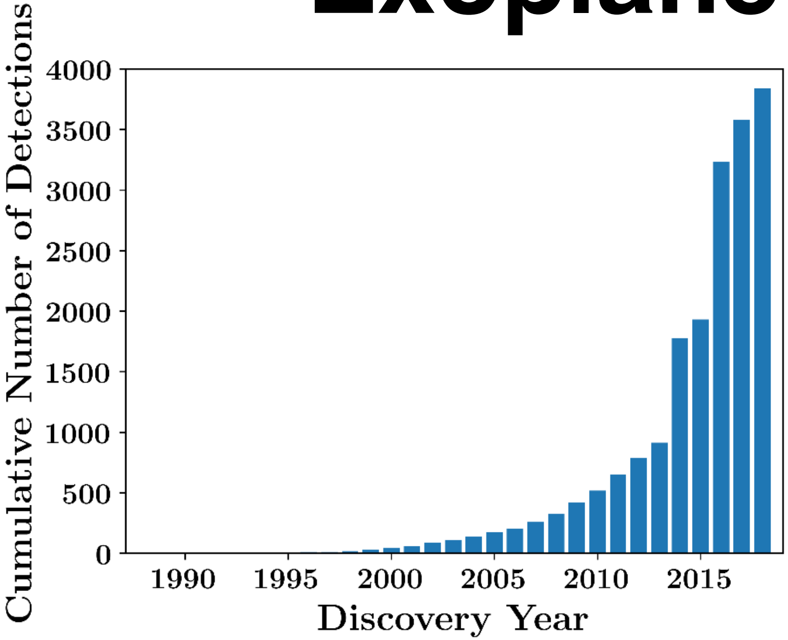
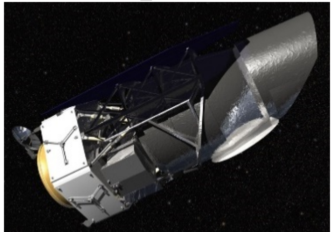
Overview

- Exoplanet Direct Imaging
- Bayes' Theorem
  - Completeness
  - Depth-of-Search
  - Occurrence Rate Model
- Conclusions and Future Directions

Exoplanet Detections

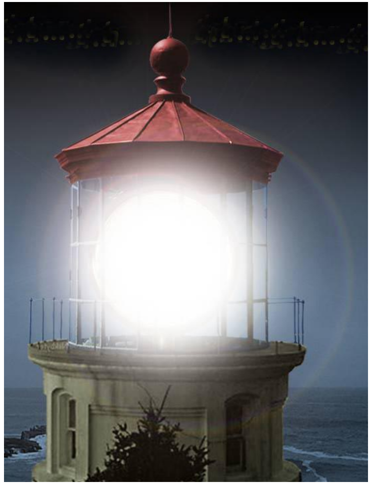


exoplanetarchive.ipac.caltech.edu



jpl.nasa.gov/spaceimages

- Which Stars?
- How many planets?
- What kind of planets?

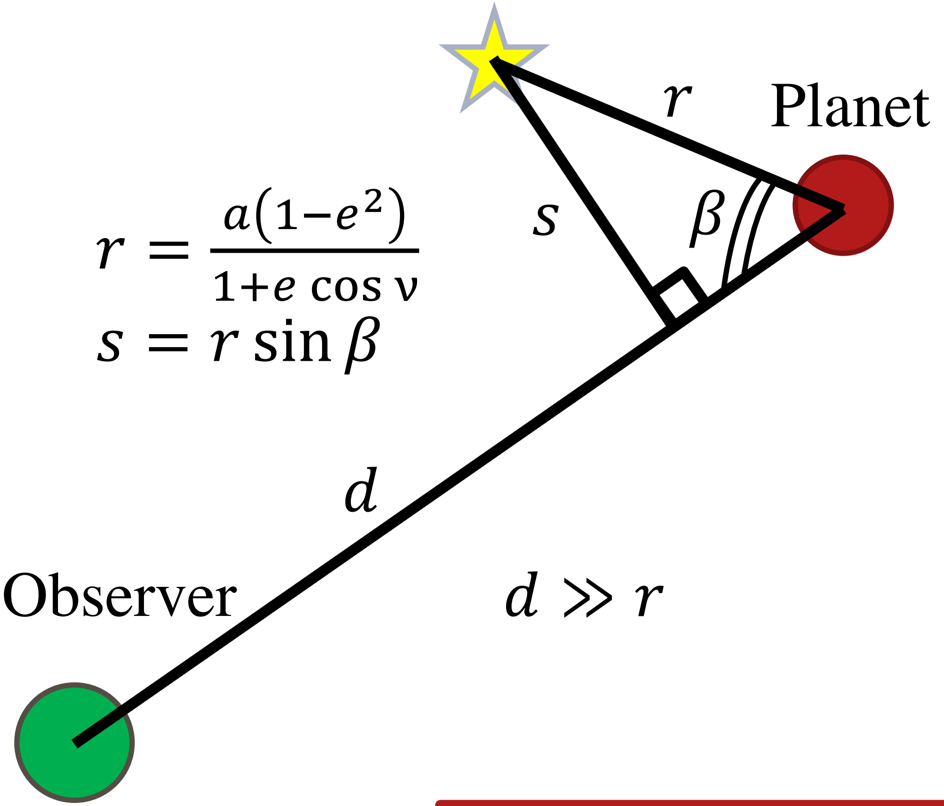


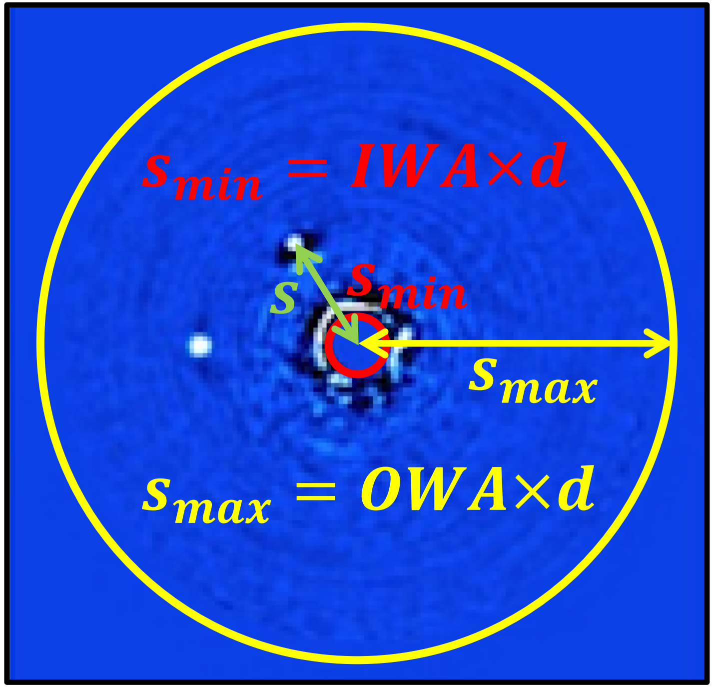
ircamera.as.arizona.edu/Astr2016/images

Detection Criteria - Geometric

$$r = \frac{a(1-e^2)}{1+e \cos \nu}$$

$$s = r \sin \beta$$





Marois et al. (2014)

Detectable if $s_{min} < s < s_{max}$

Detection Criteria - Photometric

Planet

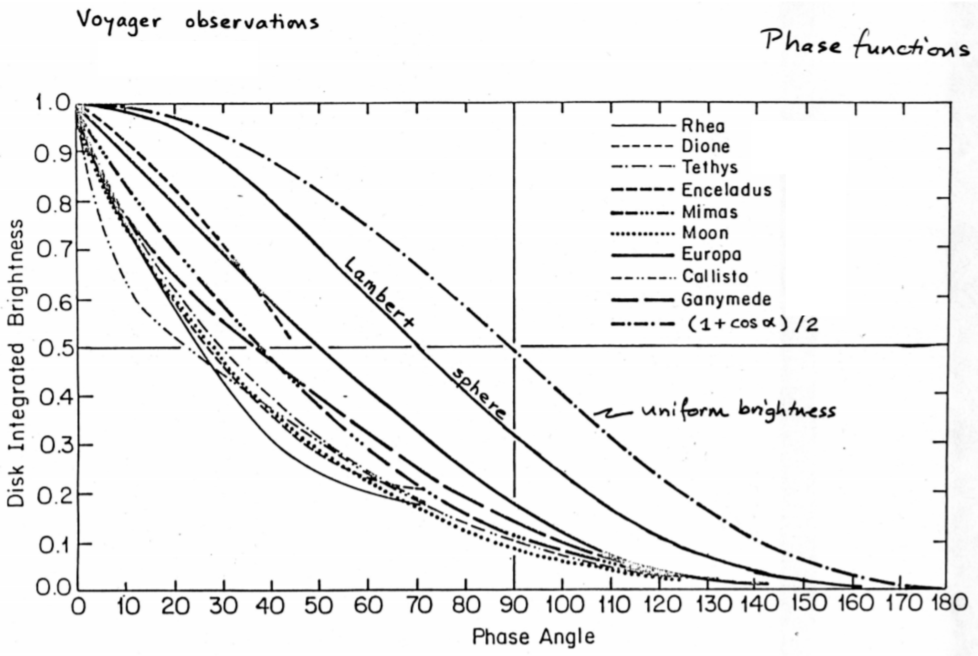
$$F_R = p\left(\frac{R_p}{r}\right)^2 \Phi(\beta)$$

$$\Delta\text{mag} = -2.5\log_{10} F_R$$

Instrument

$$C_{min}$$

$$\Delta\text{mag}_{lim} = -2.5\log_{10} C_{min}$$



http://astro.cornell.edu/academics/courses/astro6570/Light_Scattering_Photometry.pdf

Detectable if $F_R > C_{min}$ or $\Delta\text{mag} < \Delta\text{mag}_{lim}$

Bayes' Theorem

$$f_{\bar{x}|\bar{y}=y}(x|y)f_{\bar{y}}(y) = f_{\bar{x},\bar{y}}(x,y) = f_{\bar{y}|\bar{x}=x}(y|x)f_{\bar{x}}(x)$$

$$f_{\boldsymbol{\theta}|\boldsymbol{D}}(\boldsymbol{\theta}|\boldsymbol{D}) = \frac{f_{\boldsymbol{D}|\boldsymbol{\theta}}(\boldsymbol{D}|\boldsymbol{\theta})\, f_{\boldsymbol{\theta}}(\boldsymbol{\theta})}{f_{\boldsymbol{D}}(\boldsymbol{D})}$$

Posterior: $f_{\boldsymbol{\theta}|\boldsymbol{D}}(\boldsymbol{\theta}|\boldsymbol{D})$

Likelihood: $f_{\boldsymbol{D}|\boldsymbol{\theta}}(\boldsymbol{D}|\boldsymbol{\theta})$

Prior: $f_{\boldsymbol{\theta}}(\boldsymbol{\theta})$

Evidence: $f_{\boldsymbol{D}}(\boldsymbol{D})$

Depth-of-Search

Occurrence Rate Model

$$f_{\boldsymbol{\theta}|\boldsymbol{D}}(\boldsymbol{\theta}|\boldsymbol{D}) = \frac{f_{\boldsymbol{D}|\boldsymbol{\theta}}(\boldsymbol{D}|\boldsymbol{\theta})\, f_{\boldsymbol{\theta}}(\boldsymbol{\theta})}{f_{\boldsymbol{D}}(\boldsymbol{D})}$$

Completeness

Depth-of-Search

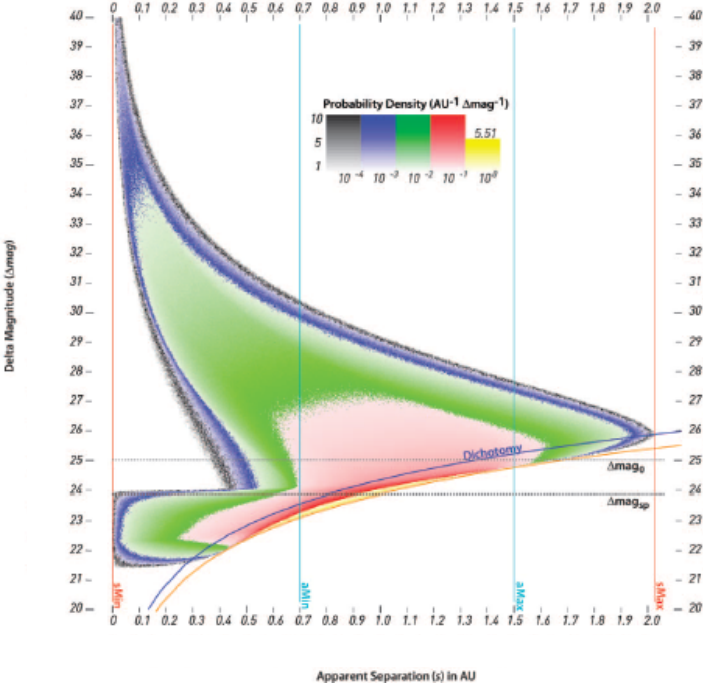
Occurrence Rate Model

Completeness

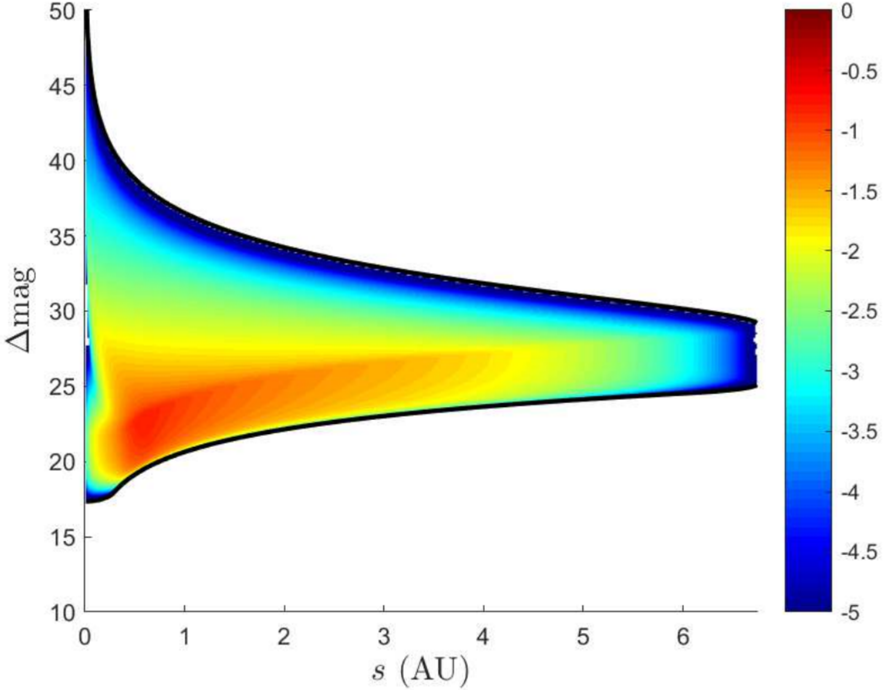
$$f_{\theta|D}(\theta|D) = \frac{f_{D|\theta}(D|\theta) f_{\theta}(\theta)}{f_D(D)}$$

- Garrett, D., Savransky, D. "Analytical Formulation of the Single-Visit Completeness Joint Probability Density Function." *ApJ* (2016).
- Garrett, D., Savransky, D. "Detected Exoplanet Population Distributions Found Analytically." In *Techniques and Instrumentation for Detection of Exoplanets VIII*, SPIE (2017).
- github.com/dgarrett622/FuncComp, github.com/dgarrett622/ObsDist
- github.com/dsavransky/EXOSIMS

Completeness



Brown (2005)



Garrett & Savransky (2016)

Color: powers of 10 with units $AU^{-1}\Delta mag^{-1}$

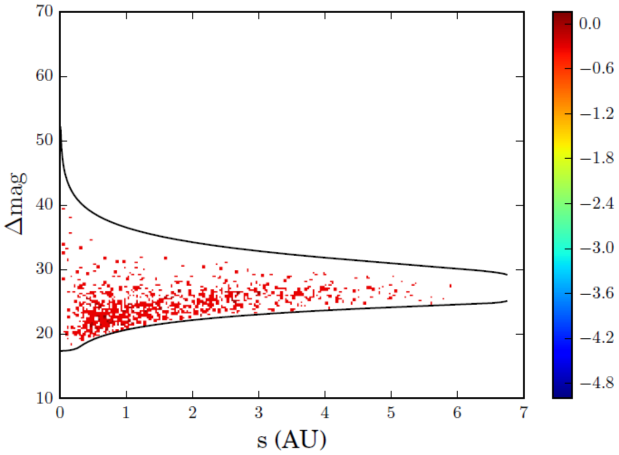
How Many Samples?

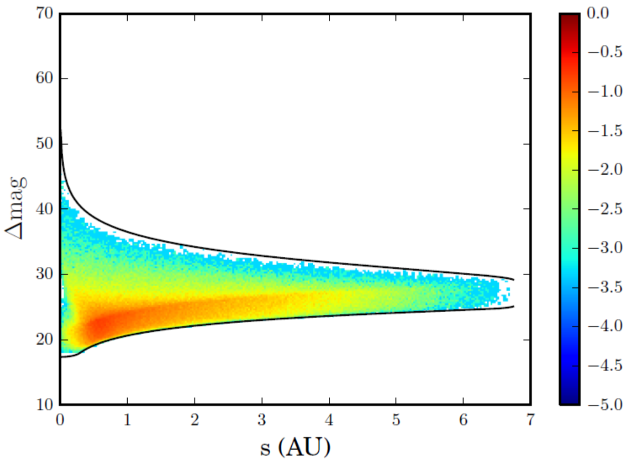
Quantity	Variable	Min	Max	PDF
Geometric Albedo	p	0.2	0.3	Log-uniform
Planet Radius (km)	R_p	6000	30000	Log-uniform
Distance to Star (AU)	r	0.325	6.75	Savransky et al. (2011)
Semi-major Axis (AU)	a	0.5	5	Log-uniform
Eccentricity	e	0	0.35	Rayleigh
Phase Angle	β	0	π	$f_{\bar{\beta}}(\beta) = \frac{\sin\beta}{2}$

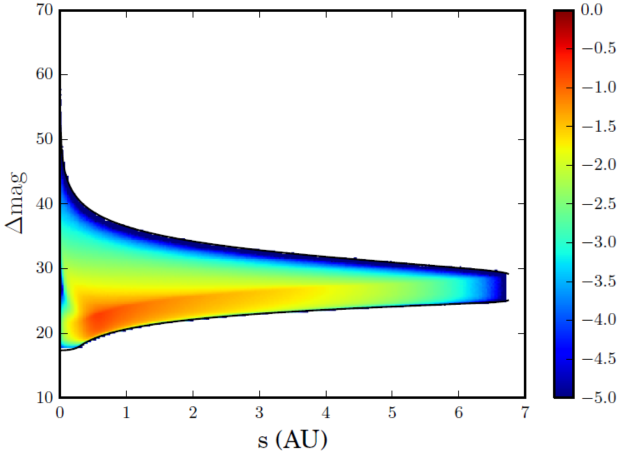
$$\Phi(\beta) = \frac{1}{\pi}[\sin\beta + (\pi - \beta)\cos\beta] \quad \text{(Sobolev 1975)}$$

Monte Carlo error: $\boldsymbol{O\left(\frac{1}{\sqrt{n}}\right)}$

Sample Comparison







1,000 samples

$$\frac{1}{\sqrt{n}} \sim 0.032$$

< 1 s

1,000,000 samples

$$\frac{1}{\sqrt{n}} \sim 0.001$$

~ 15 s

1,000,000,000 samples

$$\frac{1}{\sqrt{n}} \sim 3.2\times 10^{-5}$$

~4 hr

Analytical Method

- Joint PDF:

$$f_{\bar{p},\bar{R}_p,\bar{\beta},\bar{r}}(p, R_p, \beta, r) = f_{\bar{p}}(p) f_{\bar{R}_p}(R_p) f_{\bar{\beta}}(\beta) f_{\bar{r}}(r)$$

- Change of variables:

$$f_{\overline{\Delta \mathrm{mag}},\bar{s},\bar{p},\bar{R}_p}(\Delta \mathrm{mag}, s, p, R_p)$$

- Marginalize over p and R_p:

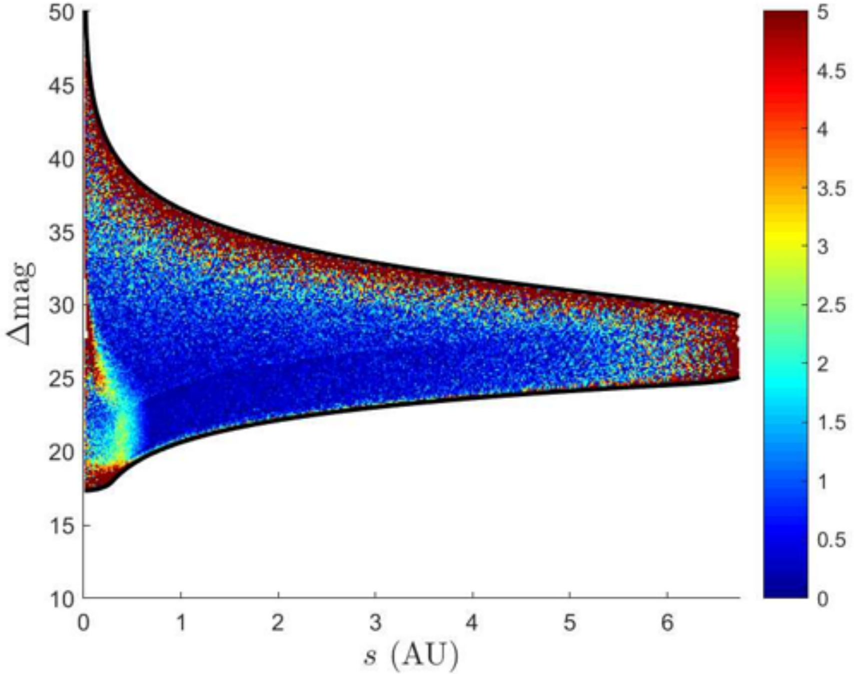
$$f_{\overline{\Delta \mathrm{mag}},\bar{s}}(\Delta \mathrm{mag}, s) = \int_{R_{p,min}}^{R_{p,max}} \int_{p_{min}}^{p_{max}} f_{\overline{\Delta \mathrm{mag}},\bar{s},\bar{p},\bar{R}_p}(\Delta \mathrm{mag}, s, p, R_p) dp dR_p$$

- Marginalize over instrument constraints:

$$Comp = \int_{s_{min}}^{s_{max}} \int_{\Delta \mathrm{mag}_{min}(s)}^{\Delta \mathrm{mag}_{lim}} f_{\overline{\Delta \mathrm{mag}},\bar{s}}(\Delta \mathrm{mag}, s) d\Delta \mathrm{mag} ds$$

Garrett & Savransky (2016)

Comparison



% difference of 1e9 Monte Carlo samples

Monte Carlo (1e9)

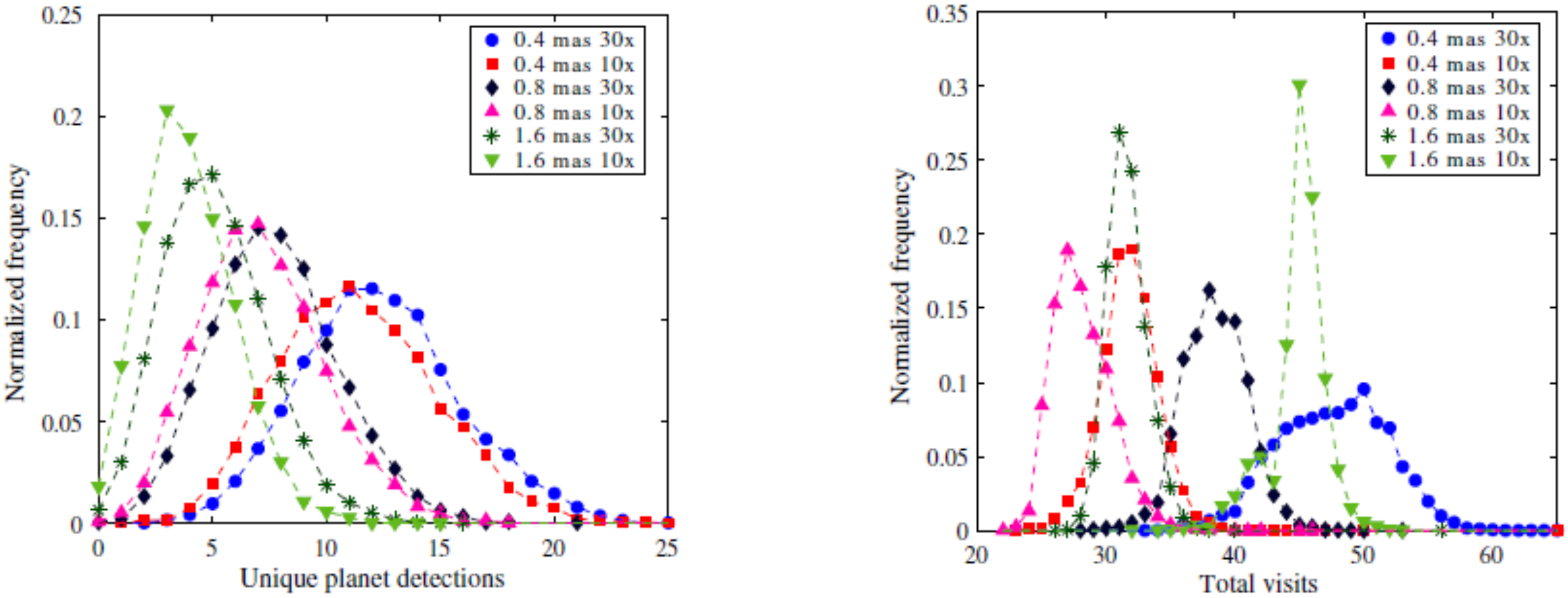
- Time: ~4 hr
- Error: $O\left(\frac{1}{\sqrt{n}}\right)$

Analytical

- Time: ~20 min
- Error: Better than $O(m^{-1})$

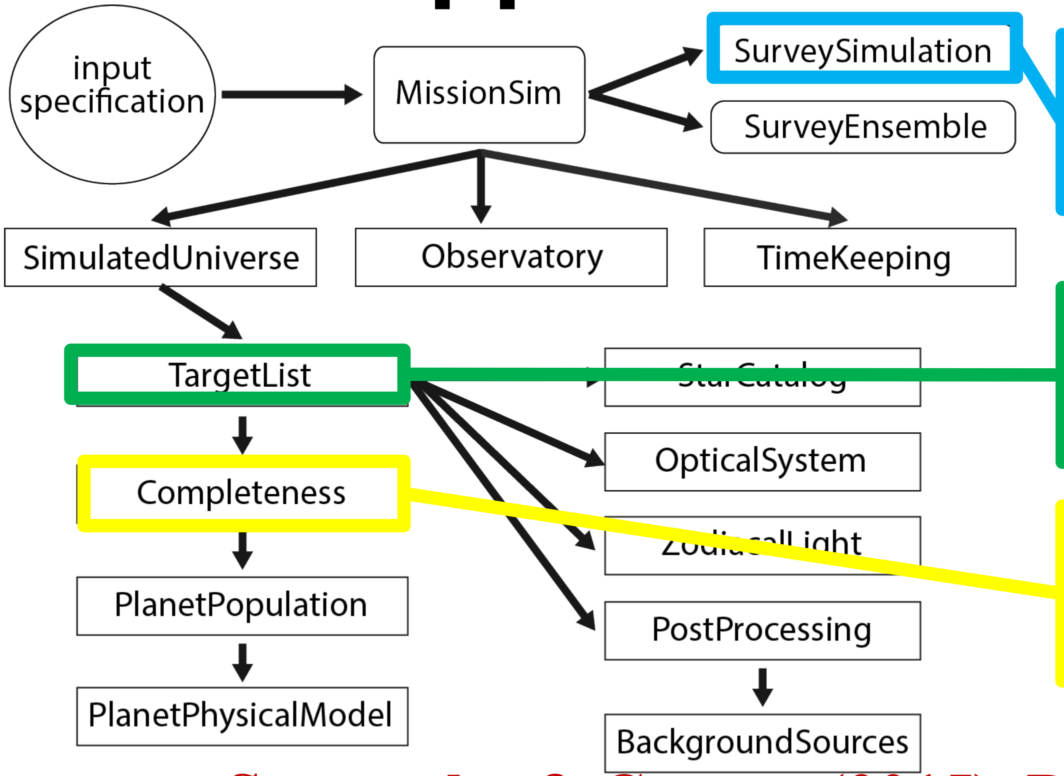
Garrett & Savransky (2016)

Application - EXOSIMS



Savransky & Garrett (2015), Delacroix et al. (2016), github.com/dsavransky/EXOSIMS

Application - EXOSIMS



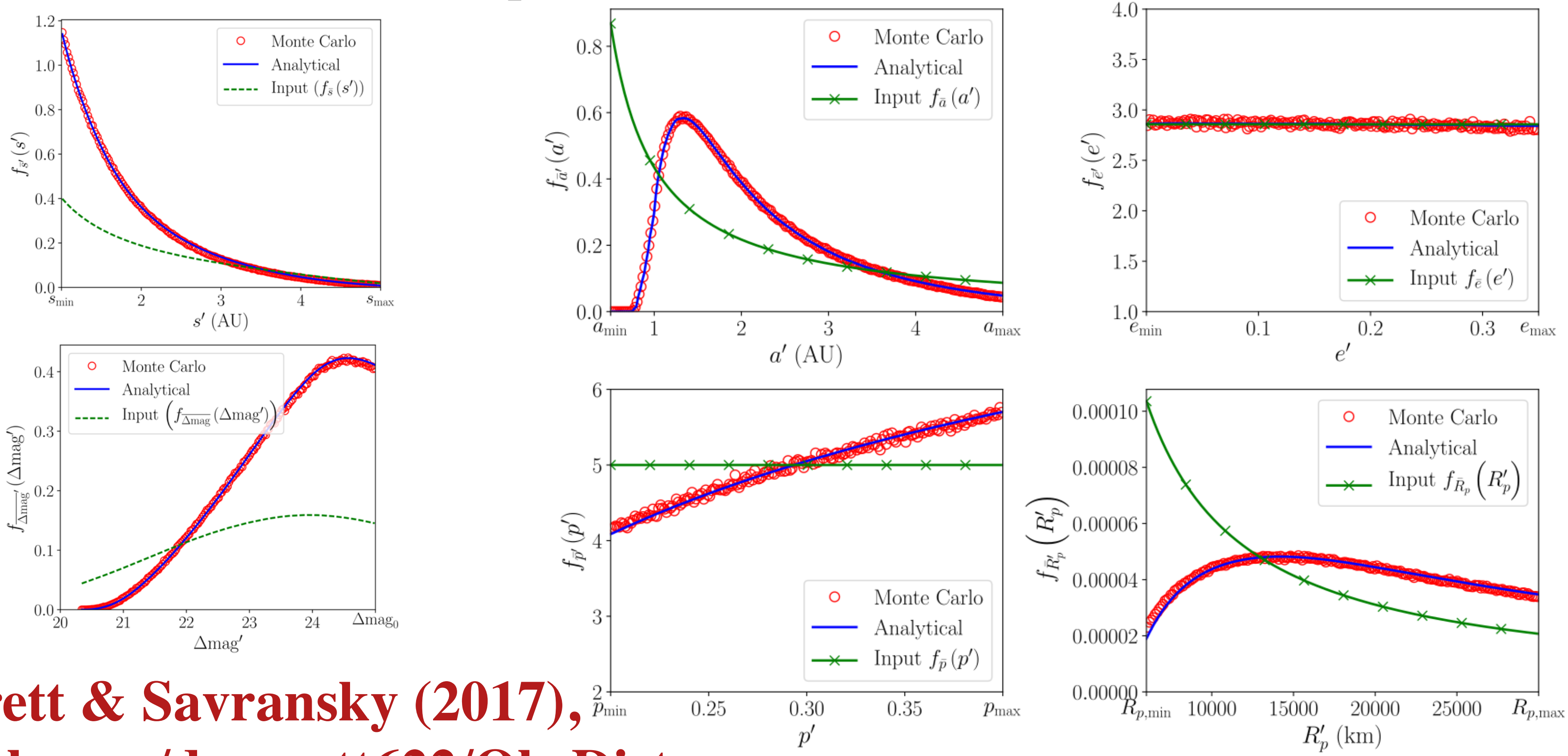
Metric helps schedule or choose next target

Filters targets on completeness metric

Calculated via Monte Carlo or analytical method

Savransky & Garrett (2015), Delacroix et al. (2016), github.com/dsavransky/EXOSIMS

Detected Population Distributions



Garrett & Savransky (2017), github.com/dgarrett622/ObsDist

Completeness Summary

- Probability of detecting planets belonging to assumed population
- **Can now determine analytically**
- Helps answer:
  - –Which stars should I include in the target list for my instrument?
    - **Stars with high completeness values**
  - –What are the biasing or filtering effects of my instrument?
    - **Detected planet population distributions**

Depth-of-Search

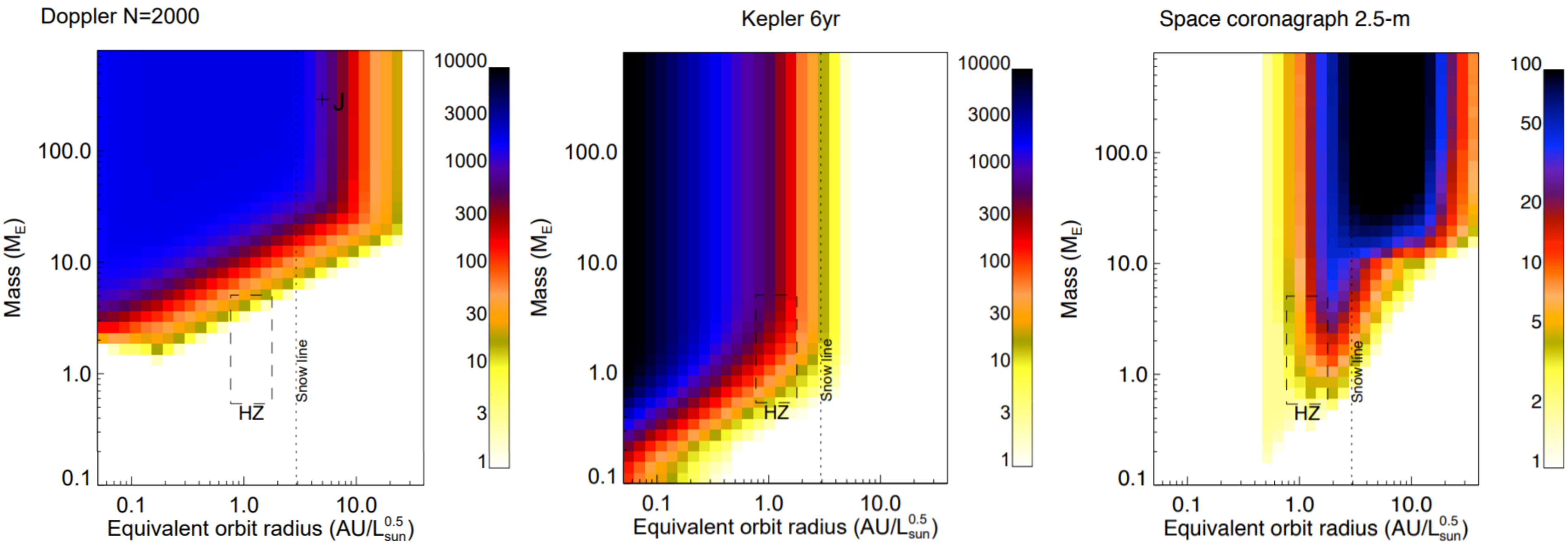
Occurrence Rate Model

$$f_{\theta|D}(\theta|D) = \frac{f_{D|\theta}(D|\theta)\, f_{\theta}(\theta)}{f_D(D)}$$

Completeness

- Garrett, D., et al. “A Simple Depth-of-Search Metric for Exoplanet Imaging Surveys.” AJ (2017).
- github.com/dgarrett622/DoS

Depth-of-Search



Lunine et al. (2008)

Assumptions

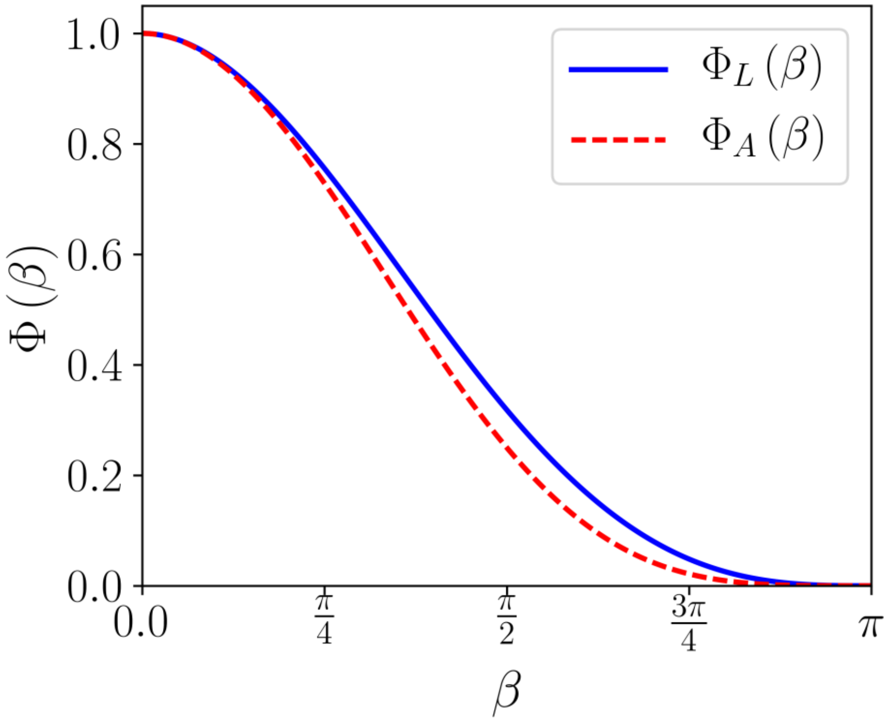
- Circular orbits: $\mathrm{e} = 0 \Rightarrow r = a$
- Phase function (Agol 2007):

$$\Phi(\beta) = \Phi_A(\beta) = \cos^4\left(\frac{\beta}{2}\right)$$

- Albedo: $p = p_{ave}$

$$s = a \sin\beta$$

$$F_R = p\left(\frac{R_p}{a}\right)^2 \cos^4\left(\frac{\beta}{2}\right)$$



Garrett et al. (2017)

Completeness Calculation Issue

β only random variable

$$s = a \sin \beta \qquad F_R = p \left(\frac{R_p}{a}\right)^2 \cos^4\left(\frac{\beta}{2}\right)$$

3 instrument constraints: $s_{min}, s_{max}, C_{min}$

Can't form $f_{\bar{s},\overline{\Delta\mathrm{mag}}}(s, \Delta\mathrm{mag})$ like before

Need alternative method

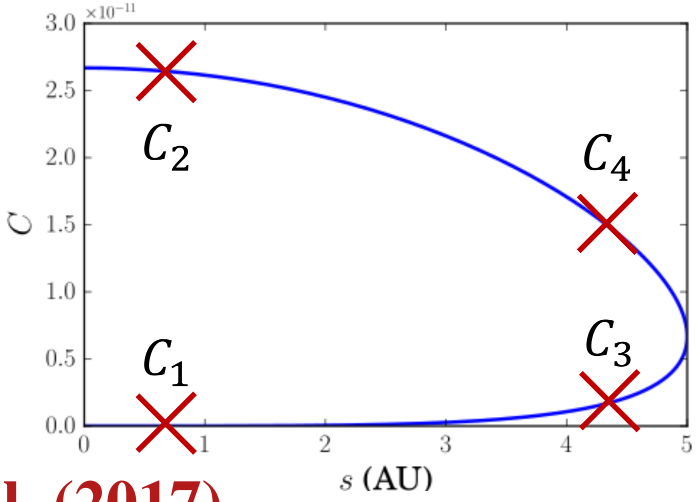
Garrett et al. (2017)

Completeness Calculation

- Find conditional PDF of F_R:

$$f_{\overline{F_R}|\bar{a}=a,\bar{R}_p=R_p,\bar{p}=p}\left(F_R|a,R_p,p\right) = \frac{a}{2\sqrt{pR_p^2F_R}}$$

- Express geometric constraints as equivalent contrasts:
  - $C_1 = \frac{pR_p^2}{a^2}\Phi\left(\pi - \sin^{-1}\left(\frac{s_{min}}{a}\right)\right)$
  - $C_2 = \frac{pR_p^2}{a^2}\Phi\left(\sin^{-1}\left(\frac{s_{min}}{a}\right)\right)$
  - $C_3 = \frac{pR_p^2}{a^2}\Phi\left(\pi - \sin^{-1}\left(\frac{s_{max}}{a}\right)\right)$
  - $C_4 = \frac{pR_p^2}{a^2}\Phi\left(\sin^{-1}\left(\frac{s_{max}}{a}\right)\right)$



Garrett et al. (2017)

Completeness Calculation

- Order constraints properly:

$$C_2 > C_4 > C_3 > C_1 > C_{min}, \text{ If } C_{min} > C_i: C_i = C_{min}$$

- Marginalize over constraints:

$$F(a, R_p, p) = \frac{a}{\sqrt{pR_p^2}} \begin{cases} \left(\sqrt{C_3} - \sqrt{C_1} + \sqrt{C_2} - \sqrt{C_4}\right) & s_{max} < a \\ \left(\sqrt{C_2} - \sqrt{C_1}\right) & s_{max} > a \\ 0 & s_{min} > a \end{cases}$$

Garrett et al. (2017)

Depth-of-Search Construction

- On semi-major axis—planetary radius grid:
  - For each bin of each target star:

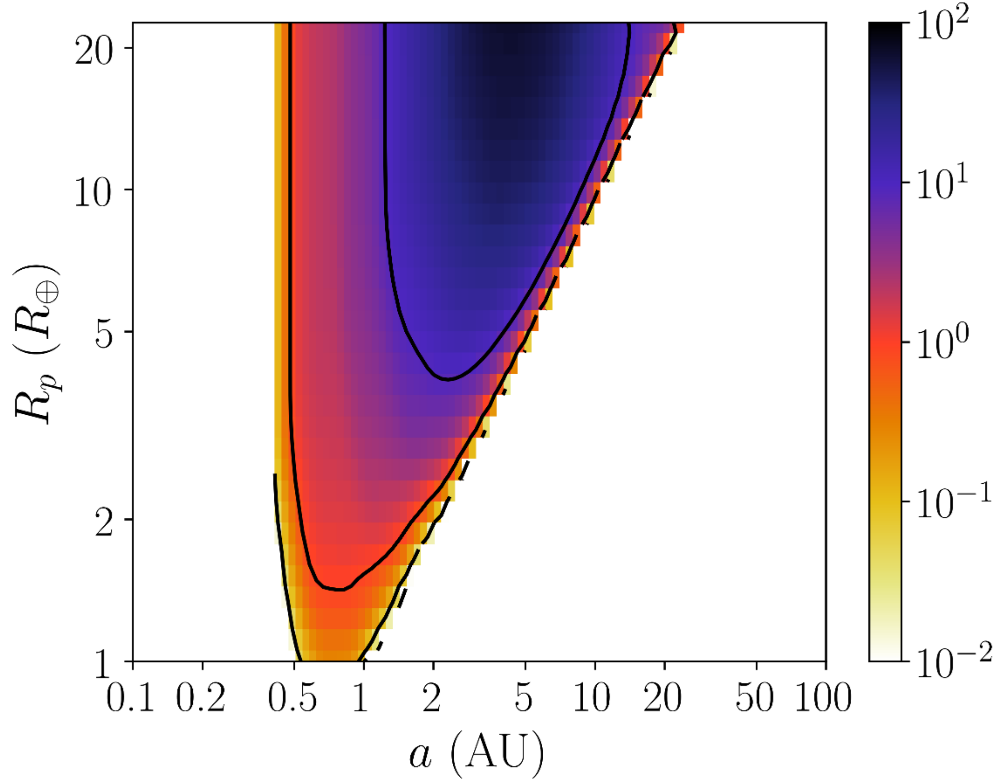
$$DoS = \left(\int_{R_{p,l}}^{R_{p,u}} \int_{a_l}^{a_u} F\left(a, R_p, p\right) da dR_p \right) A^{-1}$$

$$A = \left(R_{p,u} - R_{p,l}\right)\left(a_u - a_l\right)$$

- Element-wise sum grids from all target stars

Garrett et al. (2017)

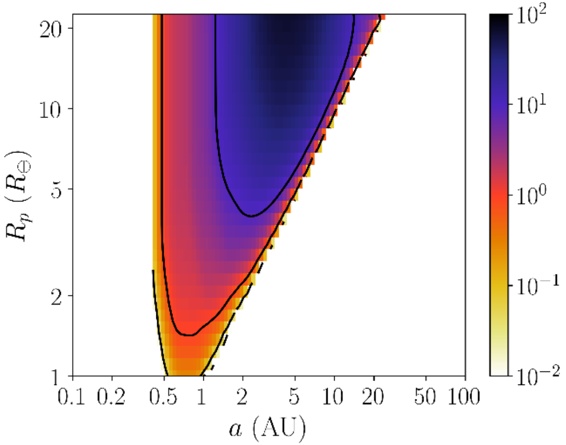
Depth-of-Search Example



- WFIRST HLC
- 108 stars

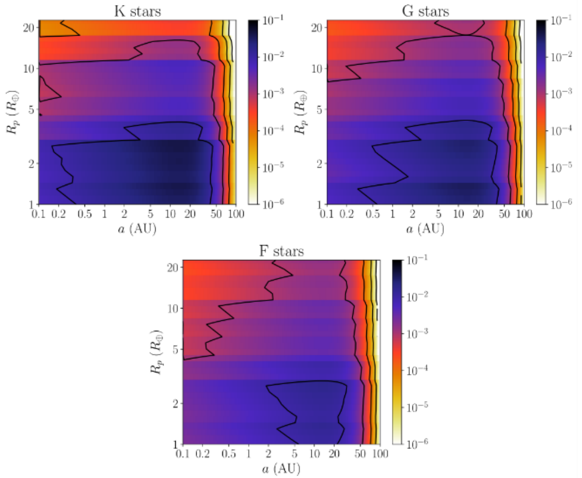
Garrett et al. (2017), github.com/dgarrett622/DoS

Number of Planets Detected



Depth-of-Search

×



Planet Occurrence Rates (Mulders et al. 2015)

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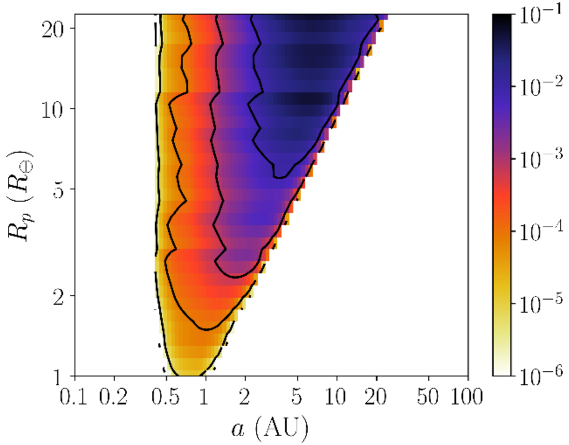


Detected Planets

Garrett et al. (2017)

Depth-of-Search Summary

- Instrument performance metric independent of planet population
- **Simple, closed-form method**
- Helps answer:
  - What kind of planets will my instrument detect?
    - **Easier = large planetary radius, smaller separation**
    - **Harder = small planetary radius, larger separation**
  - How many planets will my instrument detect?
    - **Element-wise multiplication of depth-of-search grid with occurrence rate grid**

Depth-of-Search

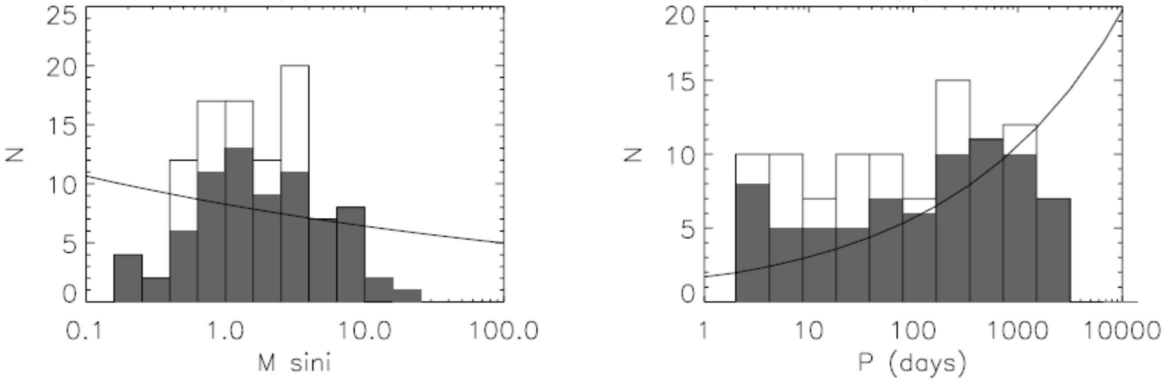
Occurrence Rate Model

$$f_{\boldsymbol{\theta}|\boldsymbol{D}}(\boldsymbol{\theta}|\boldsymbol{D}) = \frac{f_{\boldsymbol{D}|\boldsymbol{\theta}}(\boldsymbol{D}|\boldsymbol{\theta})\, f_{\boldsymbol{\theta}}(\boldsymbol{\theta})}{f_{\boldsymbol{D}}(\boldsymbol{D})}$$

Completeness

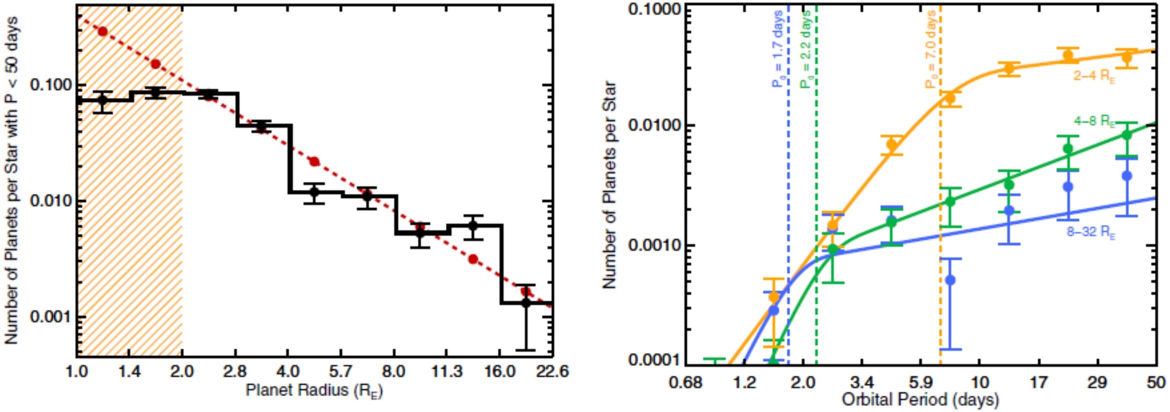
- Garrett, D., et al. "Planet Occurrence Rate Density Models Including Stellar Effective Temperature." PASP (2018).
- github.com/dgarrett622/Occurrence

Occurrence Rate Models

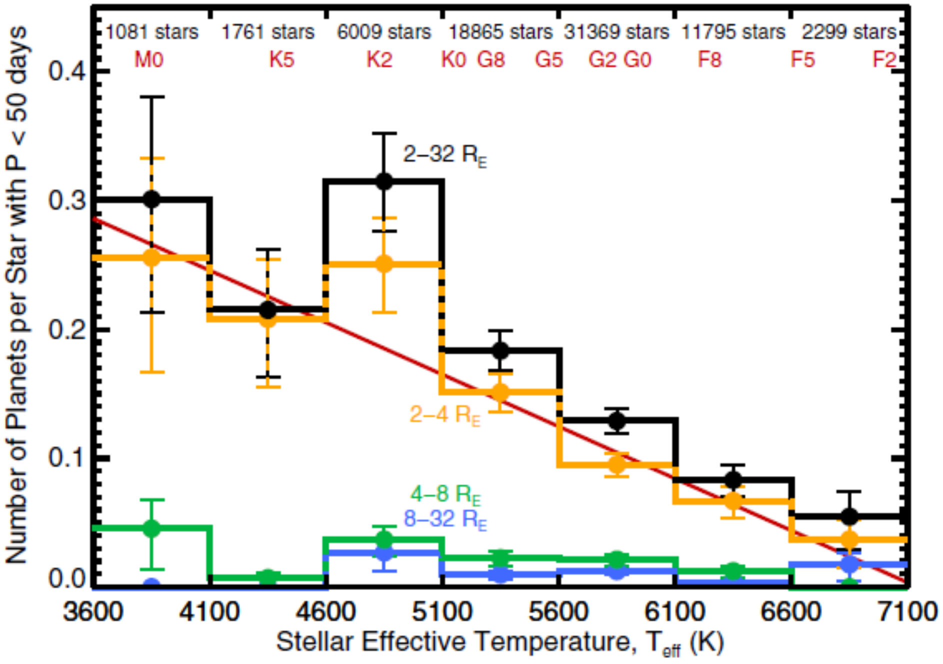


Radial Velocity
Tabachnik & Tremaine (2002)

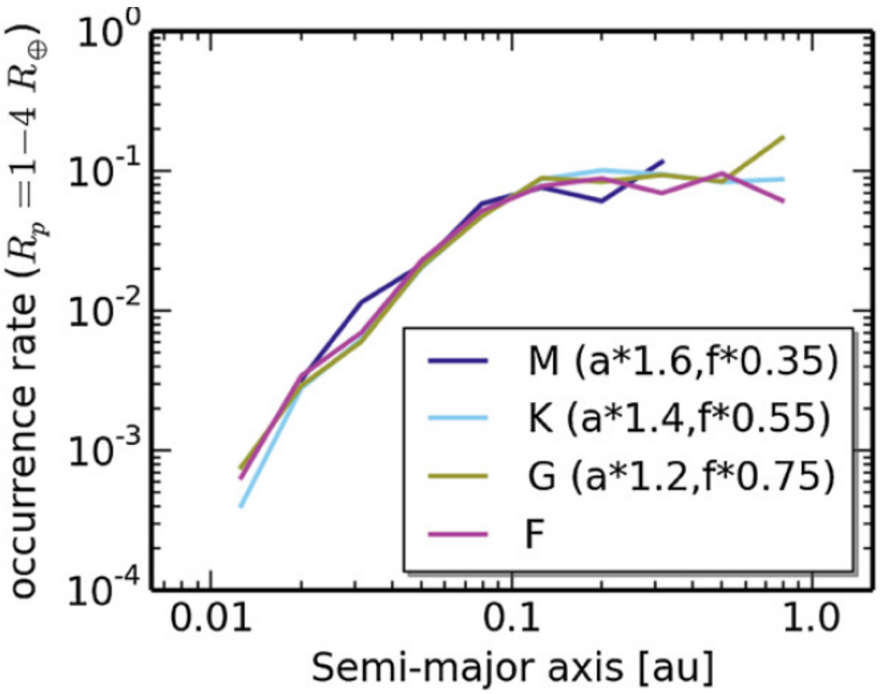
Transit
Howard et al. (2012)



Stellar Parameters



Howard et al. (2012)



Mulders et al. (2015)

New Model

- Simple Model:

$$\frac{\partial^2 \eta}{\partial \ln a \, \partial \ln R_p} = \Omega \left(\frac{a}{a_\oplus}\right)^{\alpha} \left(\frac{R_p}{R_\oplus}\right)^{\rho} u(\tau)$$

- Break Radius Model:

$$\frac{\partial^2 \eta}{\partial \ln a \, \partial \ln R_p} = \begin{cases} \Omega_0 \left(\frac{a}{a_\oplus}\right)^{\alpha_0} \left(\frac{R_p}{R_\oplus}\right)^{\rho_0} u_0(\tau), & R_p < R_b \\ \Omega_1 \left(\frac{a}{a_\oplus}\right)^{\alpha_1} \left(\frac{R_p}{R_\oplus}\right)^{\rho_1} u_1(\tau), & R_p \geq R_b \end{cases}$$

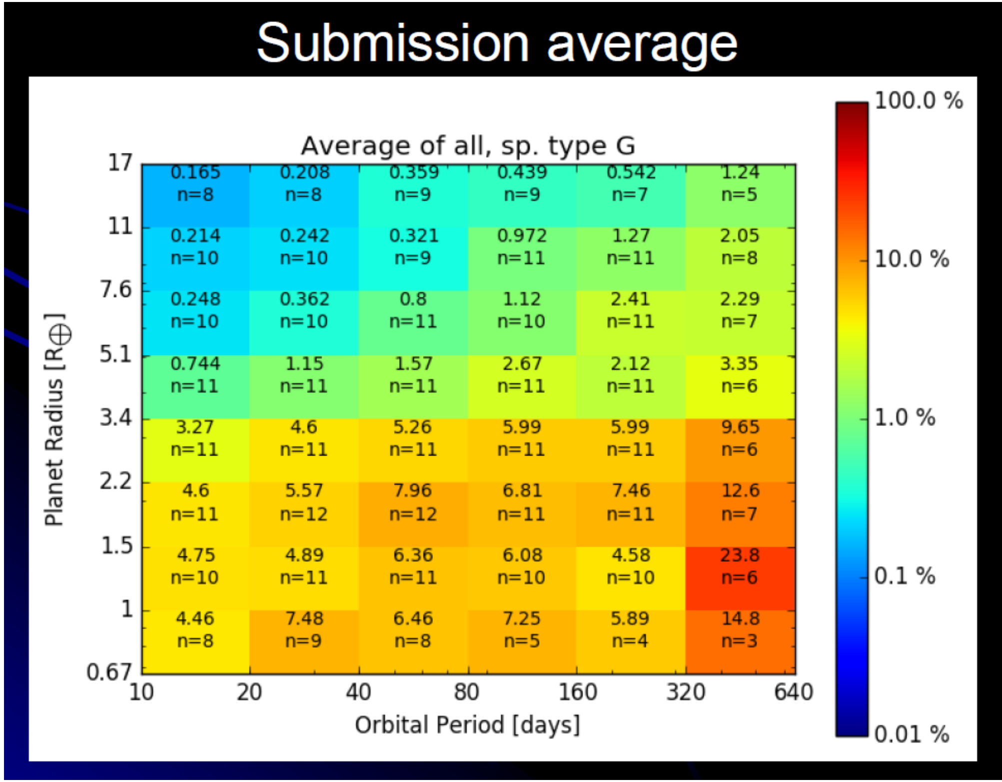
$$\tau = \frac{T_{eff}}{T_{eff,\odot}} - 1, \qquad u_i(\tau) = 1 + \lambda_i \tau + \omega_i \tau^2 + \xi_i \tau^3 + \cdots$$

$$T_{eff,\odot} = 5772K, a_\oplus = 1AU, R_\oplus = 6371km$$

Garrett et al. (2018)

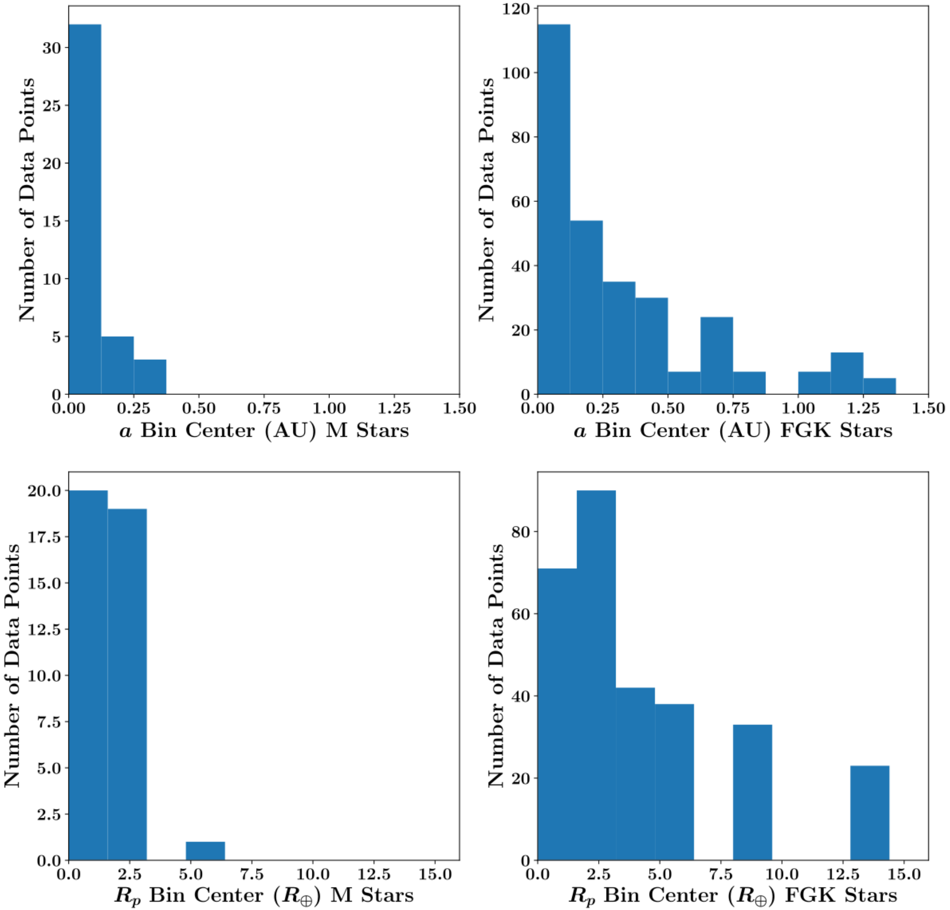
SAG13 Data Set

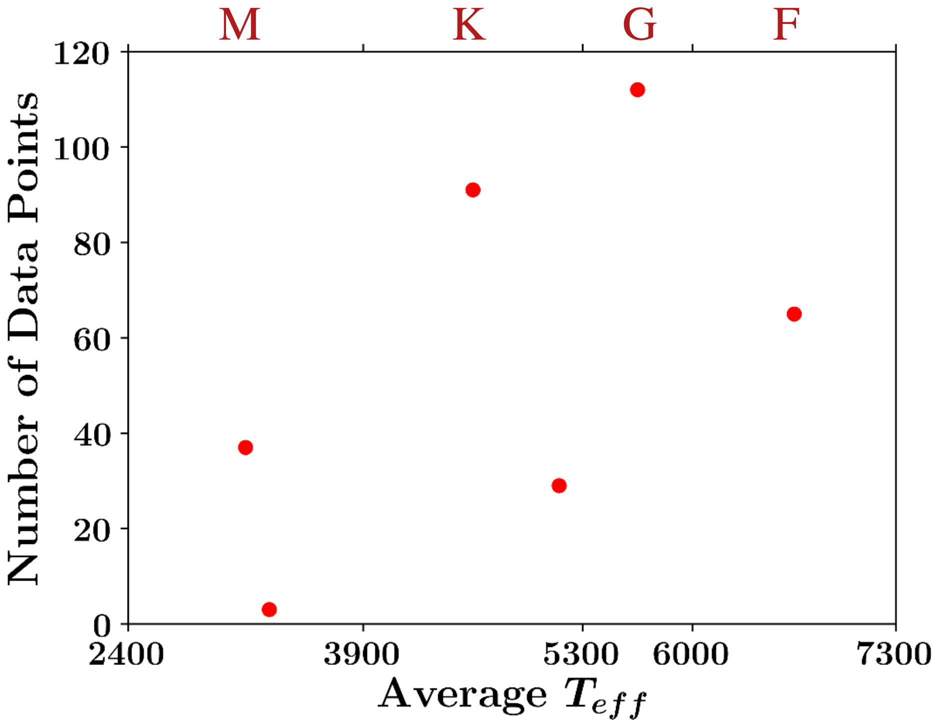
Type	T_{eff} Min (K)	T_{eff} Max (K)
M	2400	3900
K	3900	5300
G	5300	6000
F	6000	7300



exoplanets.nasa.gov/exep/exopag/sag/#sag13

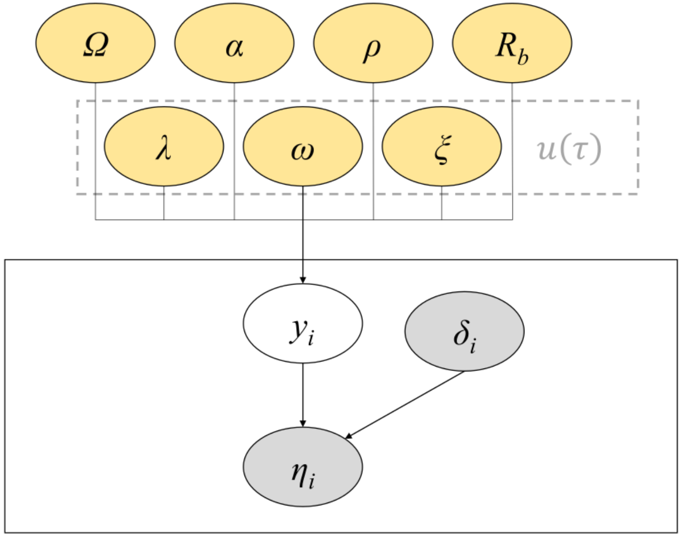
Selected Data





Bayesian Parameter Estimation

$$f_{\theta|D}(\theta|D) = \frac{f_{D|\theta}(D|\theta) f_{\theta}(\theta)}{f_D(D)}$$



emcee
The MCMC Hammer

Foreman-Mackey et al. (2013), github.com/dfm/emcee

Likelihood and Priors

Likelihood

$$\hat{L} = -\frac{1}{2}\sum_{i=1}^{N}\left[\ln\left(2\pi\delta_i^2\right) + \frac{(\eta_i - y_i)^2}{\delta_i^2}\right]$$

$$y_i = \int_{\ln R_{p,l_i}}^{\ln R_{p,u_i}} \int_{\ln a_{l_i}}^{\ln a_{u_i}} \frac{\partial^2 \eta}{\partial \ln a \ \partial \ln R_p} d \ln a \ d \ln R_p$$

Priors

$\ln \Omega \sim U(-5,10)$ → $\eta \geq 0, \Omega \geq 0$

$\alpha \sim U(-2,2)$

$\rho \sim U(-2,2)$

$R_b \sim U(0.44,26)$ $\quad -0.6 \leq \tau \leq 0.3$

$\lambda \sim U(-100,100)$ $\quad \tau$ coefficient

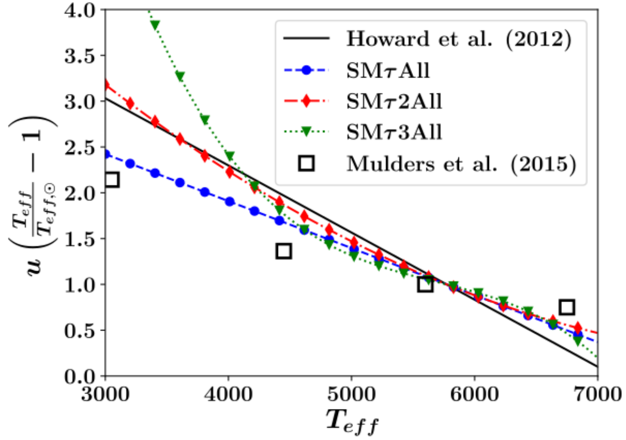
$\omega \sim U(-500,500)$ $\quad \tau^2$ coefficient

$\xi \sim U(-5000,5000)$ $\quad \tau^3$ coefficient

Garrett et al. (2018)

Model Fits – All Data

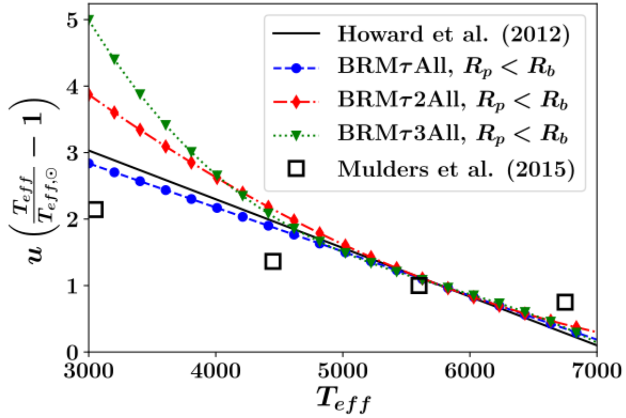
	BIC
SMAll	832.7
SMτAll	323.5
SMτ2All	323.4
SMτ3All	312.9
BRMAll	548.6
BRMτAll	-170.9
BRMτ2All	-168.3
BRMτ3All	-164.4

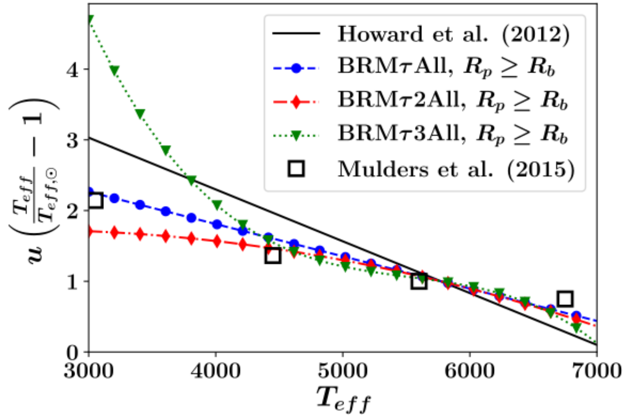


$$BIC = \ln(N)\,N_{mp} - 2\hat{L}$$

$$N \gg N_{mp}$$

Schwarz (1978)



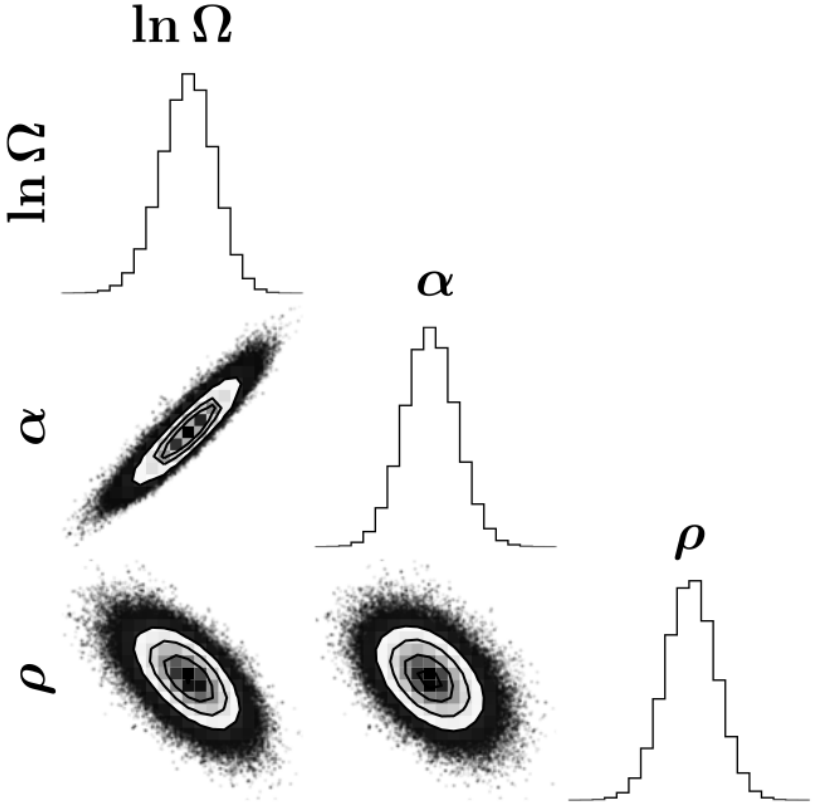


Garrett et al. (2018)

Simple Model Fit – M Data

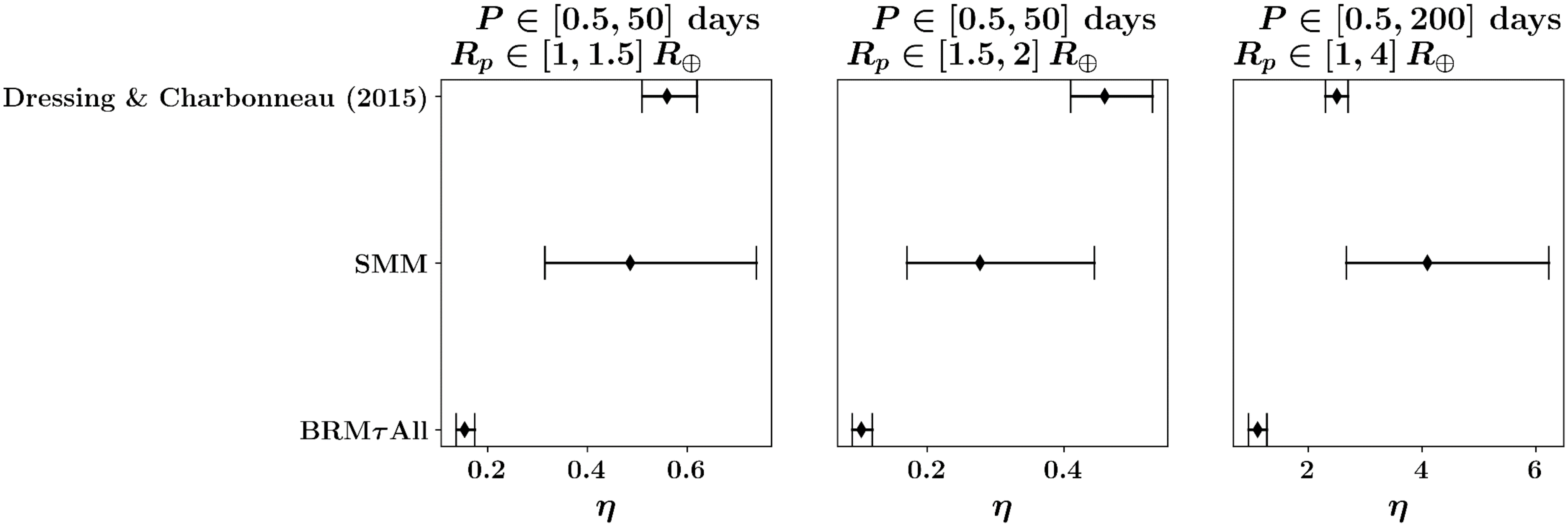
BIC	-118.4
Ω	$11.98^{+3.037}_{-2.556}$
α	$1.260^{+0.073}_{-0.072}$
ρ	$-0.623^{+0.154}_{-0.154}$

$$\frac{\partial^2 \eta}{\partial \ln a \, \partial \ln R_p} = \Omega \left(\frac{a}{a_\oplus}\right)^{\alpha} \left(\frac{R_p}{R_\oplus}\right)^{\rho}$$



Garrett et al. (2018)

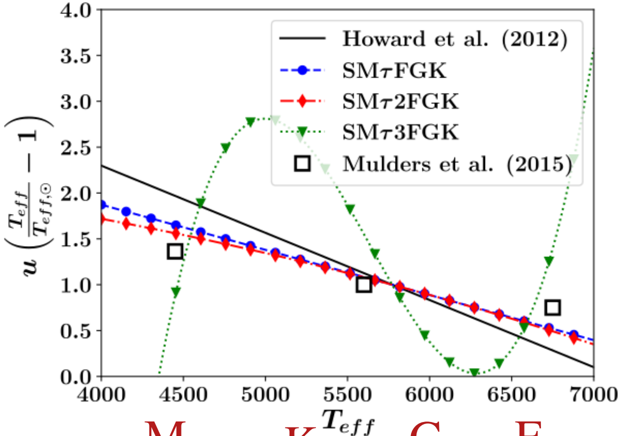
M-Type Star Comparison

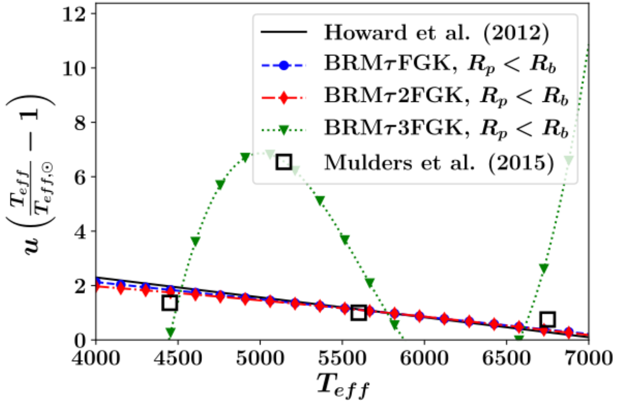


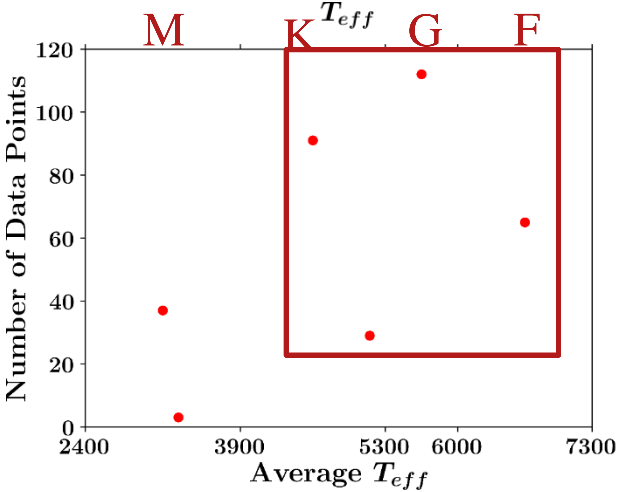
Garrett et al. (2018)

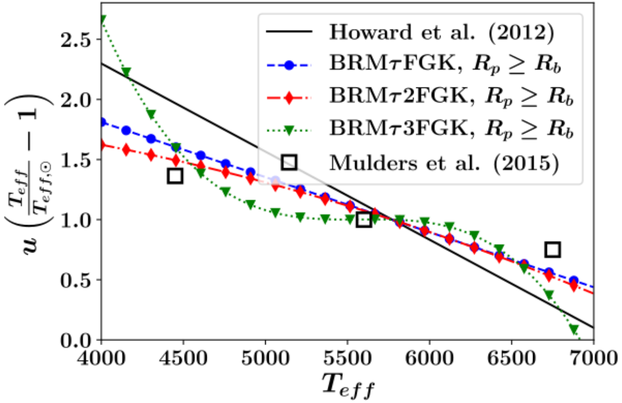
Model Fits – FGK Data

	BIC
SMFGK	807.5
SMτFGK	352.0
SMτ2FGK	356.3
SMτ3FGK	227.2
BRMFGK	525.0
BRMτFGK	-118.2
BRMτ2FGK	-108.5
BRMτ3FGK	-453.3



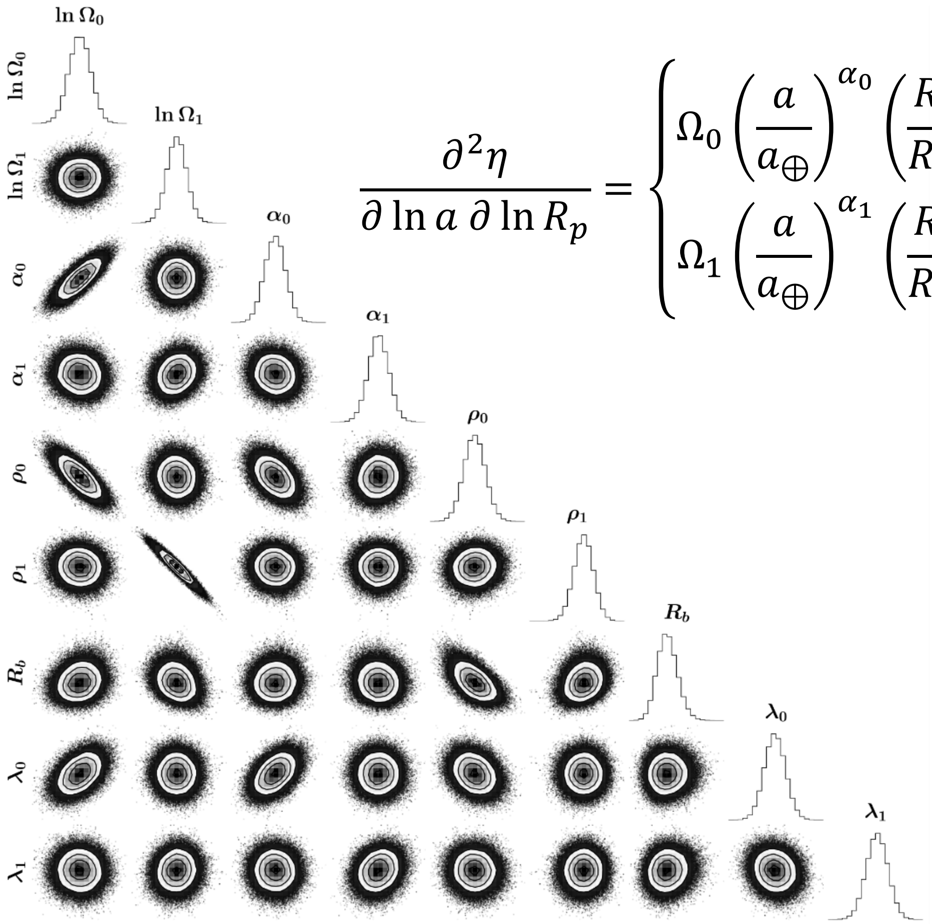






Garrett et al. (2018)

Break Radius Model – FGK Data



$$\frac{\partial^2\eta}{\partial \ln a\, \partial \ln R_p} = \begin{cases} \Omega_0 \left(\frac{a}{a_\oplus}\right)^{\alpha_0} \left(\frac{R_p}{R_\oplus}\right)^{\rho_0} u_0(\tau), & R_p < R_b \\ \Omega_1 \left(\frac{a}{a_\oplus}\right)^{\alpha_1} \left(\frac{R_p}{R_\oplus}\right)^{\rho_1} u_1(\tau), & R_p \geq R_b \end{cases}$$

$\Omega_0 = 1.027^{+0.054}_{-0.052}$

$\Omega_1 = 0.533^{+0.110}_{-0.092}$

$\alpha_0 = 1.104^{+0.013}_{-0.013}$

$\alpha_1 = 1.006^{+0.029}_{-0.028}$

$\rho_0 = -0.175^{+0.072}_{-0.071}$

$\rho_1 = -0.884^{+0.100}_{-0.101}$

$R_b = 2.766^{+0.052}_{-0.048}$

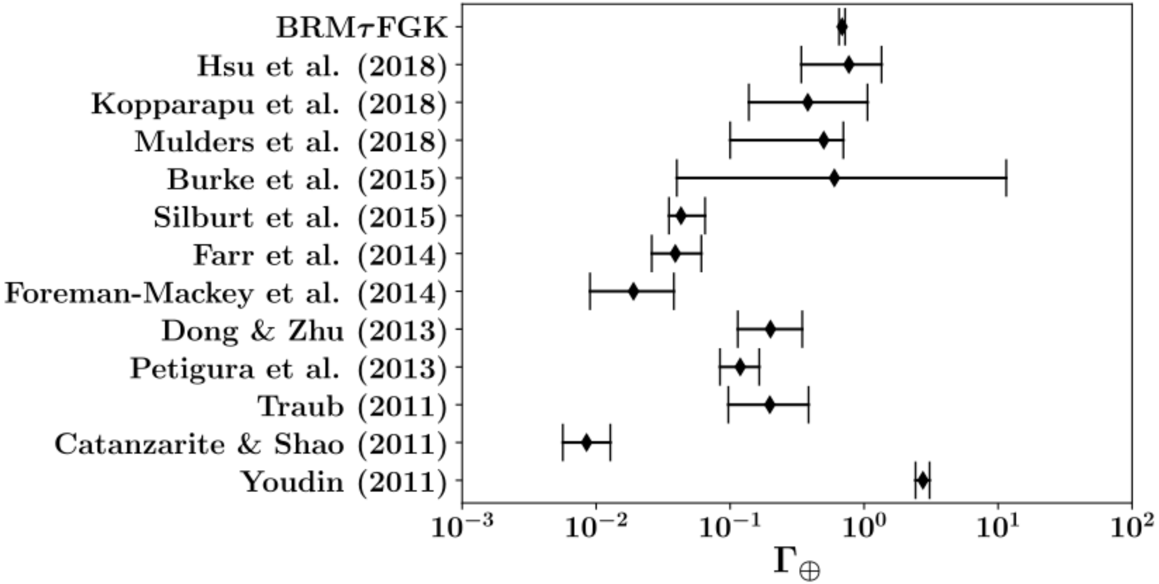
$\lambda_0 = -3.676^{+0.150}_{-0.148}$

$\lambda_1 = -2.642^{+0.291}_{-0.296}$

Garrett et al. (2018)

$\Gamma_\oplus$ Comparison

$$\Gamma_\oplus = \left.\frac{\partial^2 \eta}{\partial \ln P \;\; \partial \ln R_p}\right|_{1\text{ year},\, 1R_\oplus} = \frac{2\Omega_0}{3}$$



Garrett et al. (2018)

Occurrence Rate Model Summary

- Fit model to SAG13 occurrence rate data over T_{eff} range
  - **Explicitly include function of T_{eff}**
  - **M-type stars: don't fit T_{eff} trend**
  - **F-, G-, K-type stars: break radius model with linear T_{eff}**
- Occurrence rates comparable to literature
- **Step towards more complete model of planet occurrence**

Depth-of-Search

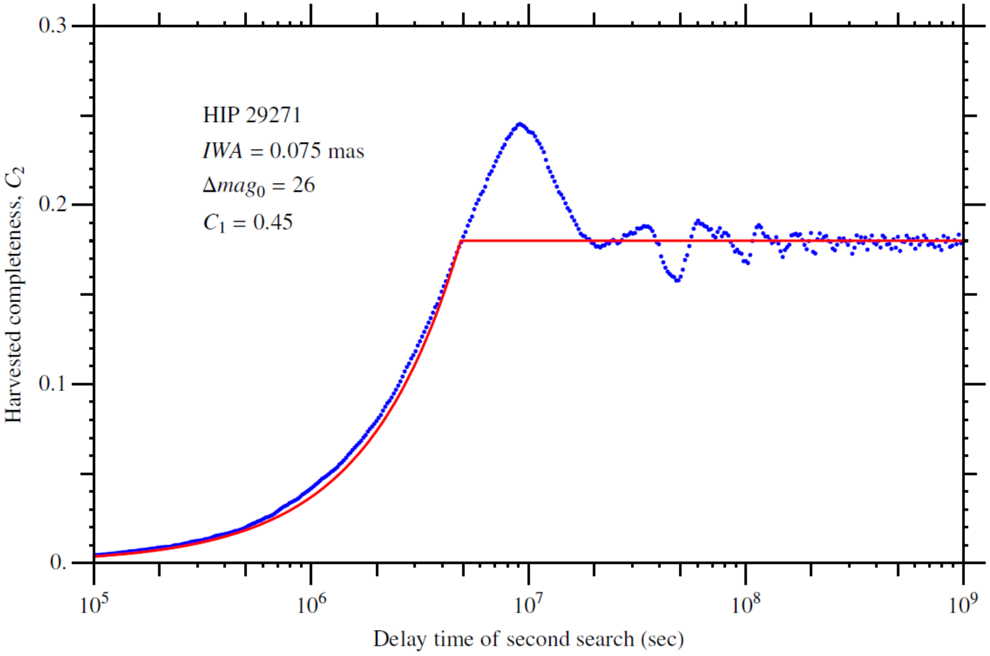
Occurrence Rate Model

$$f_{\boldsymbol{\theta}|\boldsymbol{D}}(\boldsymbol{\theta}|\boldsymbol{D}) = \frac{f_{\boldsymbol{D}|\boldsymbol{\theta}}(\boldsymbol{D}|\boldsymbol{\theta})\, f_{\boldsymbol{\theta}}(\boldsymbol{\theta})}{f_{\boldsymbol{D}}(\boldsymbol{D})}$$

Completeness

Future Directions

Completeness



Brown & Soummer (2010)

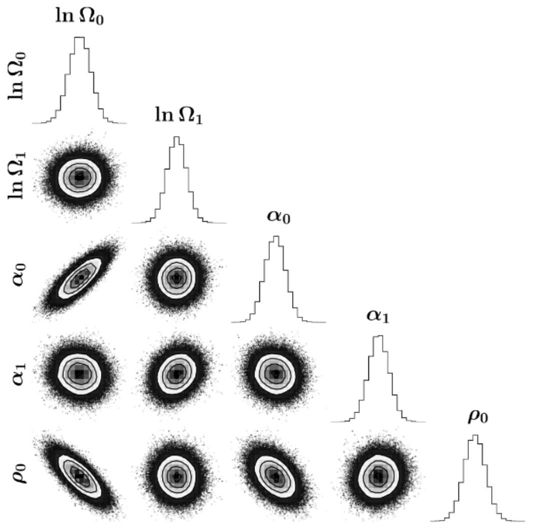
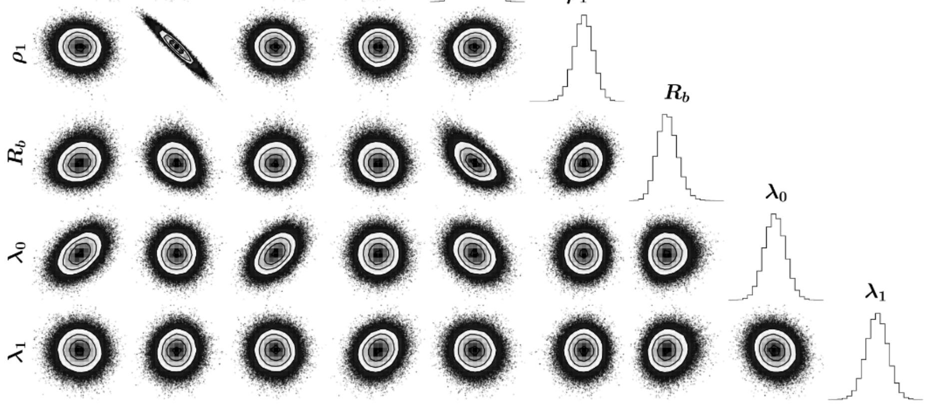
Depth-of-Search

- $e \neq 0$
- Phase functions
- Albedo

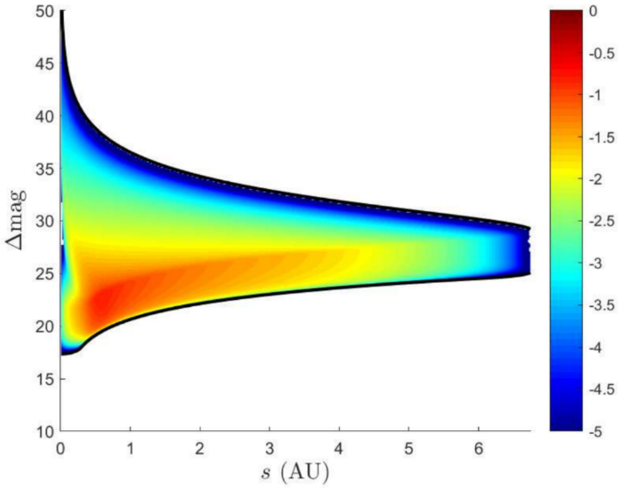
Occurrence Rate Models

- More data
- Stellar Parameters
  - $L = 4\pi R_{\star}^{2} \sigma T_{eff}^{4}$
  - Mass
  - Metallicity

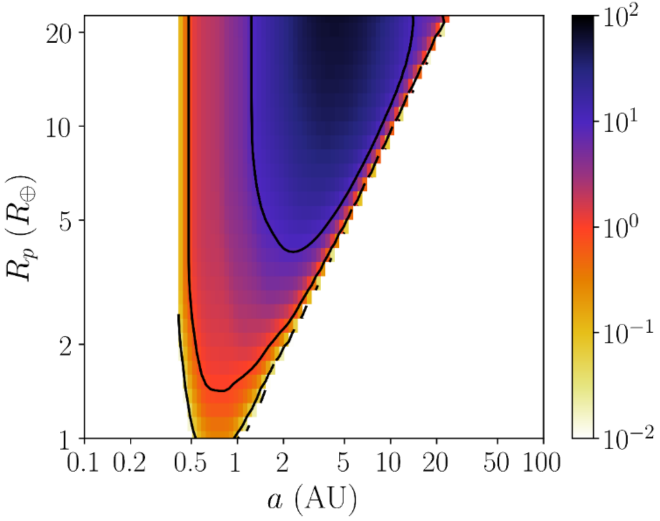
Thank You

$\ln \Omega_0$
$\ln \Omega_1$
α_0
α_1
ρ_0
ρ_1
R_b
λ_0
λ_1



50
45
40
35
30
25
20
15
10
Δmag
0 1 2 3 4 5 6
s (AU)
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5



20
10
5
2
1
R_p $(R_\oplus)$
0.1 0.2 0.5 1 2 5 10 20 50 100
a (AU)
10^2
10^1
10^0
10^{-1}
10^{-2}

Publications

- Journal
  - Savransky, D., and Garrett, D., “WFIRST-AFTA coronagraph science yield modeling with EXOSIMS.” JATIS (2016).
  - Garrett, D., and Savransky, D., “Analytical formulation of the single-visit completeness joint probability density function.” ApJ (2016)
  - Garrett, D., Savransky, D., and Macintosh, B. A., “A simple depth-of-search metric for exoplanet imaging surveys.” AJ (2017).
  - Garrett, D., Savransky, D., and Belikov, R., “Planet occurrence rate density models including stellar effective temperature.” PASP (2018).
- Conference
  - Garrett, D., and Savransky, D., “Science yield modeling with EXOSIMS.” AAS (2016).
  - Garrett, D., and Savransky, D., “Analytical methods for exoplanet imaging detection metrics.” AAS (2017).
  - Garrett, D., and Savransky, D., “Detected exoplanet population distributions found analytically.” SPIE (2017).
  - Garrett, D., and Savransky, D., “Building better planet populations with EXOSIMS.” AAS (2018).