Navigation and Orbit Phasing of Modular Spacecraft for Segmented Telescope Assembly about Sun-Earth L2

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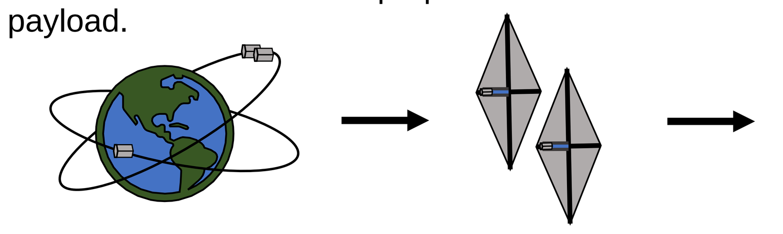
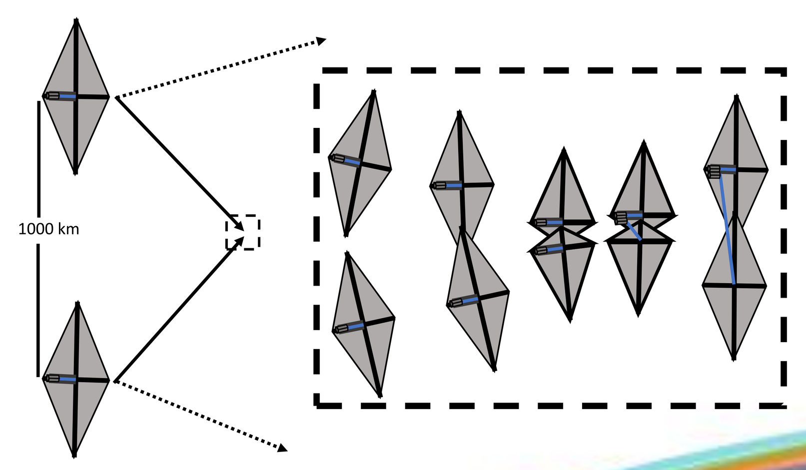
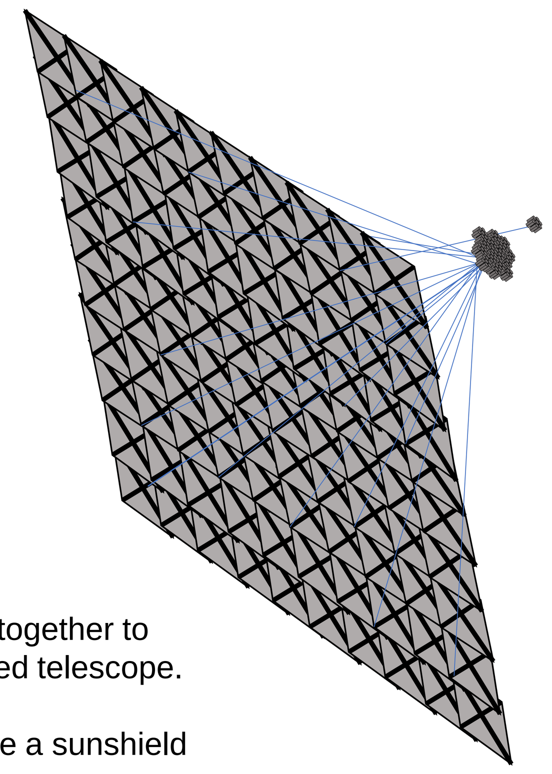
Modular spacecraft start on Earth orbits with mirror as payload. → Solar sails unfurl and propel the mirrors to L2. → Spacecraft are assembled on a Lissajous orbit.



- When within 1000 km, two spacecraft will rendezvous and dock together.
- Sails overlap, remain attached to mirror cluster through extensible tether.



- Mirrors all dock together to form a segmented telescope.
- Solar sails create a sunshield for the telescope.

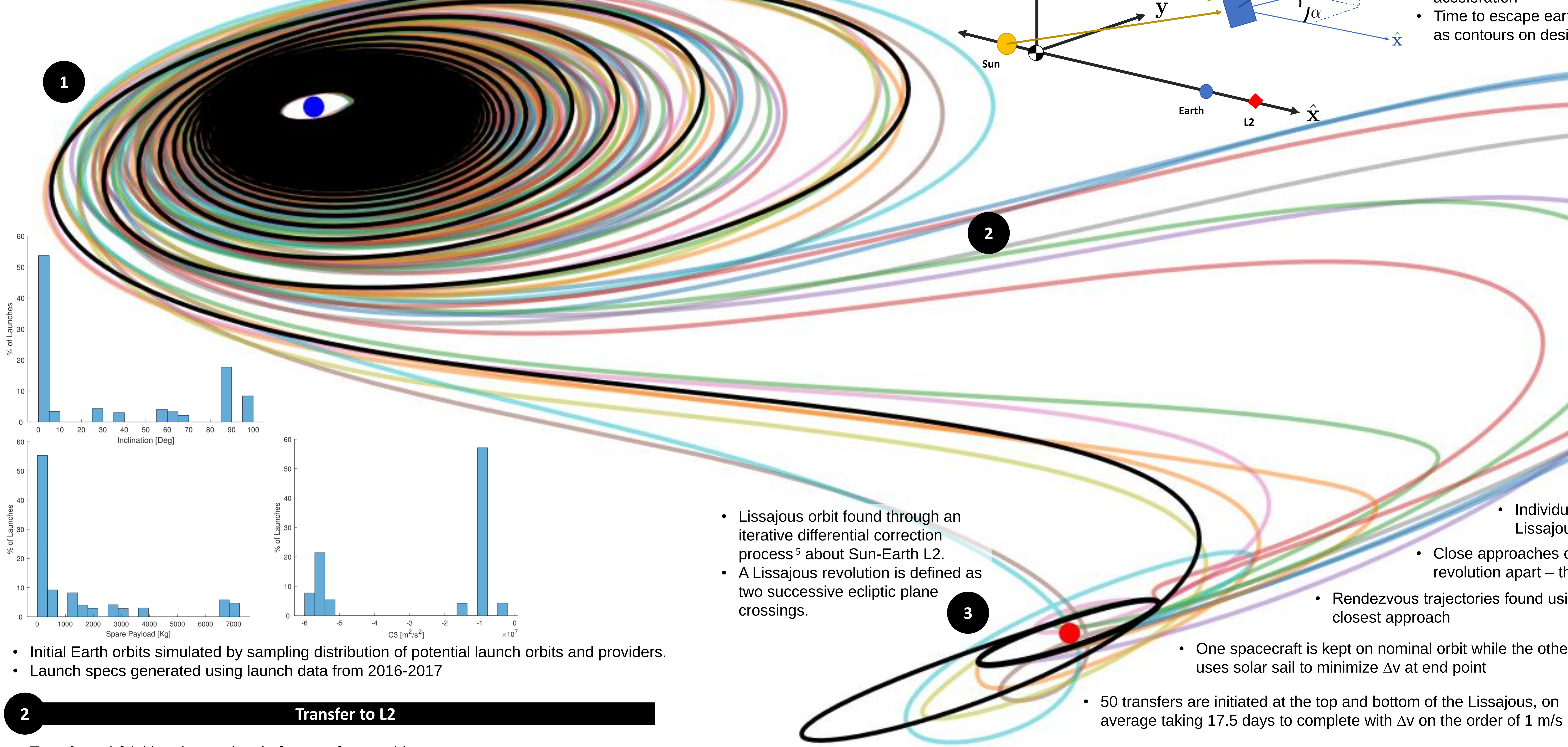


- Dynamics are modeled using the Circular Three Body formalism [1].
- Solar radiation pressure exerts specific force onto each solar sail [2].

$$\mathbf{a}_s = \beta \frac{1-\mu}{r_1^2}(\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}})^2 \hat{\mathbf{n}} \qquad \beta = \frac{\sigma^*}{\frac{m_p}{A} + \sigma_s}$$

$$\hat{\mathbf{n}} = \cos(\alpha)\cos(\delta)\hat{\mathbf{x}} + \sin(\alpha)\cos(\delta)\hat{\mathbf{y}} + \sin(\delta)\hat{\mathbf{z}}$$

- The lightness number β is used as a non-dimensionalized sail characteristic acceleration.
- It is a function of the sail area A, payload mass m_p, sail areal density σ_s of units $\frac{g}{m^2}$ and a critical lightness number σ^*.



1 Earth Escape

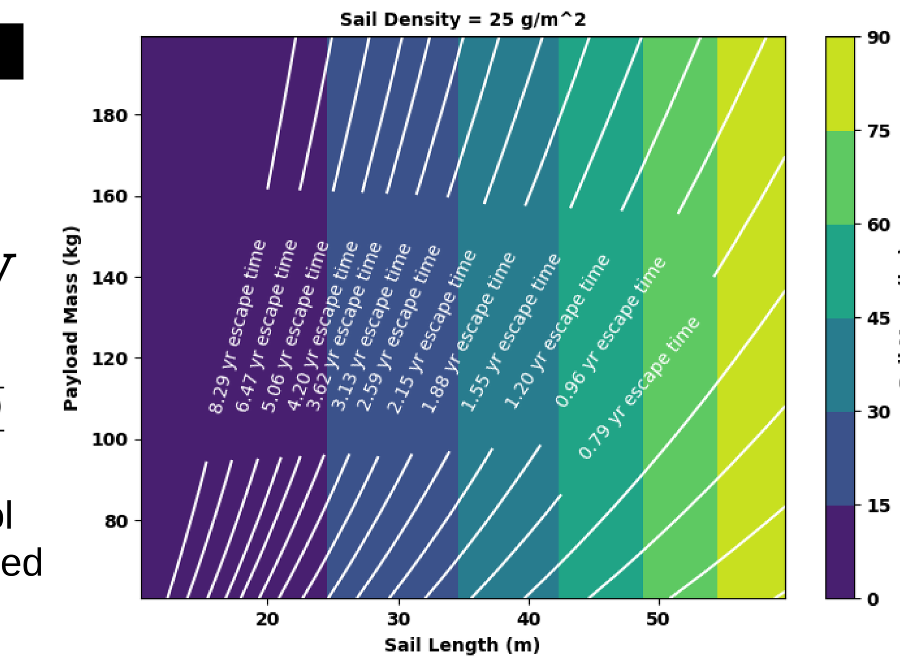
- A control law [3] for the sail angles is used to maximize energy gain:

$$n_x = \frac{|v_y|}{\sqrt{v_y^2 + \xi^2(v_y^2+v_z^2)}} \qquad \max_{\hat{\mathbf{n}}} \ \mathbf{a}_s \cdot \mathbf{v}$$

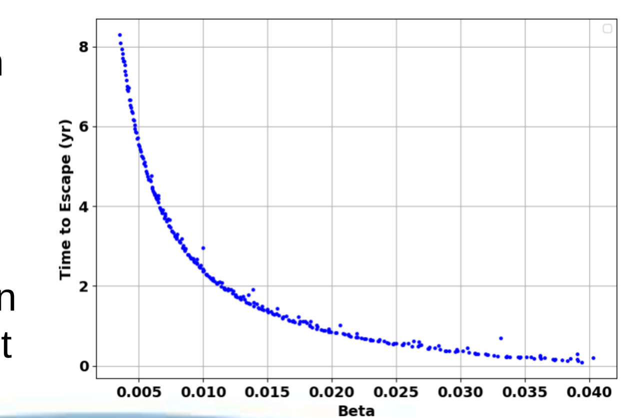
$$n_y = \xi n_x$$

$$n_z = \frac{v_z}{v_y} n_y \qquad \xi = \frac{-3v_x v_y \pm v_y\sqrt{9v_x^2 + 8(v_y^2+v_z^2)}}{4(v_y^2+v_z^2)}$$

- Equations of motion with sail normal control law integrated until escape energy is reached



- Assuming a set sail density based on JPL NEA Scout design
- Design space includes payload mass and sail size – each corresponds to a characteristic acceleration
- Time to escape earth orbit shown as contours on design space plot



- Initial Earth orbits simulated by sampling distribution of potential launch orbits and providers.
- Launch specs generated using launch data from 2016-2017

2 Transfer to L2

- Transfer to L2 initiated sometime before or after reaching escape energy
- Constrained optimization problem using $\mathbf{f}$: equations of motion perturbed by $\mathbf{a}_s$

$$H = J + \lambda \cdot \mathbf{f} \quad \Rightarrow \quad \dot{\lambda} = -\frac{\partial H}{\partial \mathbf{x}} \qquad \frac{\partial H}{\partial \hat{\mathbf{n}}} = \mathbf{0}$$

Variational Hamiltonian: minimize cost J and constrained by equations of motion — Co-states add 6 more equations to integrate — Definition of new control law

- Optimal control law points solar sail in direction of velocity co-states (Lagrange multipliers pertaining to velocity)

$$n_x = \frac{|\lambda_{v_y}|}{\sqrt{\lambda_{v_y}^2 + \xi^2(\lambda_{v_y}^2+\lambda_{v_z}^2)}} \qquad \max_{\hat{\mathbf{n}}} \ \mathbf{a}_s \cdot \lambda_{\mathbf{v}}$$

$$n_y = \xi n_x$$

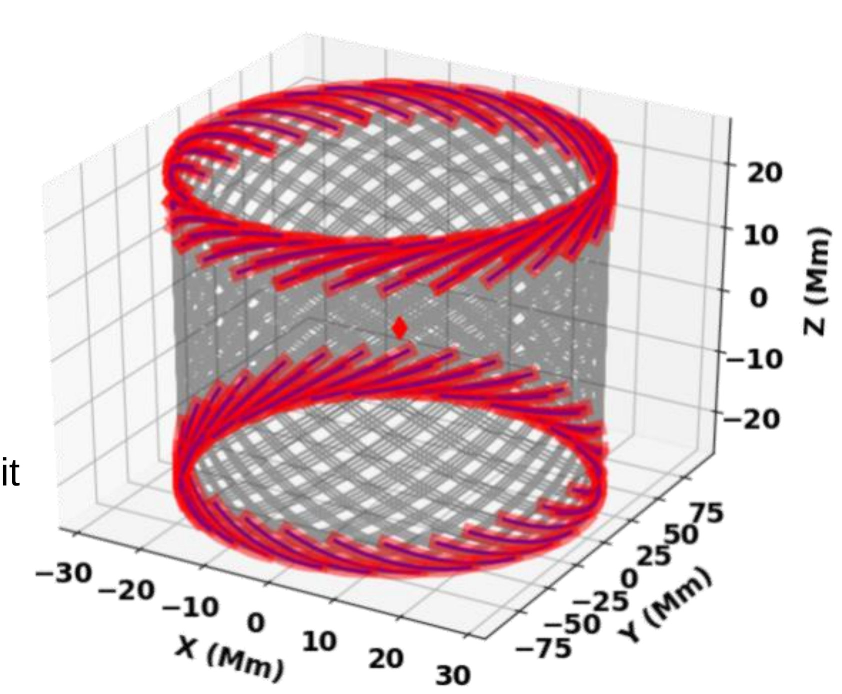
$$n_z = \frac{\lambda_{v_z}}{\lambda_{v_y}} n_y \qquad \xi = \frac{-3\lambda_{v_x}\lambda_{v_y} \pm \lambda_{v_y}\sqrt{9\lambda_{v_x}^2 + 8(\lambda_{v_y}^2+\lambda_{v_z}^2)}}{4(\lambda_{v_y}^2+\lambda_{v_z}^2)}$$

- Boundary value problem with 12 variables, 6 boundary conditions each at start and end of trajectory (position and velocity)
- Single shooting problem wrapped into optimization problem using basinhopping algorithm
- Optimization variables:

$$[t_i, t_f, \Delta t, \lambda_{x_0}, \lambda_{y_0}, \lambda_{z_0}, \lambda_{v_{x,0}}, \lambda_{v_{y,0}}, \lambda_{v_{z,0}}]$$

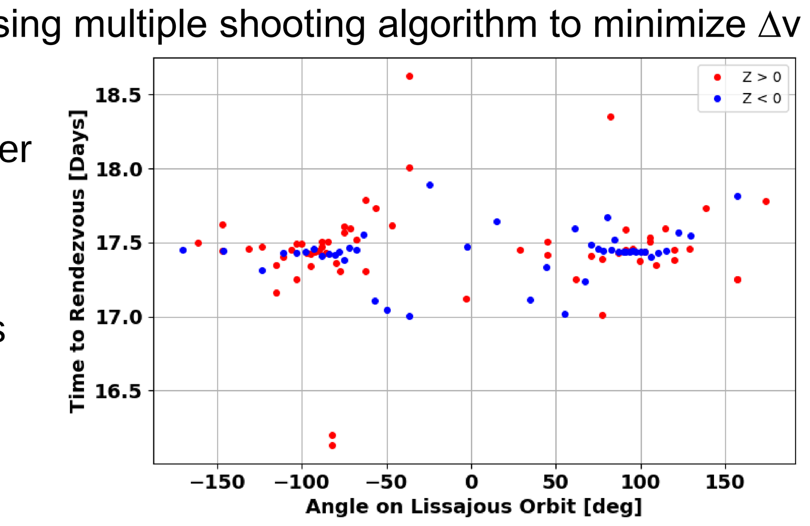
  - t_i defines time at which transfer to L2 is initiated
  - t_f defines point where spacecraft is injected into Lissajous orbit
- Objective function integrates initial states augmented with Lagrange multipliers for Δt time
  - Minimizes difference between desired final states and final result of integration

- Lissajous orbit found through an iterative differential correction process [5] about Sun-Earth L2.
- A Lissajous revolution is defined as two successive ecliptic plane crossings.



3 Rendezvous

- Individual modules inserted at different points on the Lissajous where they will eventually rendezvous and dock
- Close approaches of 1000 km occur when spacecraft are injected one revolution apart – they occur at top and bottom of Lissajous
- Rendezvous trajectories found using multiple shooting algorithm to minimize Δv at closest approach
- One spacecraft is kept on nominal orbit while the other uses solar sail to minimize Δv at end point
- 50 transfers are initiated at the top and bottom of the Lissajous, on average taking 17.5 days to complete with Δv on the order of 1 m/s



Acknowledgements and References

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