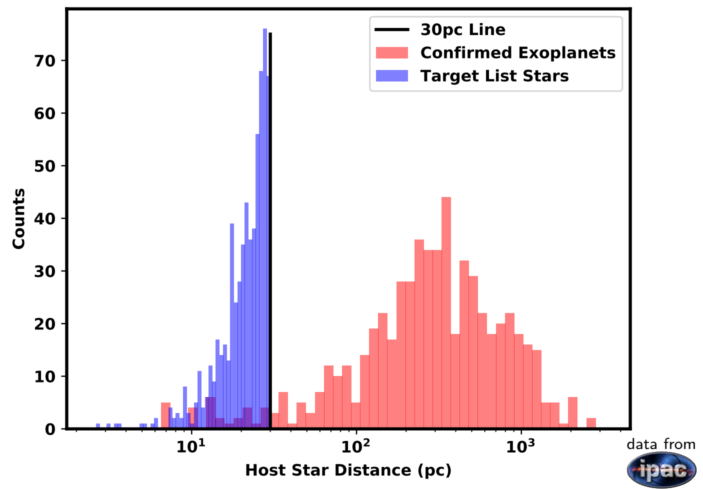
time

With thousands of possible target stars and
a limited amount of **time** on a future multi-billion dollar telescope
How do we maximize the number of new exoplanets detected?



1 pc $\approx$ 3.26156 ly

Maximizing Exoplanet Detections of a Direct Imaging Mission

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THE PALE BLUE DOT & BEYOND

Cornell University

SIOS LABORATORY

September 9, 2019

Outline

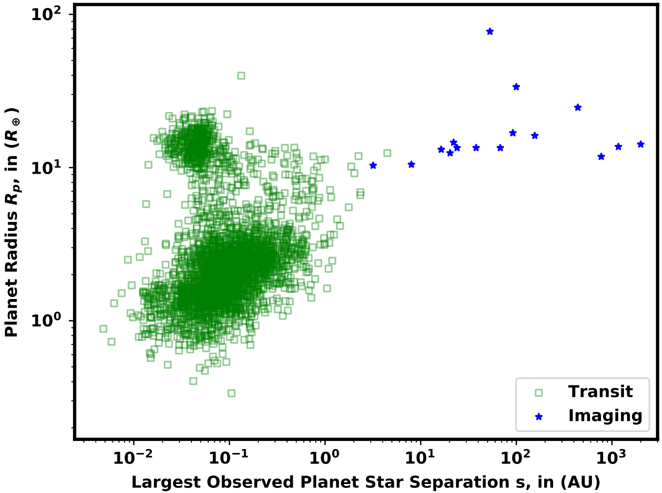
- Why Imaging?
- Δmag, s
- P(detection)

- Optimization

My contributions start here

- Validation
- WFIRST Results
- Future Work

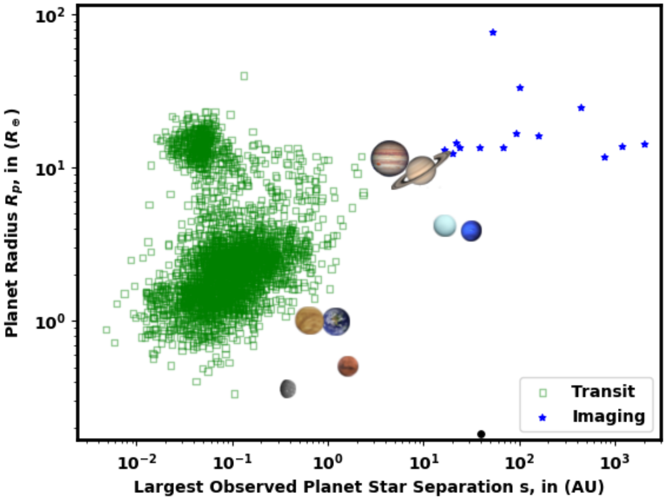
Confirmed Planets



- More transits (green) than other detections
- Imaging detects planets further fromthe host star

https://exoplanetarchive.ipac.caltech.edu/

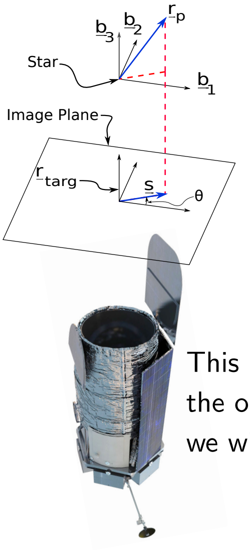
Confirmed and Solar System Planets



- More transits (green) than other detections
- Imaging detects planets further fromthe host star
- Imaging hasn't detected anything smaller than ≈Jupiter
- Earth-Like rocky bodies undiscovered

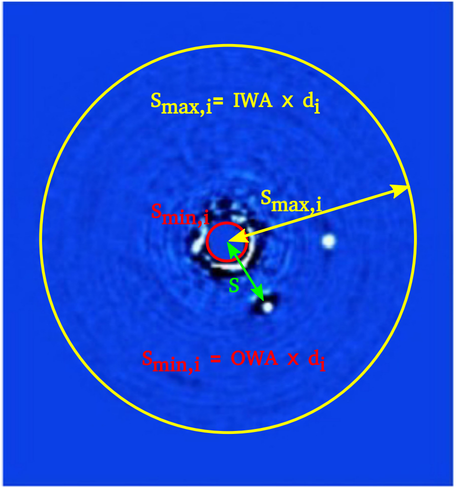
https://exoplanetarchive.ipac.caltech.edu/

Directly Imaging Exoplanets Geometry

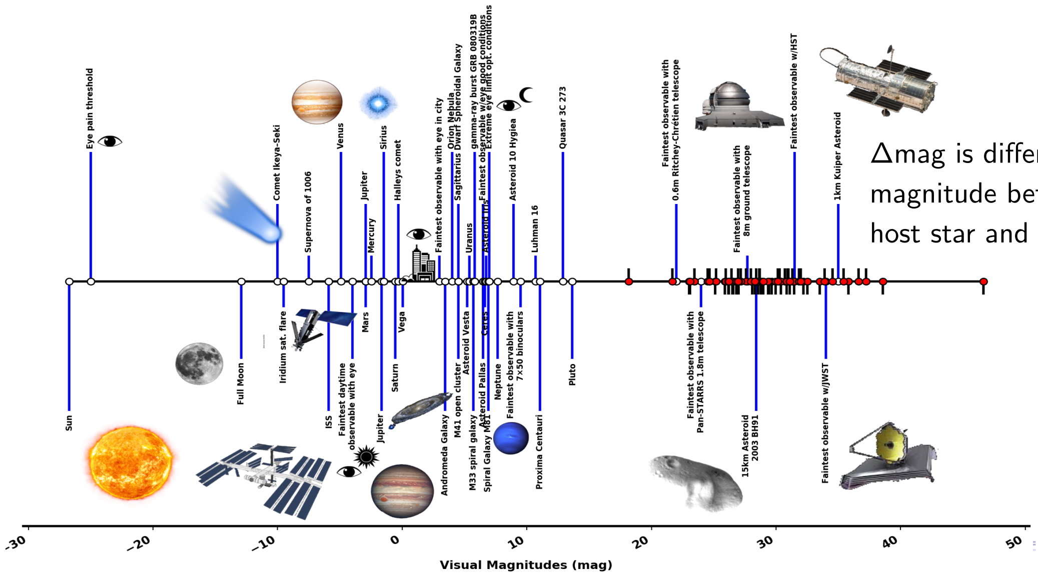


This is WFIRST; the observatory we will focus on.

(Marois et al. 2014)

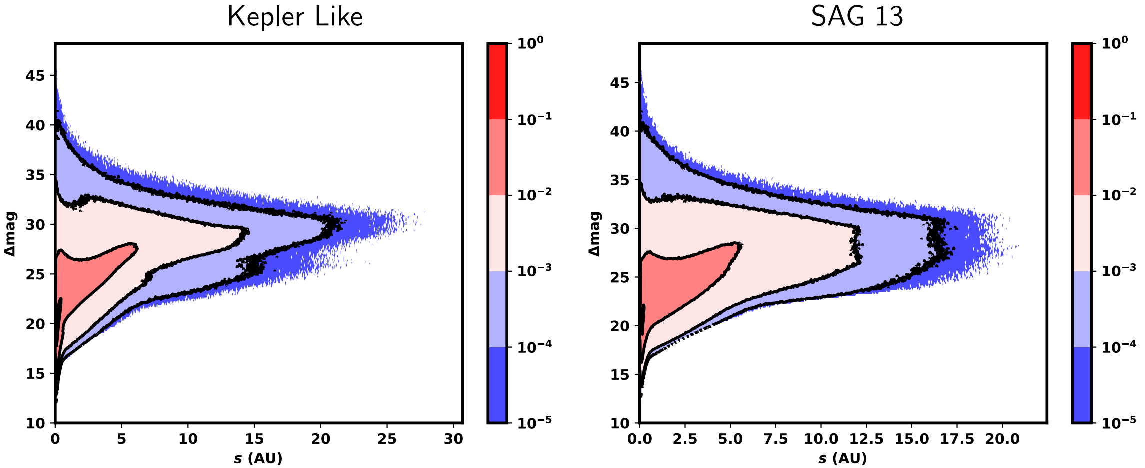


Visual Magnitude Reference



Δmag is difference in magnitude between the host star and exoplanet

Joint Probability Density Function, $f_{\bar{s},\overline{\Delta\mathrm{mag}}}(s, \Delta\mathrm{mag})$



Derived from (Garret et al., 2016)

Calculating P(detection), c_i

Completeness, c_i, is the probability of detecting an exoplanet around a host star should a planet exist around that star

$s_{max,i}$- largest planet-star separation observable

next slides - $\Delta\mathrm{mag}_i$

$$c_i = \int_0^{\Delta\mathrm{mag}_i} \int_{s_{\mathrm{min},i}}^{s_{\mathrm{max},i}} f_{\bar{s},\overline{\Delta\mathrm{mag}}}(s,\Delta\mathrm{mag})\, \mathrm{d}s\, \mathrm{d}\Delta\mathrm{mag}$$

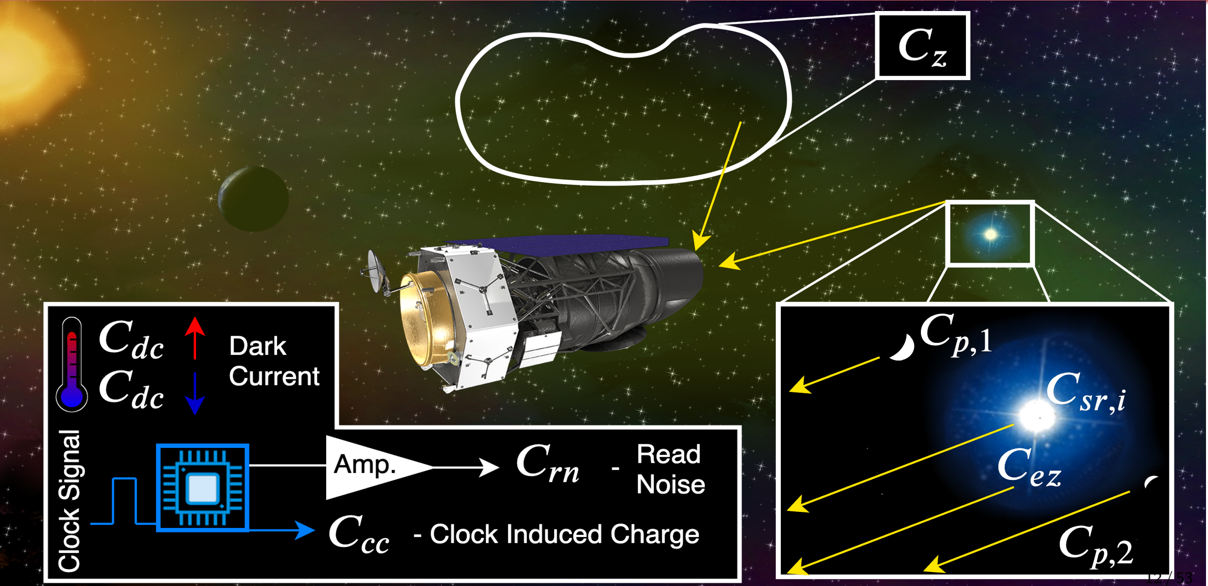
$s_{min,i}$- smallest planet-star separation observable

index of the target star - i

0- fundamental lower limit (for non-self-luminous)

(Savransky et al., 2017)

Photon Sources



C_z

$C_{p,1}$

$C_{sr,i}$

C_{ez}

$C_{p,2}$

C_{dc} ↑ Dark Current

C_{dc} ↓

Clock Signal

Amp. → C_{rn} - Read Noise

C_{cc} - Clock Induced Charge

Δmag_i and t_i

This is the background limiting Δmag

$$\Delta\text{mag}_i(t_i) = -2.5\log_{10}\frac{SNR\sqrt{\frac{C_{b,i}}{t_i}+C_{sp,i}^2}}{C_{\mathscr{F}_0}10^{-0.4v_i(\lambda)}T(\lambda,WA)\varepsilon_{PC}}$$

$$C_{b,i} = ENF^2\times(C_{sr,i}+C_{z,i}+C_{ez})+(ENF^2\times(C_{dc}+C_{cc})+C_{rn})$$

$$C_{sp,i} = C_{sr,i}\times\varepsilon_{pp}$$

$$C_{sr,i} = C_{\mathscr{F}_0}\times 10^{-0.4\times v_i}\times\Psi(\lambda,WA)\times N_{pix}$$

$$C_{\mathscr{F}_0}(\lambda) = \mathscr{F}_0(\lambda)A\Delta\lambda\varepsilon_q(\lambda)\varepsilon_{inst}\varepsilon_{syst}$$

- $\lambda = 565$nm wavelength
- $v_i(\lambda)$ - target star B-V color
- $SNR = 5$ - minimum required for detection
- $\varepsilon_{PC} = 0.8$ - photon counting efficiency
- $C_{b,i}$ - net background count rate
- $C_{sp,i}$ - speckle residual count rate
- $C_{sr,i}$ - starlight residual
- $C_{\mathscr{F}_0}$ - spectral flux density

(Nemati, 2014), (Nemati et al., 2017)

Reward & Cost

We now have a metric describing the "reward" for observing each star.

We related the "reward" to the time "cost" of making that observation.

How do I determine integration time for each star, i in $\mathbf{I}$ (t_i)?

Reward & Cost

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We have a set of time constraints to consider:

$T_{settling}$ - time reserved for vibration damping, reaching thermal equilibrium, “digging the dark hole” (0.5d)

T_{OH} - time reserved for momentum dumping, orbit maintenance, dark hole maintenance (0.5d)

T_{max} - total mission time (30d)

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T_{max} - total mission time (30d)

What is an initial feasible solution to the full non-linear optimization problem?

Optimization: Part 1

Algorithm 1: Binary Integer Program - $\mathbf{x}_1^* = \mathrm{BIP}(\mathbf{c}_0, \mathbf{t}_0)$

Input: $\mathbf{I}$, $\mathbf{c}_0$, $\mathbf{t}_0$, T_{OH}, $T_{settling}$, $T_{\max}$, and an optimization time limit maximum of 5 minutes

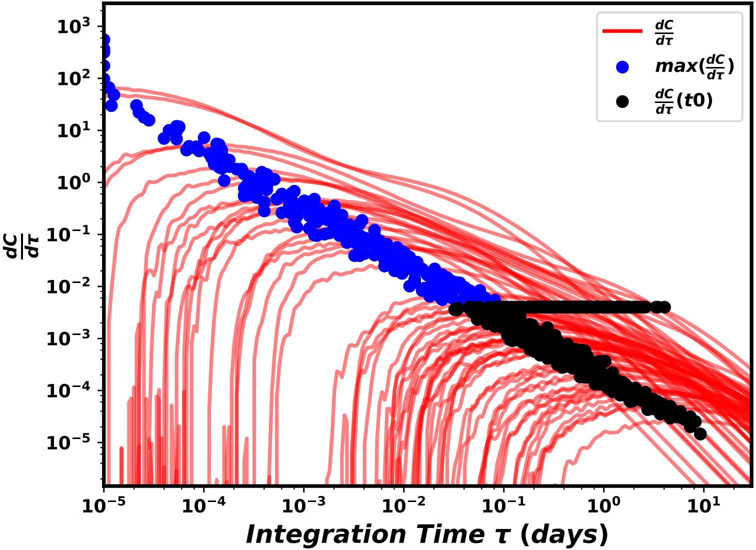
Output: $\mathbf{x}_1^*$, the list of binary values signaling to keep (1) or remove (0) each target

$$
\begin{aligned}
\mathbf{x}_1^* = \quad & \arg\min_{\mathbf{x}} \quad -\sum_{i=0}^{N-1} x_i c_{0,i} \\
& \text{s.t.} \\
& \sum_{i \in \mathbf{I}} x_i \left(t_{0,i} + T_{OH} + T_{settling}\right) \leq T_{max} \quad , \\
& \qquad\qquad\qquad x_i \in \{0,1\}, \qquad \forall\ i\ \in \mathbf{I}
\end{aligned}
$$

Coin-OR MIP - (Lougee-Heimer, 2003), (Savransky et al., 2017), (Keithly et al., 2019)

Initial t_i, Maximizing dc/dt

What is a good initial feasible solution to the full non-linear optimization problem?



Slope of Reward/Cost, $\varepsilon = \mathrm{d}c/\mathrm{d}t$

1. Assemble the expression and take the derivative w.r.t time

$$\varepsilon = \left.\frac{\mathrm{d}c_i}{\mathrm{d}t_i}\right|_{t_i} = \frac{\mathrm{d}}{\mathrm{d}t_i}\left.\left[\int_0^{\Delta\mathrm{mag}_i(t_i)}\int_{s_{\mathrm{min},i}}^{s_{\mathrm{max},i}} f_{\bar{s},\overline{\Delta\mathrm{mag}}}\left(s,\Delta\mathrm{mag}\right)\mathrm{d}s\,\mathrm{d}\Delta\mathrm{mag}\right]\right|_{t_i}$$

Slope of Reward/Cost, $\varepsilon = \mathrm{d}c/\mathrm{d}t$

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2. Multiply by $d\Delta\mathrm{mag}_i/d\Delta\mathrm{mag}_i$

$$= \left.\frac{\mathrm{d}}{\mathrm{d}\Delta\mathrm{mag}_i}\left[\int_0^{\Delta\mathrm{mag}_i(t_i)}\int_{s_{\mathrm{min},i}}^{s_{\mathrm{max},i}} f_{\bar{s},\overline{\Delta\mathrm{mag}}}(s,\Delta\mathrm{mag})\,\mathrm{d}s\,\mathrm{d}\Delta\mathrm{mag}\right]\frac{\mathrm{d}\Delta\mathrm{mag}_i}{\mathrm{d}t_i}\right|_{t_i}$$

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3. Apply Fundamental Theorem of Calculus

$$= \left[\int_{s_{\mathrm{min},i}}^{s_{\mathrm{max},i}} f_{\bar{s},\overline{\Delta\mathrm{mag}}}(s,\Delta\mathrm{mag}(t_i))\,\mathrm{d}s\right]\left.\frac{\mathrm{d}\Delta\mathrm{mag}_i}{\mathrm{d}t_i}\right|_{t_i}$$

Slope of Reward/Cost, $\varepsilon = \mathrm{d}c/\mathrm{d}t$

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2. Multiply by $d\Delta\mathrm{mag}_i/d\Delta\mathrm{mag}_i$

$$= \left.\frac{\mathrm{d}}{\mathrm{d}\Delta\mathrm{mag}_i}\left[\int_0^{\Delta\mathrm{mag}_i(t_i)}\int_{s_{\mathrm{min},i}}^{s_{\mathrm{max},i}} f_{\bar{s},\overline{\Delta\mathrm{mag}}}(s,\Delta\mathrm{mag})\,\mathrm{d}s\,\mathrm{d}\Delta\mathrm{mag}\right]\frac{\mathrm{d}\Delta\mathrm{mag}_i}{\mathrm{d}t_i}\right|_{t_i}$$

3. Apply Fundamental Theorem of Calculus

$$= \left.\left[\int_{s_{\mathrm{min},i}}^{s_{\mathrm{max},i}} f_{\bar{s},\overline{\Delta\mathrm{mag}}}(s,\Delta\mathrm{mag}(t_i))\,\mathrm{d}s\right]\frac{\mathrm{d}\Delta\mathrm{mag}_i}{\mathrm{d}t_i}\right|_{t_i}$$

$$\frac{\mathrm{d}\Delta\mathrm{mag}_i}{\mathrm{d}t_i}(t_i) = \frac{5C_{b,i}}{4\ln(10)}\frac{1}{C_{b,i}t_i+\left(C_{sp,i}t_i\right)^2}$$

From (Nemati, 2014)

Optimization: Part 2

```
Algorithm 2: Bounded Scalar Minimization Wrapping Binary Integer Program
Input: $\mathbf{I}$, $\mathbf{C}_{p0}$, $\mathbf{C}_{b0}$, $\mathbf{C}_{sp0}$, $T_{OH}$, $T_{settling}$, $T_{\max}$, and an optimization time limit
    maximum of 5 minutes
Output: $\varepsilon^*$, the value of $\mathrm{d}c/\mathrm{d}t$ evaluated for each target which maximizes yield
Output: $\mathbf{t}^*$, integration times for each target evaluated at $\varepsilon^*$
Output: $\mathbf{x}_2^*$, the list of binary values signaling to keep or remove each target

    $\varepsilon^* = \underset{\varepsilon}{\arg\min} \quad -\sum_{i \in \mathbf{I}} \mathrm{BIP}(c_i(t_i^*(\varepsilon)), t_i^*(\varepsilon), T_{OH}, T_{settling}, T_{\max})_i c_i(t_i^*(\varepsilon))$
        s.t.
          $\varepsilon \leq 7,$
          $-\varepsilon \leq 0$

  $\mathbf{t}_2^* \Leftarrow [t_i^*(\varepsilon^*), \ \forall \ i \ \in \ \mathbf{I}]$
  $\mathbf{x}_2^* \Leftarrow [\mathrm{BIP}(c_i(t_i^*(\varepsilon^*)), t_i^*(\varepsilon^*), T_{OH}, T_{settling}, T_{\max}), \ \forall \ i \ \in \ \mathbf{I}]$
```

(Savransky et al., 2017), (Keithly et al., 2019)

Optimization: Part 3

Algorithm 3: SLSQP Optimization

Input: $\mathbf{I}$, $\mathbf{f}_Z$, $\mathbf{t}$, T_{OH}, $T_{settling}$ and $T_{\max}$

Output: $\mathbf{t}_3^*$, the integration times to spend on each star

$$\mathbf{t}^* = \underset{\mathbf{t}}{\arg\min} \quad -\sum_{i=0}^{N-1} c_i(t_i)$$

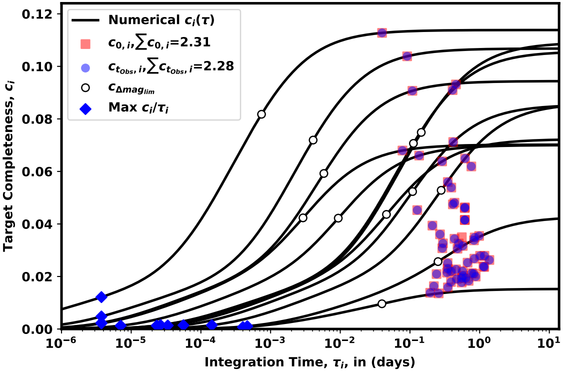
s.t.

$$\begin{aligned} t_i &< T_{\max}, \quad \forall\, i \in \mathbf{I}, \\ -t_i &< 0, \quad \forall\, i \in \mathbf{I}, \\ \sum_{i\in\mathbf{I}} x_i(T_{OH}+T_{settling})+t_i &< T_{\max} \end{aligned}$$

Scipy SLSQP - (Boggs and Tolle, 1995), (Savransky et al., 2017), (Keithly et al., 2019)

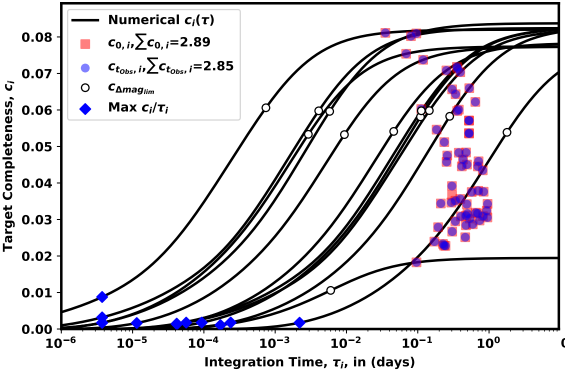
Completeness vs Integration Time - Kepler Like Planet Population

Kepler Like



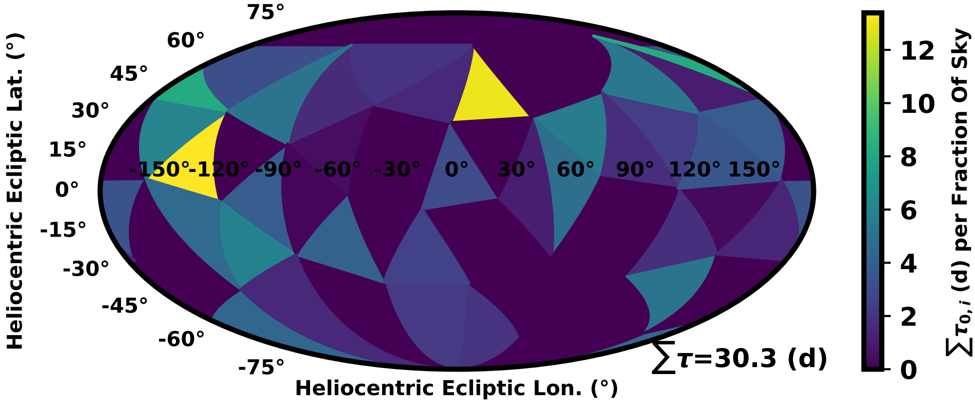
59/60 observations made

SAG13

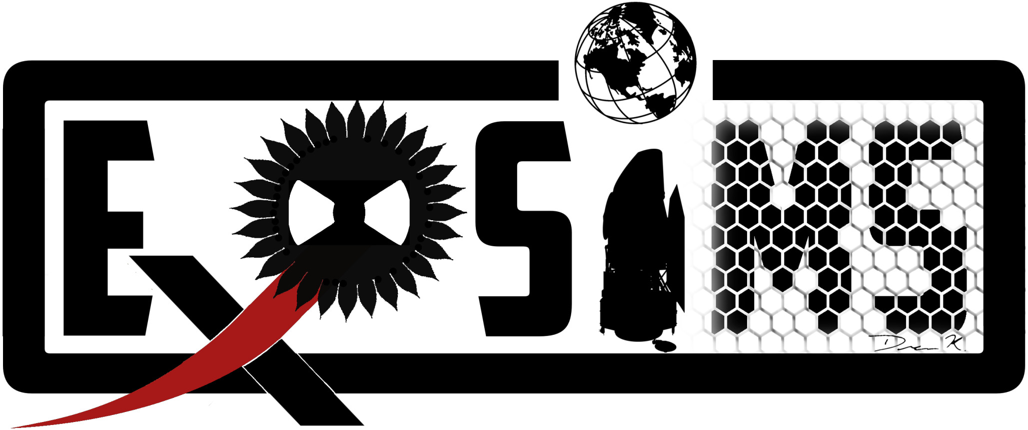


63/64 observations made

Sky Distribution of Target List Time

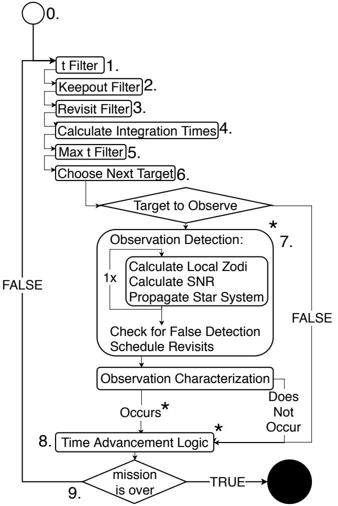


EXOSIMS



Simulating Single Missions

0. Start
1. Filter targets with large t_i
2. Filter targets in keepout region
3. Filter previously visited targets
4. Calculate integration times
5. Filter targets not observable in remaining mission time
6. Choose the next target or choose to wait
7. Perform detection
8. Advance mission time
9. Check mission termination criteria


0.
t Filter 1.
Keepout Filter 2.
Revisit Filter 3.
Calculate Integration Times 4.
Max t Filter 5.
Choose Next Target 6.
Target to Observe
Observation Detection:
1x
Calculate Local Zodi
Calculate SNR
Propagate Star System
Check for False Detection
Schedule Revisits
* 7.
FALSE
FALSE
Observation Characterization
Occurs*
Does Not Occur
*
8. Time Advancement Logic
mission is over
9.
TRUE

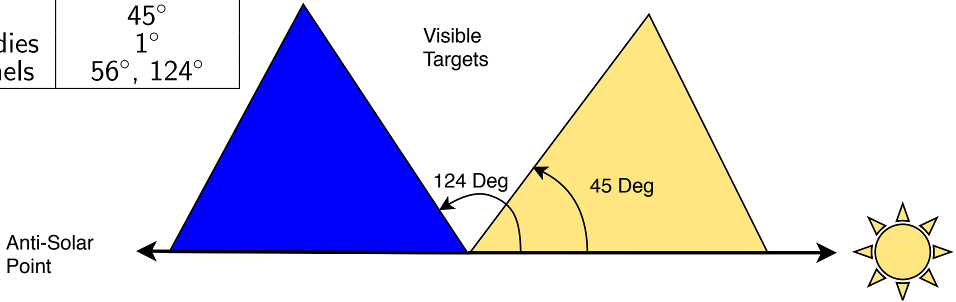
* Denotes an operation where mission time is advanced

Keep-out Regions

Problem: Sensors saturate when looking at bright objects

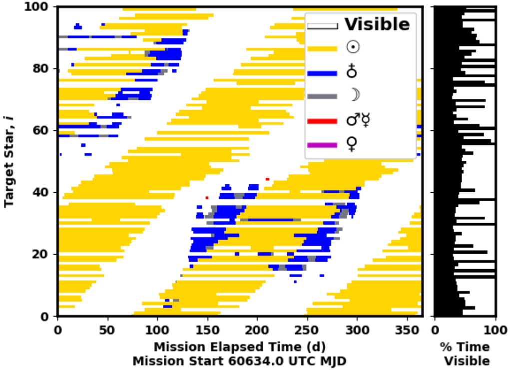
Solution: Designate regions the telescope is not allowed to look at

Body	**Keep-out Angle** (deg)
Earth	45°
Moon	45°
Sun	45°
Small Bodies	1°
Solar Panels	56°, 124°



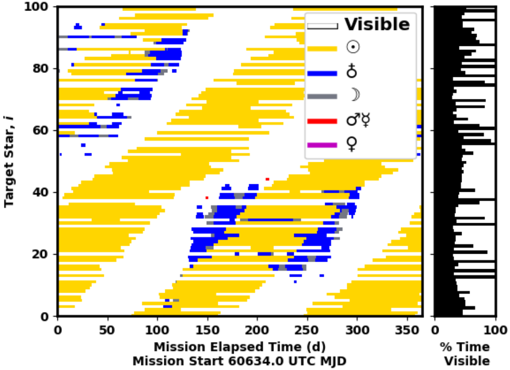
(WFIRST SDT, 2015)

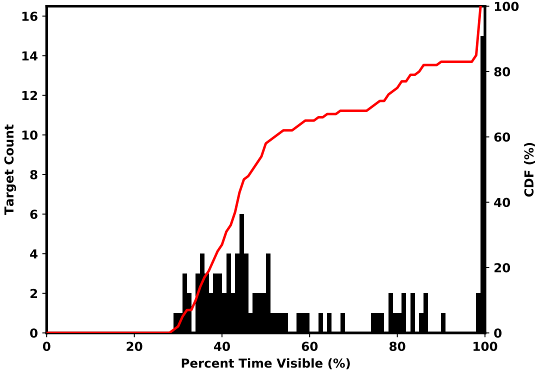
Keep-out Map



(Soto et al., 2019)

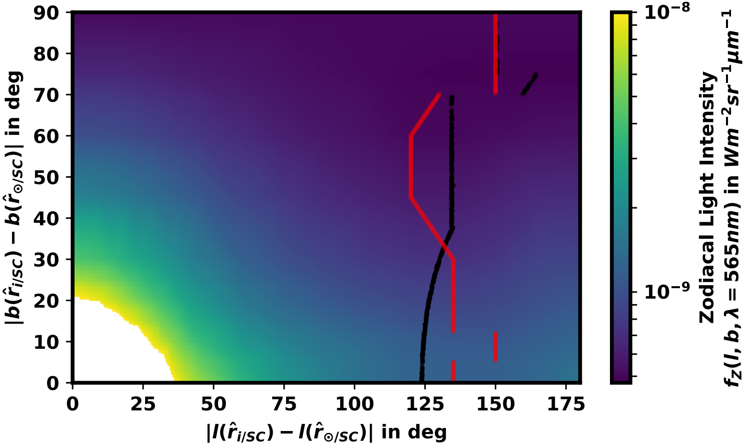
Keep-out Map





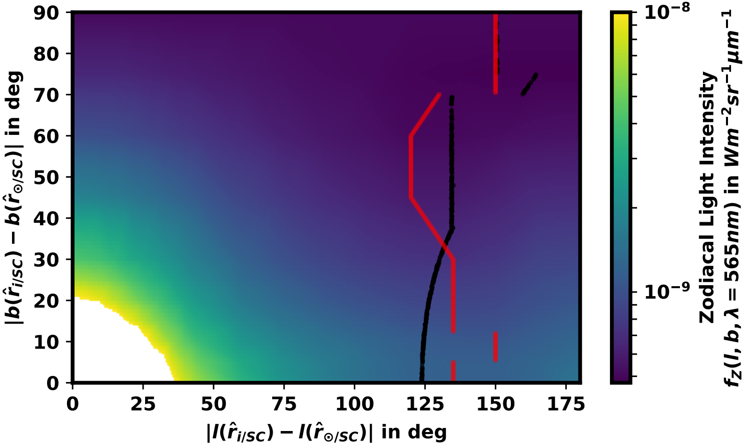
(Soto et al., 2019)

Zodiacal Light



(Leinert et al., 1998)

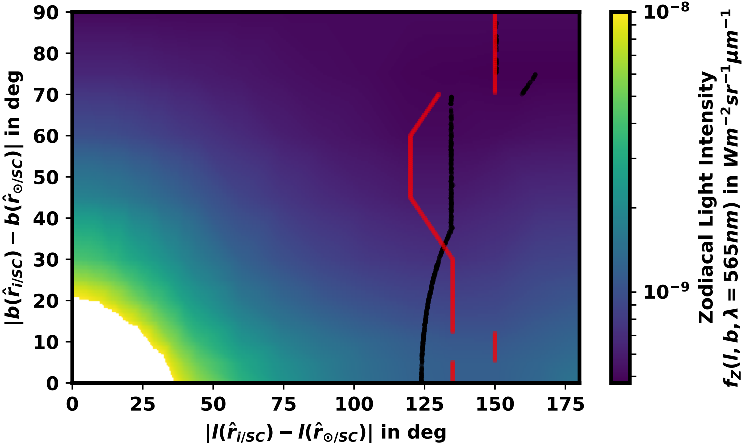
Zodiacal Light



- Red dots - linear interpolant minimums for each latitude
- 15d deviation from minimum has marginal value change

(Leinert et al., 1998)

Zodiacal Light



- Red dots - linear interpolant minimums for each latitude
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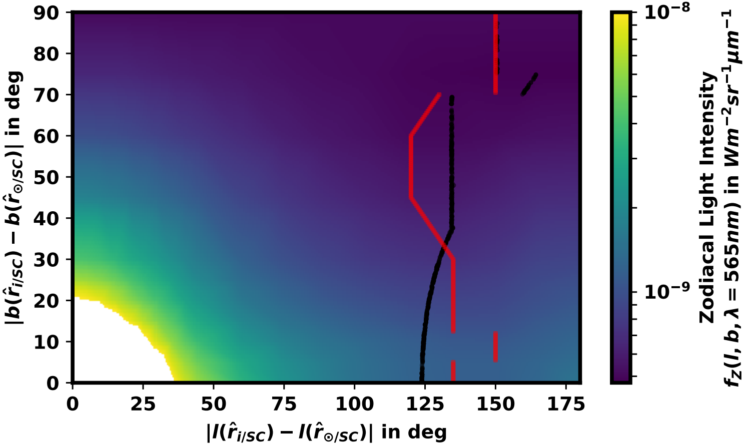
  Idea! lets make observations at minimums!

(Leinert et al., 1998)

Zodiacal Light

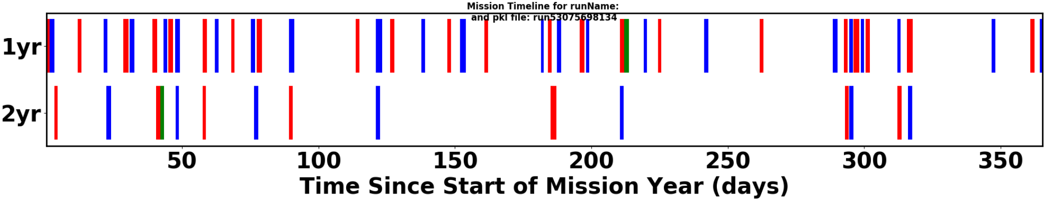


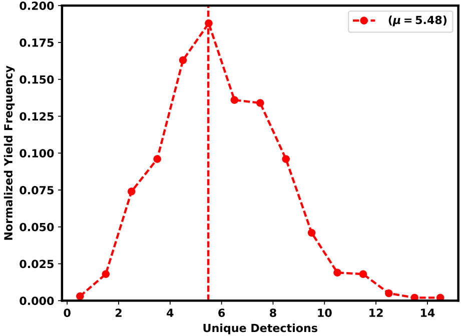
- Red dots - linear interpolant minimums for each latitude
- 15d deviation from minimum has marginal value change

  Idea! lets make observations at minimums!
- Black dots - implemented observations

(Leinert et al., 1998)

A Resulting Schedule



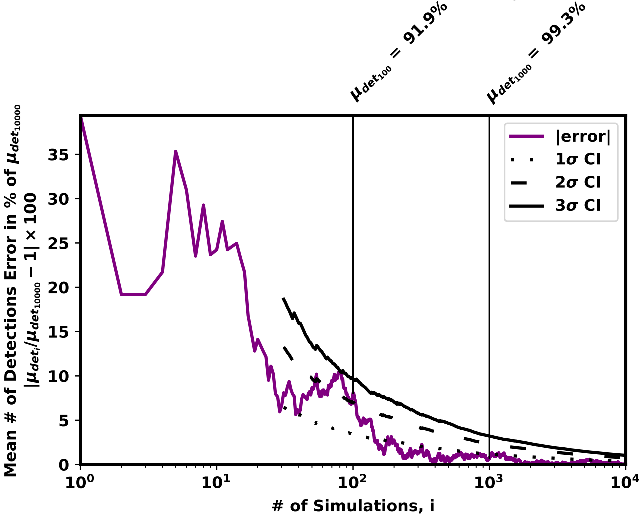


red and blue lines alternate

(Used to emphasize different observations)

Convergence

How many simulations do I need to run? (to get within XX% of the mean)

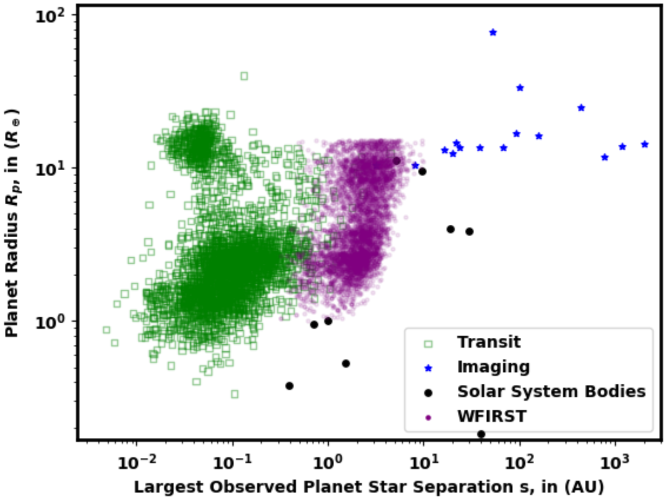


$$\mu_i = \frac{(i-1)\mu_{i-1} + x_i}{i}$$

(Savransky et al. 2015)

# Sims	CI	|%| error
1000	1σ	1.16
1000	2σ	2.33
1000	3σ	3.19
100	1σ	3.45
100	2σ	6.95
100	3σ	9.58

Confirmed with WFIRST Detected Planets



- All planets detected in **WFIRST** all simulations (purple)
- WFIRST might detect $\approx 2R_\oplus$ planets
- WFIRST is likely to detect planets with $0.5\text{AU} \lesssim s \lesssim 5\text{AU}$

https://exoplanetarchive.ipac.caltech.edu/

Mean Unique Detections

Unique Detections

	Planet Population	
Completeness	Kepler Like	SAG 13
Kepler Like	5.484	
SAG13		

The summed completeness of the planned observation list was 2.31.

Multiplying by the planet occurrence rate (2.375) predicts 5.48 detections will be made.

Mean Unique Detections

Unique Detections

	Planet Population	
Completeness	Kepler Like	SAG 13
Kepler Like	5.484	
SAG13		16.266

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Mean Unique Detections

Unique Detections

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Kepler Like	5.484	16.117
SAG13	5.206	16.266

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Mean Unique Detections

Unique Detections

	Planet Population	
Completeness	Kepler Like	SAG 13
Kepler Like	5.484	16.117
SAG13	5.206	16.266

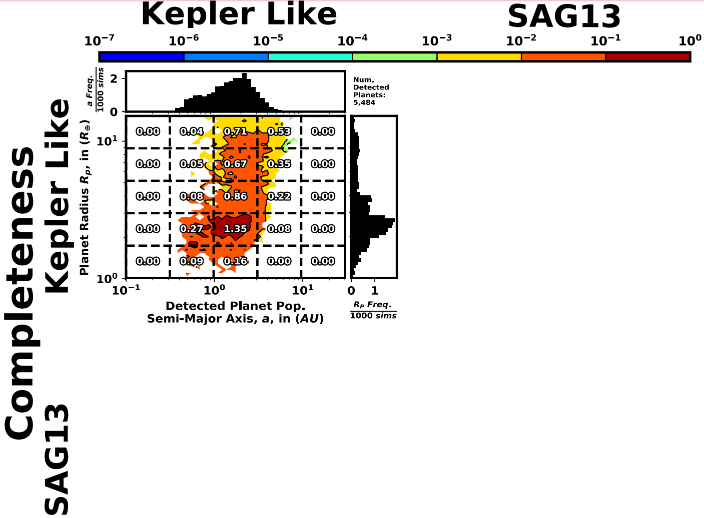
Characterizations

	Planet Population	
Completeness	Kepler Like	SAG 13
Kepler Like	0.214	1.003
SAG13	0.217	0.718

The summed completeness of the planned observation list was 2.31.

Multiplying by the planet occurrence rate (2.375) predicts 5.48 detections will be made.

Observed Planet Populations

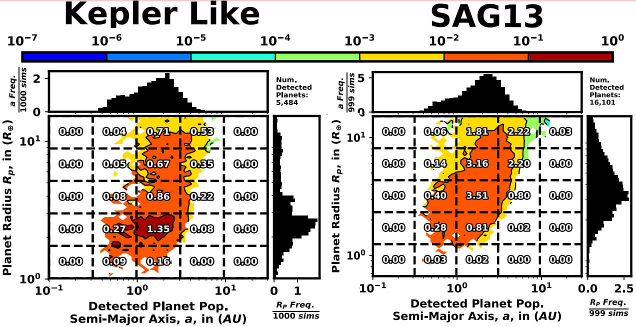


Observed Planet Populations

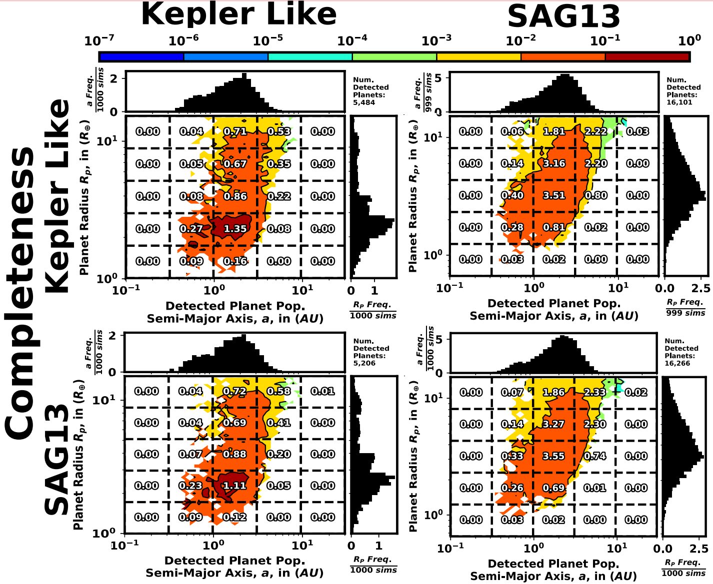
Completeness

Kepler Like

SAG13



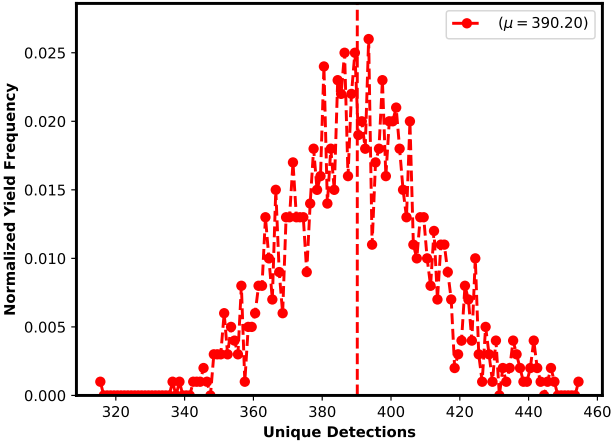
Observed Planet Populations



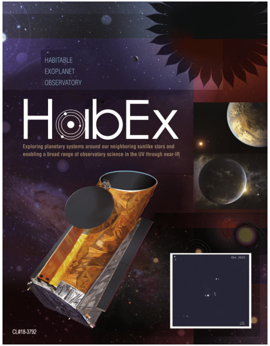
Summary

1. Target list optimization method
2. $\sum c$ planned $\approx \sum c$ implemented
3. Target t_i distribution on sky is uneven
4. WFIRST can detect unique planets in R_p vs a space
5. EXOSIMS simulates universes, validates the planned target list
6. Optimizing with Kepler-Like population is preferred
7. Optimizing with Kepler-Like leads to detections of smaller R_p planets
8. WFIRST can detect planets in the regime between "imaging" and "transits"
9. Running 1000x simulations → ≈3% uncertainty
10. WFIRST can detect ≈5.48 exoplanets in a blind-search survey

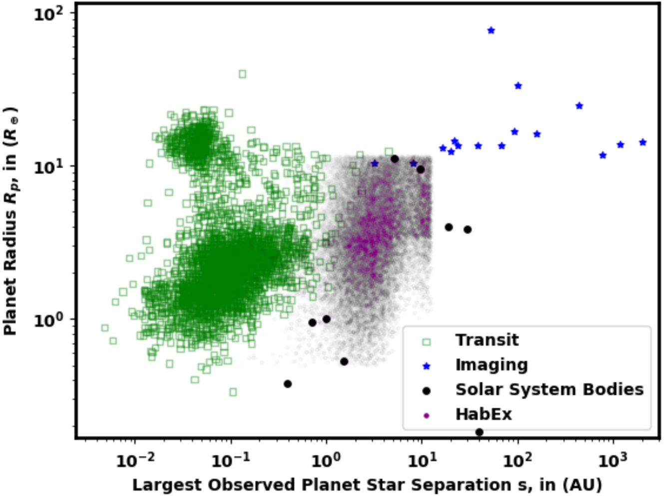
HabEx



- HabEx is one of 4 future flagship telescope concepts
- Designed to image exo-Earths



Confirmed with HabEx Detected Planets



- All planets detected in **HabEx** simulations (purple)
- HabEx might detect $\approx R_\oplus$ at 1AU
- "lines" are limits of simulated planets

https://exoplanetarchive.ipac.caltech.edu/

Future Work

Detecting and Characterizing Earth-Like Exoplanets, Revisiting targets, Characterizing Orbits

- Dynamic program rewarding only confirmed & characterized Earth-Like planets
- $P(planet\ \ type|s_0, \Delta\mathrm{mag}_0)$ - what is the probability a detected planet is of a given planet type?
- $P(s_1, \Delta\mathrm{mag}_1, \theta_1|planet\ \ type, MET + \Delta t, s_0, \Delta\mathrm{mag}_0)$ - when is the earliest I can take my next image?
- simulating stable star systems
- Decompose completeness by planet-type
- Decompose dynamic-completeness by planet type

Contributions

Journal Publications:

[1] Keithly D., et al., (2018) "A cephalopod-inspired combustion powered hydro-jet engine using soft actuators." Extreme Mechanics Letters.

[2] Keithly D., et al., (In Review) "Optimal Scheduling of Exoplanet Direct Imaging Single-Visit Observations of a Blind Search Survey." Journal of Astronomical Telescopes, Instruments, and Systems.

Conference Presentations:

[1] Keithly D., et al., (2019) "Blind Search Single-Visit Exoplanet Direct Imaging Yield for Space Based Telescopes." American Astronomical Society Meeting 233.

[2] Keithly D., et al., (2018) "Scheduling and target selection optimization for exoplanet imaging spacecraft." International Society for Optics and Photonics.

[3] Keithly D., et al., (2018) "WFIRST: Exoplanet Target Selection and Scheduling with Greedy Optimization." American Astronomical Society Meeting 231.

Code Contribution: github.com/dsavransky/EXOSIMS

Report: Savransky et al., (2019) "Modular Active Self-Assembling Space Telescope Swarms," NIAC - Future conference paper (Mirro Force Opt.)

Coursework

Completed:

SYSEN 5400 - System Architecture
SYSEN 5100 - Model Based Systems Engineering
MAE 5160 - Spacecraft Technology & Systems
MAE 6060 - Spacecraft Dynamics, Estimation, & Control
SYSEN 5200 - Analysis Behavior & Optimization
MAE 5730 - Intermediate Dynamics & Vibrations
MAE 5780 - Feedback Control Systems
MAE 6700 - Advanced Dynamics
ASTRO 6525 - Optical, Infrared, and Sub-millimeter Telescopes
MAE 6720 - Celestial Mechanics
ORIE 6125 - Computational Methods in Operation Research
ORIE 5300 - Optimization I
ORIE 5310 - Optimization II

Future:

Multivariable Control
Celestial Mechanics
Global Positioning System

Super Awesome Side-Work

Internships:

Marshall Space Flight Center	2015	
Jet Propulsion Lab	2016	(Atkinson et al., 2016)
Jet Propulsion Lab	2017	Lander Launched Impact Probe - Future Conference
Jet Propulsion Lab	2018	Procedural Thermal Model Generation
Air Force Research Lab	2019	Valuing Ground Station Images - Future Conference
Ball Aerospace?	2020	GOAL

Extra:

SPLASH	2018-19	Teaching Space Classes (obviously)
FIRST	2018	FRC 5254 Trumansburg

Acknowledgements

- NASA's NAIF planetary data system kernels.
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- Astropy (Astropy Collaboration, 2018)
- OR-Tools, an optimization utility package made by Google Inc. with community support.
- Imaging Mission Database (IMD), which is operated by the Space Imaging and Optical Systems Lab at Cornell University.
- NASA Exoplanet Archive, operated by California Institute of Technology, under contract with the National Aeronautics and Space Administration under the Exoplanet Exploration Program, and from the SIMBAD database, operated at CDS, Strasbourg, France.
- EXOSIMS contributors: Christian Delacroix, Daniel Garrett, Dean Keithly, Gabriel Soto, and Dmitry Savransky, with contributions by Rhonda Morgan, Michael Turmon, Walker Dula, Patrick Lowrance, and Neil Zimmerman

Constructing Joint Probability Distributions: Kepler Like

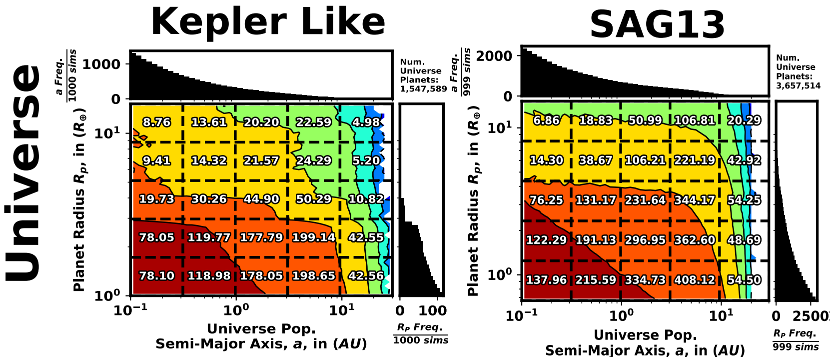
$$f_{\bar{a}}(a) = \frac{a^{-0.62}}{a_{\mathrm{norm}}} \exp\left(-\frac{a^2}{a_{\mathrm{knee}}^2}\right)$$

$$a_{\mathrm{norm}} = \int_{a_{\mathrm{min}}}^{a_{\mathrm{max}}} a^{-0.62} \exp\left(-\frac{a^2}{a_{\mathrm{knee}}^2}\right) \mathrm{d}a$$

$$a_{\mathrm{min}} = 0.1 \text{ AU}$$

$$a_{\mathrm{max}} = 30 \text{ AU}$$

Constructing Joint Probability Distributions: Kepler Like



Nemati 2014 SNR Equation

$$SNR = \frac{r_{pl}\,t}{\sqrt{r_{noise}\,t + \sigma_{spstr}^2}}$$

r_{pl} - Electron count rate from the planet
r_{noise} - noise "rate" from planet, speckle, zodi, exo-zodi, DC, CIC, RN
σ_{spstr} - variance of the residual speckle structure
ENF - Excess Noise Factor caused by signal gain

WFIRST Optics: Shaped pupil coronagraph

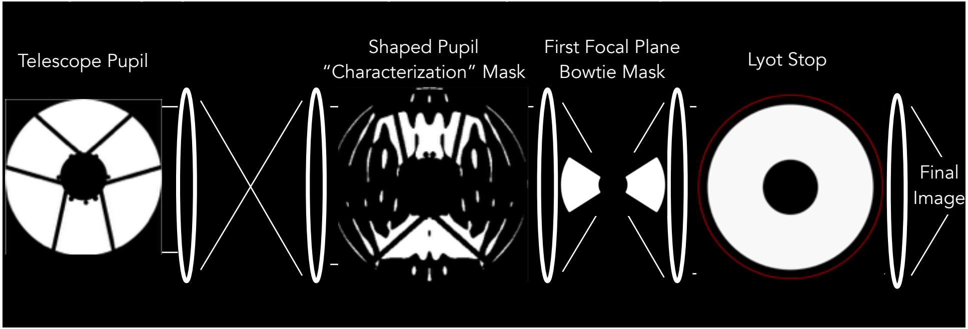
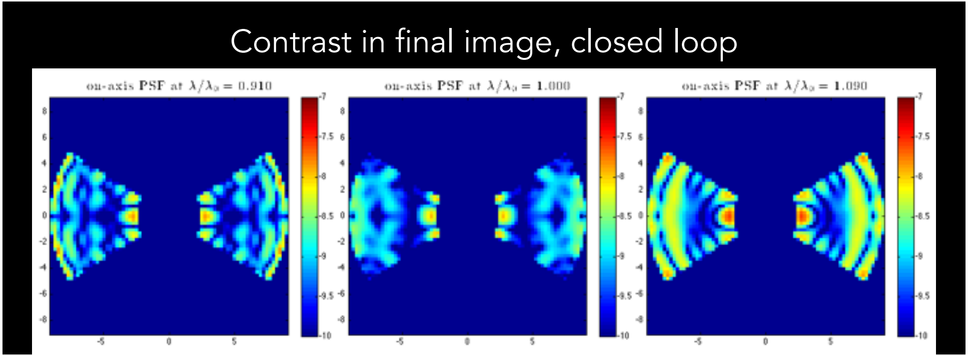


Image Credit: Jeremy Kasdin 2014

WFIRST Optics: Final image contrast


Contrast in final image, closed loop

on-axis PSF at $\lambda/\lambda_0 = 0.910$

on-axis PSF at $\lambda/\lambda_0 = 1.000$

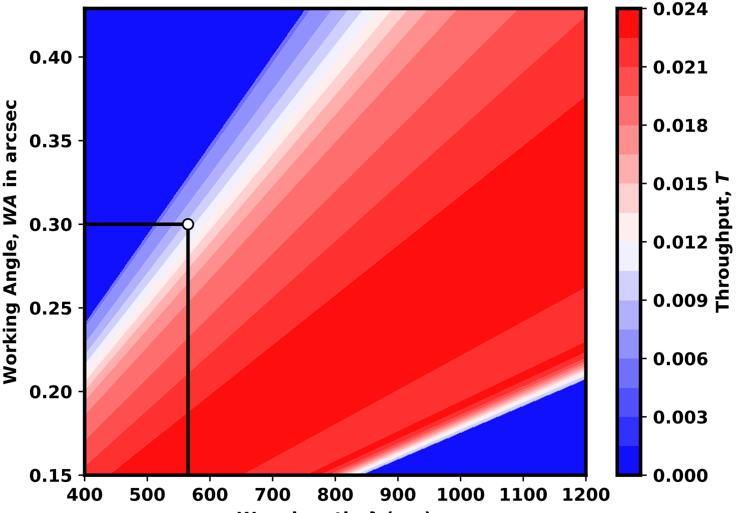
on-axis PSF at $\lambda/\lambda_0 = 1.090$

Image Credit: Jeremy Kasdin 2014

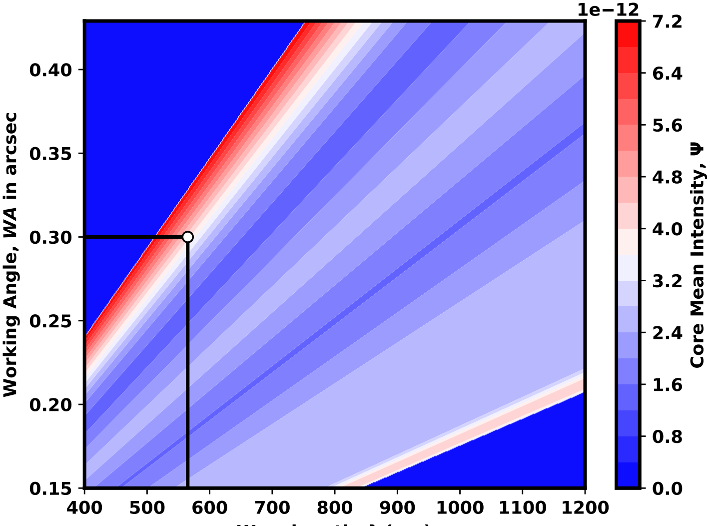
Zero-Magnitude Flux $C_{\mathscr{F}_0}$

$$\mathscr{F}_0(\lambda) = 10^4 \times 10^{\left(4.01 - \frac{\lambda - 550\mathrm{nm}}{770\mathrm{nm}}\right)} ph/s/m^2/nm$$

Core Throughput



Core Mean Intensity



Transit Detection Diagram

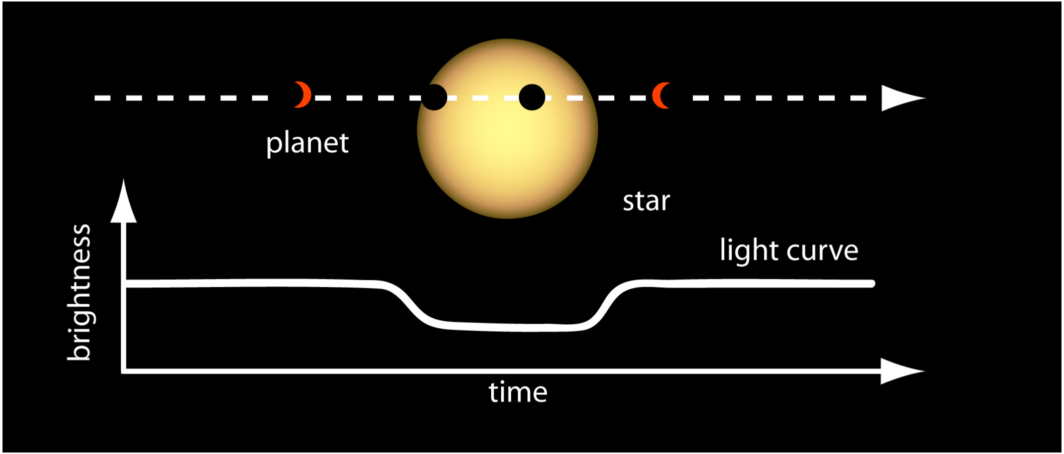


Image courtesy of NASA

Planet Occurrence Rates

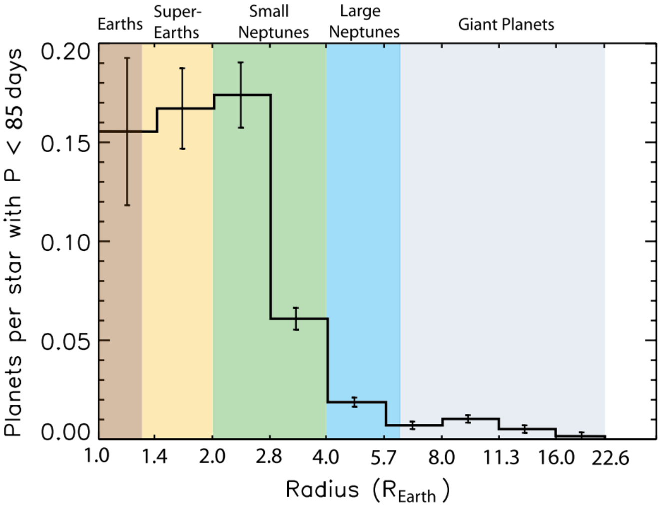


Figure 7. Average number of planets per size bin for main-sequence FGKM stars, determined here from the Q1–Q6 *Kepler* data and corrected for false

Orbital Elements to **r**

ω	argument of perigee
I	Inclination of the orbital plane
O	Right ascension of the ascending node

Table: Caption

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = \begin{bmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos I & \sin I \\ 0 & -\sin I & \cos I \end{bmatrix} \cdot \begin{bmatrix} \cos O & \sin O & 0 \\ -\sin O & \cos O & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

After expanding

Planet Semi-major axis

Kepler-Like: Modified power-law distribution for semi-major axis (a) of the form

$$f_{\bar{a}}(a) = \frac{a^{-0.62}}{a_{\mathrm{norm}}} \exp\left(-\frac{a^2}{a_{\mathrm{knee}}^2}\right) \tag{1}$$

where -0.62 is adopted from refnumMoorhead2011 derived from the power law fit from refnumCumming2008.

In this model, we include an exponential decline in semi-major axis past a “semi-major axis knee” (a_{knee}), which we place at 10 AU, based on the observed, sharp decline in detected planets with period $\approx 10^4$ d around an assumed solar mass star (Cumming et. al. 2008). The normalization factor is given by the integrating the un-normalized distribution over a specific a range

$$a_{\mathrm{norm}} = \int_{a_{\mathrm{min}}}^{a_{\mathrm{max}}} a^{-0.62} \exp\left(-\frac{a^2}{a_{\mathrm{knee}}^2}\right) \mathrm{d}a\,, \tag{2}$$

where we consider values of a range in $a_{\mathrm{min}} = 0.1$ AU to $a_{\mathrm{max}} = 30$ AU, again based on the paucity of wide-separation planets discovered to date.

We note, however, that for WFIRST, which has an inner working angle (IWA) of 0.15 arcsec, the closest target list star has distance, d_i, of 2.63 pc and would have the

McBride2011

1. The goal of direct detection is to spatially separate the exo- planet light from that of its primary. This affords access to exo- planet atmospheres, which yields fundamental information including effective temperature, gravity, atmospheric composi- tion and abundances, orbital motion, and perhaps even weather and and planetary spin. 2. The goal of direct imaging is to assem- ble the first statistically significant sample of exoplanets that probes beyond the reach of indirect searches and quantifies the abundance of solar systems like our own. (McBride et al., 2011)Read Section 1 of McBride2011 for all other scientific motivation.

Significance of Orbital Eccentricity

A significant orbital eccentricity effects a planet's climate (i.e. equilibrium temperature, amplitude of seasonal variability and potentially its habitability due to variations in the incident stellar flux) (Moorhead et al., 2011) From Williams and Pollard 2002, Gaidos and Williams 2004

dMag vs s of Different Solar System Planets

EXTRA