Using Modern Mathematical and Computational Tools for Image Processing

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B-EXAM

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Overview of Presentation

Identifying Exoplanets in Direct Imaging Data with Common Spatial Pattern Filtering

- “Blind source separation algorithms for PSF subtraction from direct imaging.” Poster Presentation. AAS 2017.
- “Planet signal extraction from direct imaging using common spatial pattern filtering.**”** Oral Presentation. SPIE Optics and Photonics, 2017.
- “Common spatial pattern filtering for detection of circumstellar discs.**”** Poster Presentation. SPIE Telescopes and Instrumentation, 2018
- Shapiro, J., Savransky, D., Ruffio, J.B., Ranganathan, N., and Macintosh, B. **Detecting Planets from Direct Imaging Observations Using Common Spatial Pattern Filtering.** *The Astronomical Journal*. (2019).
- “Identifying Exoplanets with CSP Filtering and a Forward Model Matched Filter.” Oral Presentation. AAS #235, 2020.
- Shapiro, J., and Savransky, D. **Statistical Properties of the Common Spatial Pattern Filtering with a Forward Model Matched Filter technique for Direct Imaging.** *The Astronomical Journal. (In Prep).*

Optical Design of a Large, Segmented, Space Telescope

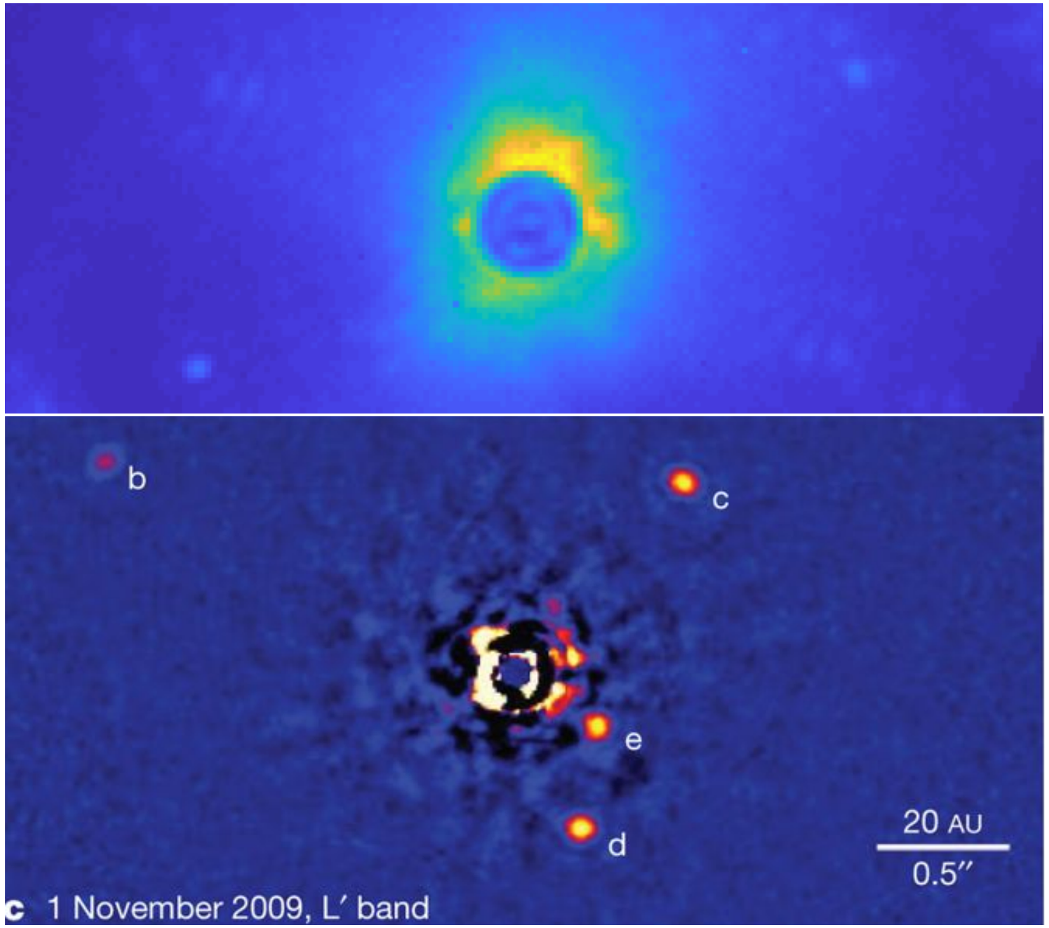
- “Optical design of a large segmented space telescope.” Poster Presentation. AAS 2019.
- “Optical design of a modular segmented space telescope.” Oral presentation. SPIE Optics and Photonics 2019
- NASA NIAC Report: https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20190018062.pdf

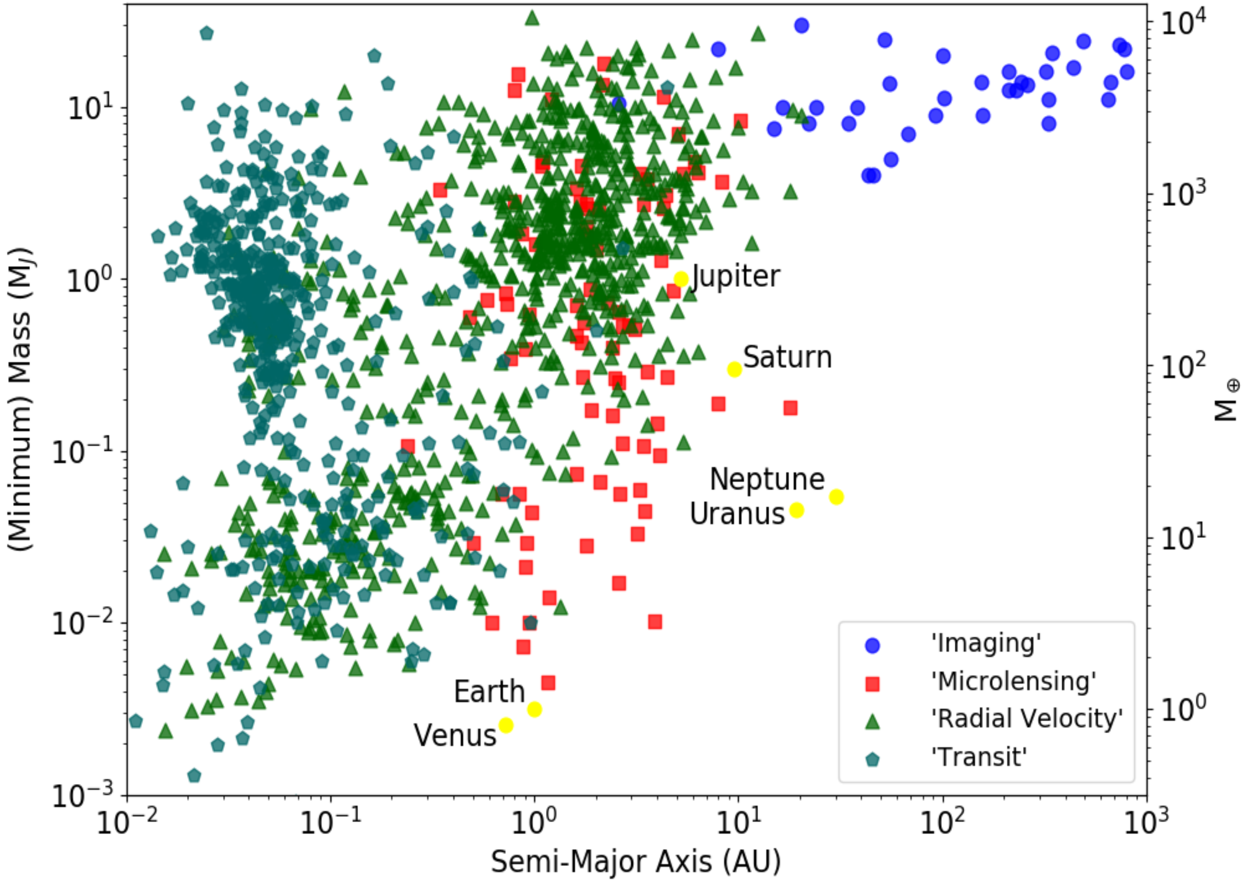
Satellite Imagery Calibration via Dynamic Filtering

Direct Imaging

POST-PROCESSING WITH COMMON SPATIAL PATTERN FILTERING

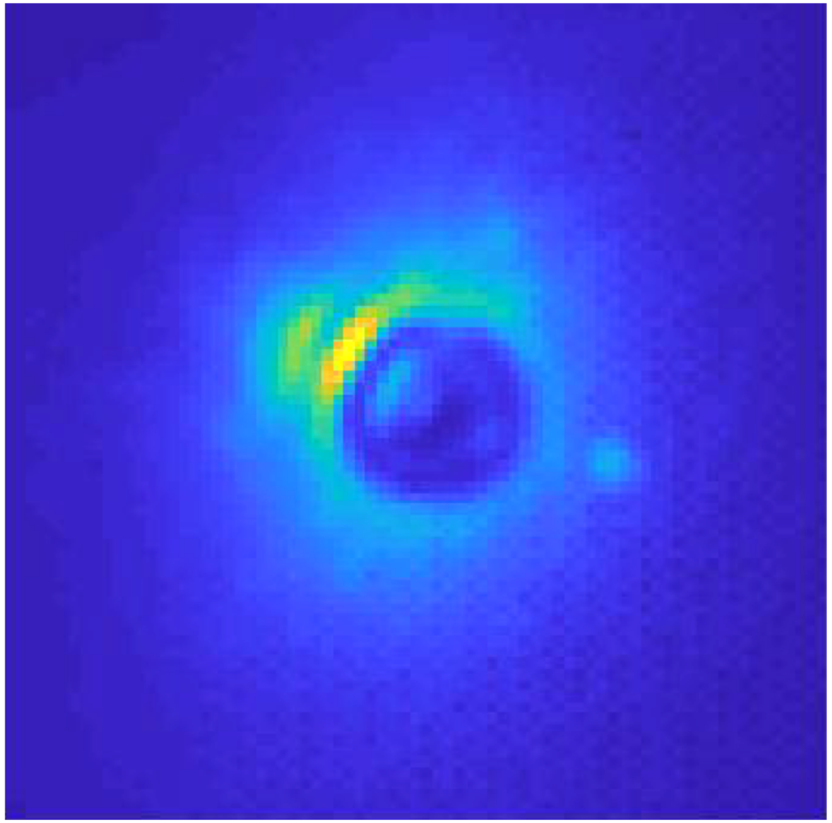
Direct Imaging Background: Overview



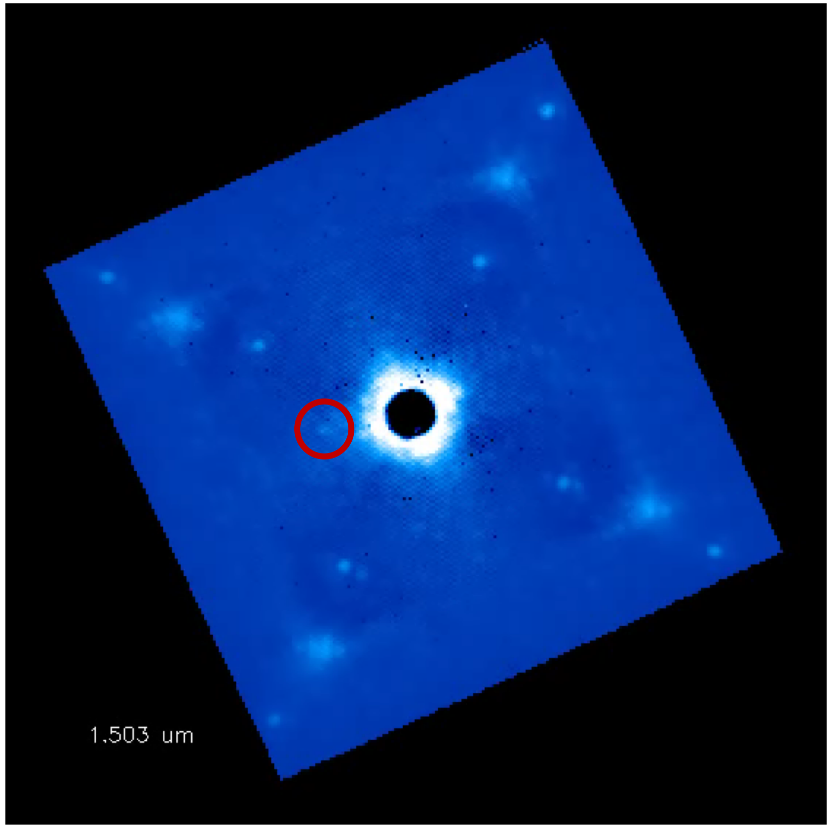


Direct Imaging Background: Spatial Diversity of the Planet Signal

Temporal



Wavelength



Technical Approach: PCA and CSP

Principal Component Analysis

(Rao & Yip, 2000)

$$\arg\max_{\mathbf{w}} \left\|\mathbf{X}^T\mathbf{w}\right\|^2 \quad \text{s.t.} \quad \|\mathbf{w}\| = 1$$

- Finds direction of maximum variance
- Models the noise
- Subtracts the noise

Common Spatial Pattern Filtering

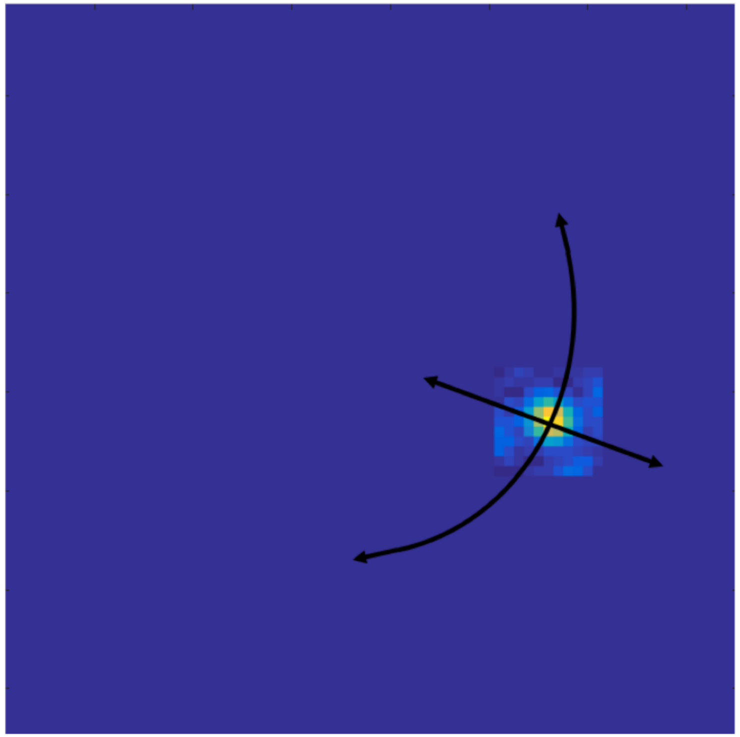
(Ang et al, 2008)

$$\arg\max_{\mathbf{w}} \frac{\left\|\mathbf{X}_1^T\mathbf{w}\right\|^2}{\left\|\mathbf{X}_2^T\mathbf{w}\right\|^2} \quad \text{s.t.} \quad \|\mathbf{w}\| = 1$$

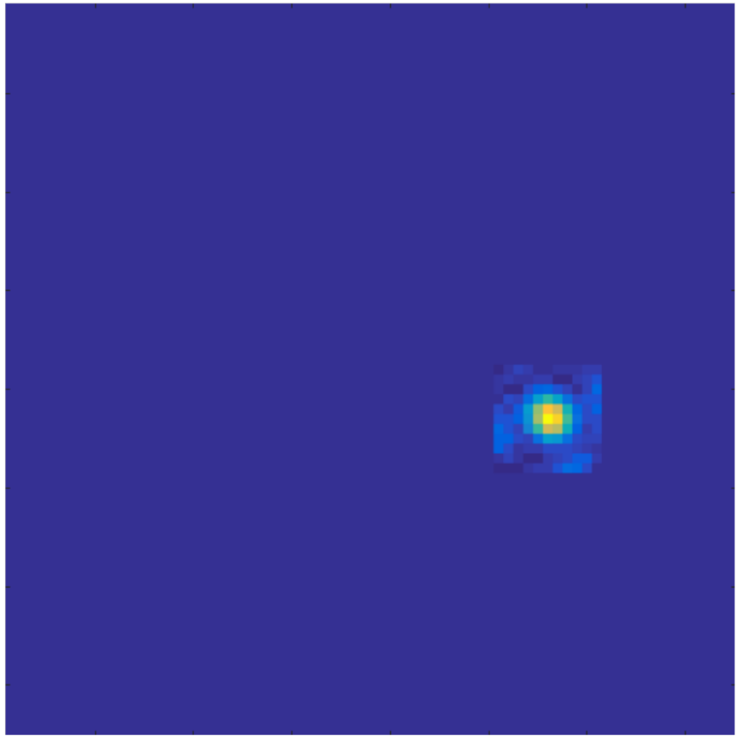
- Finds direction of maximum difference
- Planet should be part of the "difference"
- Models the planet signal

Technical Approach: Implementation

Dataset 1:



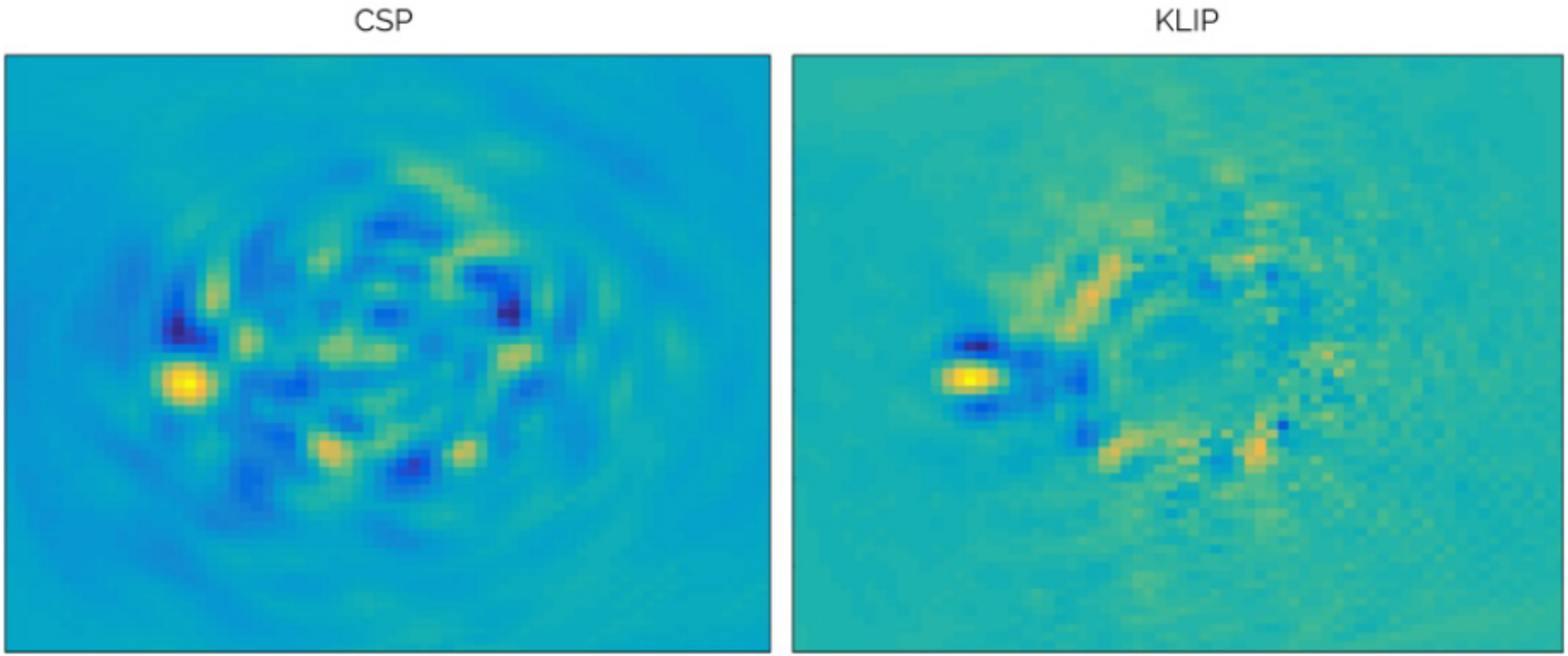
Dataset 2:



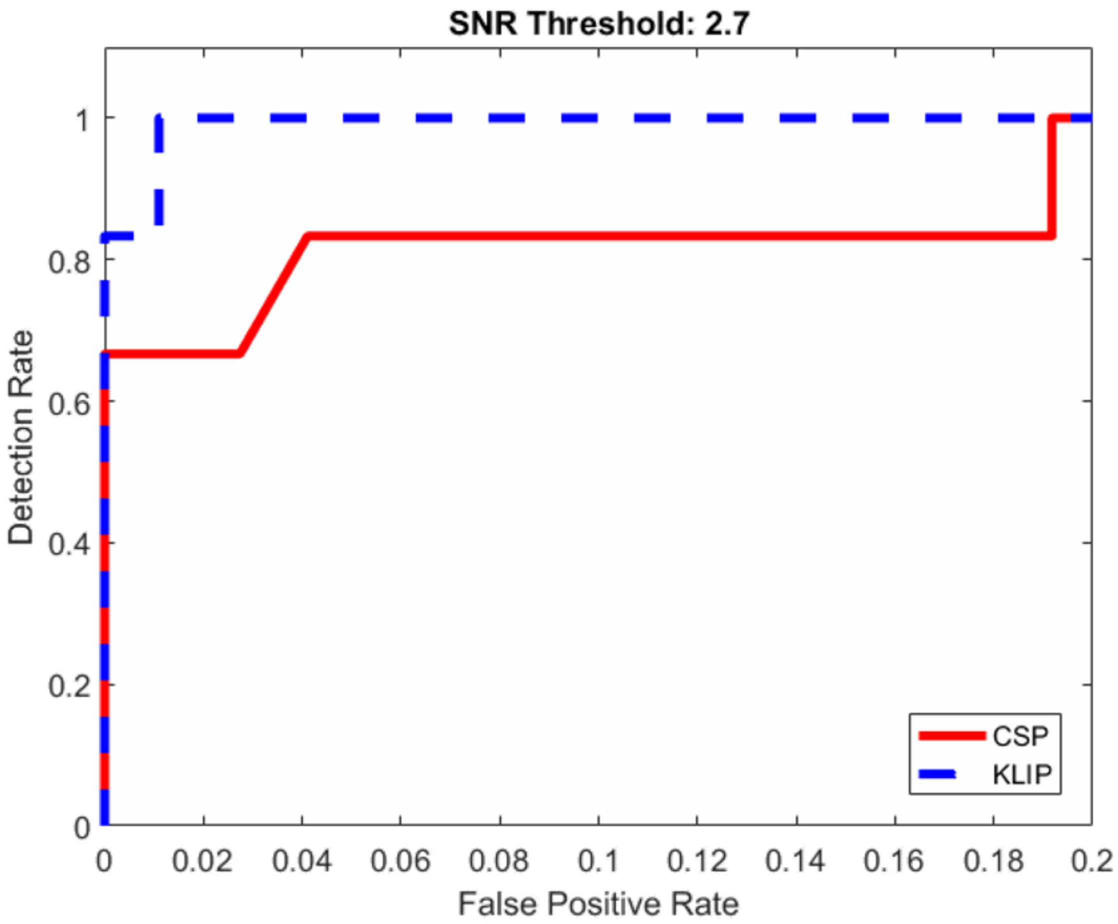
$$Z = \Phi^{-1/2} W^T P X_1$$

Final modes contain planet signal
Sum together final k modes

Preliminary Qualitative results



Preliminary Quantitative Results: Receiver Operating Characteristic Curve

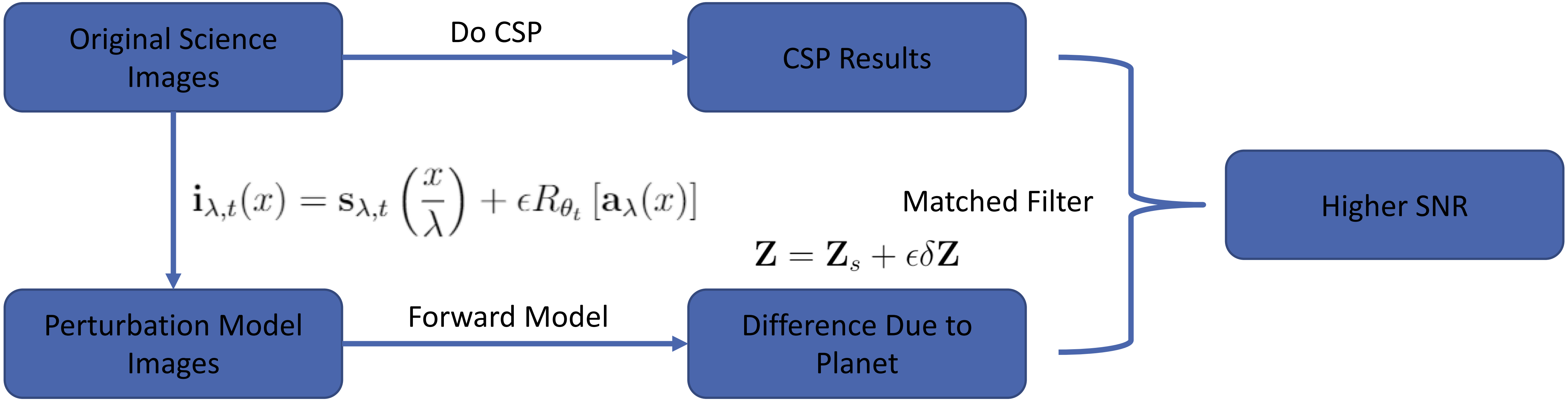


Shapiro et al, 2017

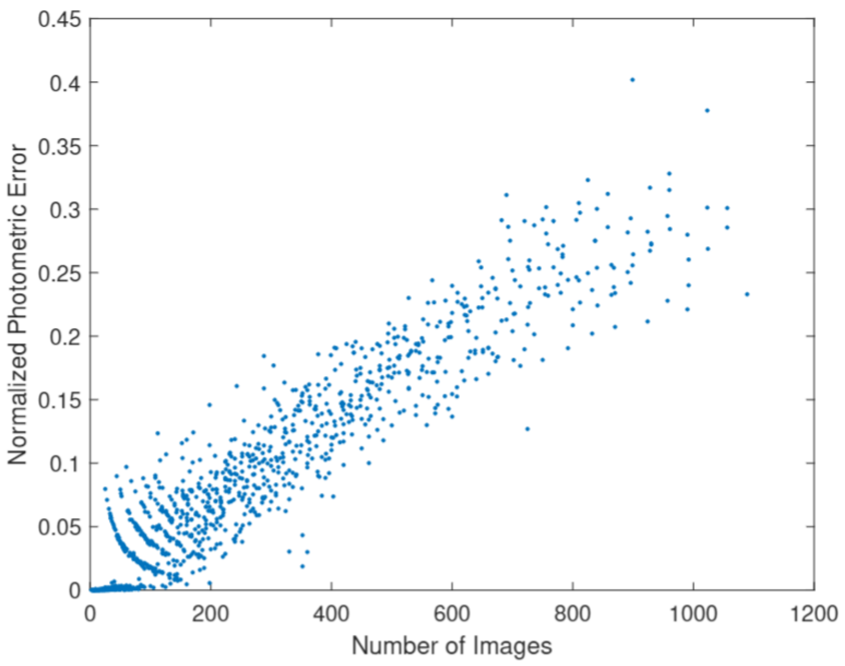
New Technical Approach: Forward-Model Matched Filter

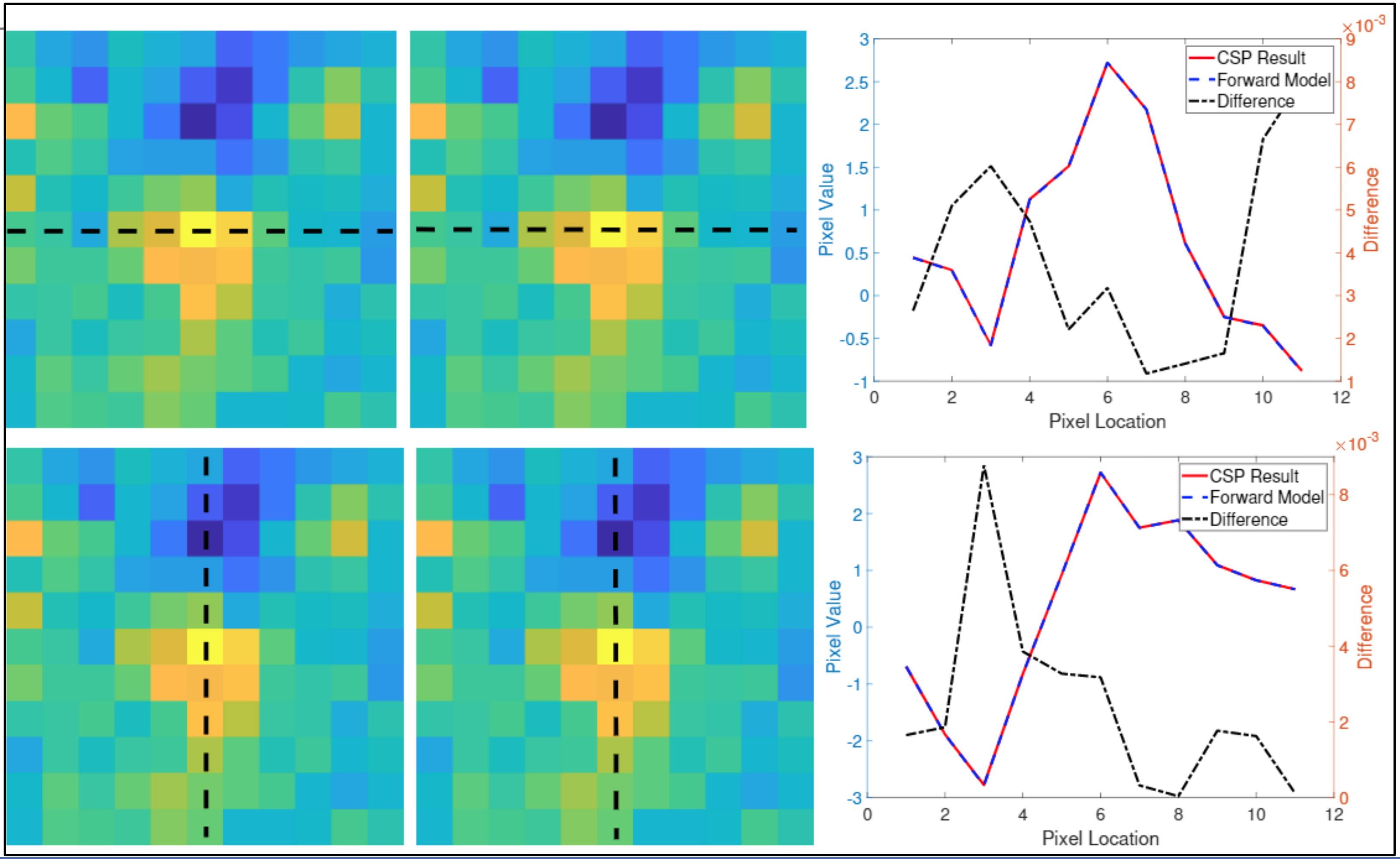
We want:

- Increase signal to noise ratio
- Remove false positives
- Model only the effects of the planet

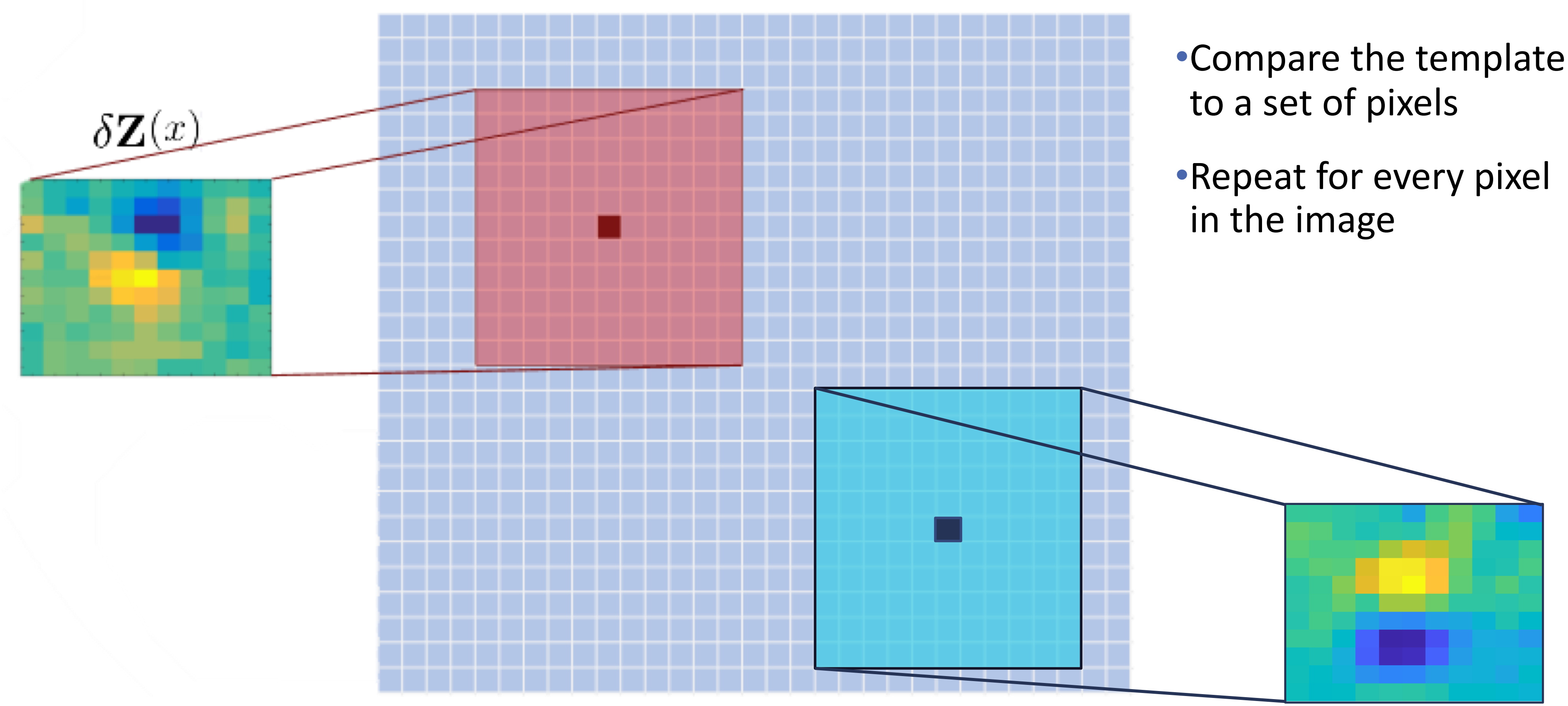


Results: FM Accuracy





Matched Filter



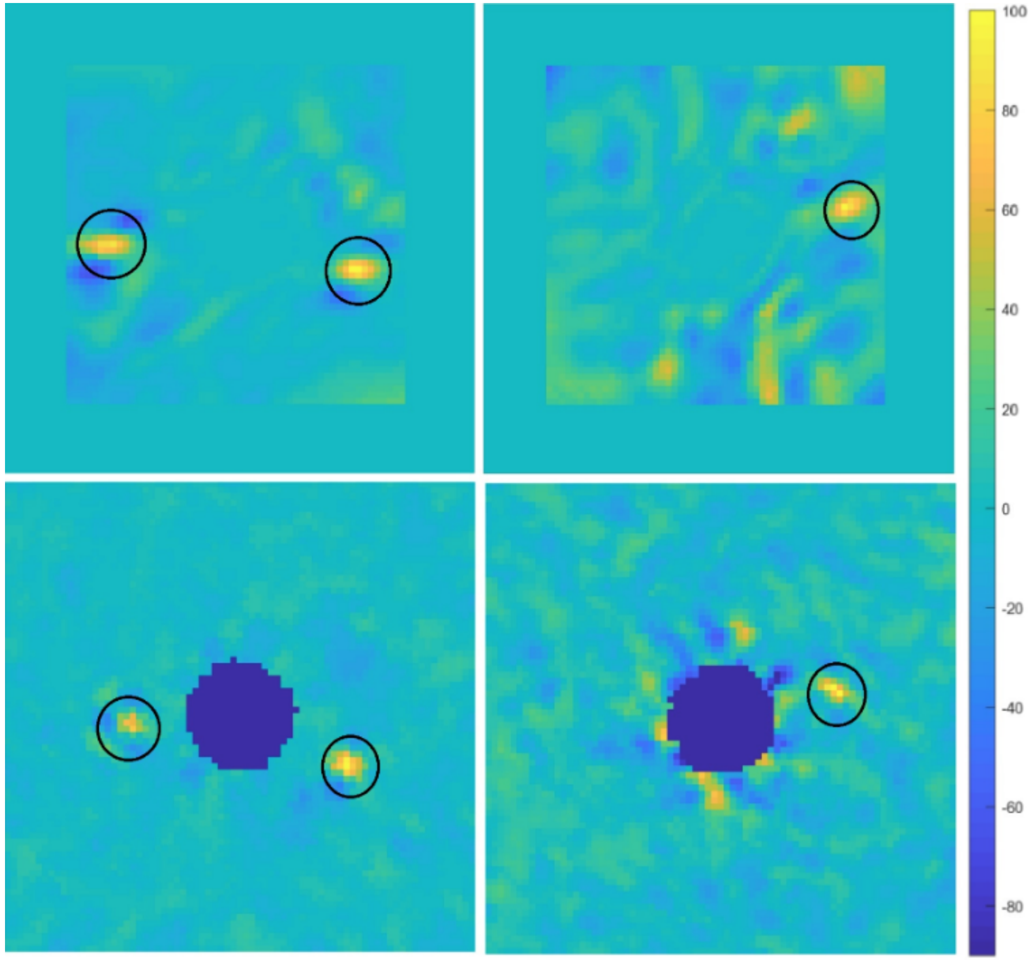
- Compare the template to a set of pixels
- Repeat for every pixel in the image

Results

β Pictoris HD 14706

CSP

PCA

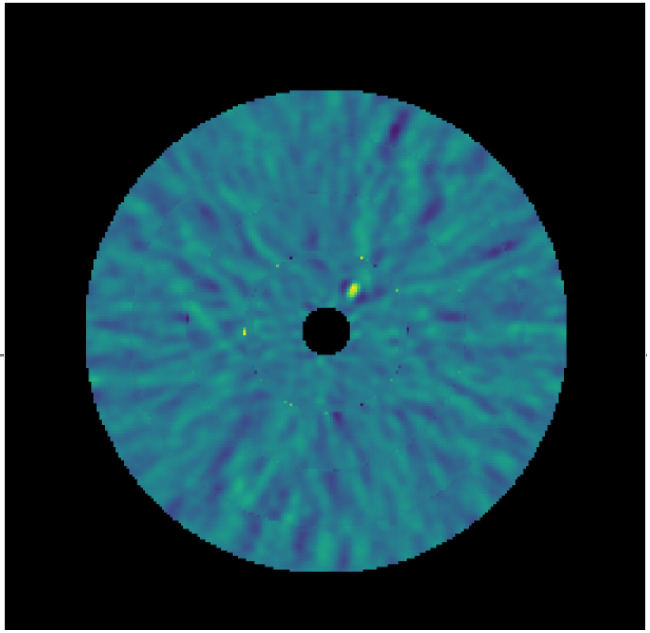


Signal to Noise Ratios

	CSP	PCA
B Pic 1	14.47	13.05
B Pic 2	11.79	9.97
HD 14706	6.90	10.65

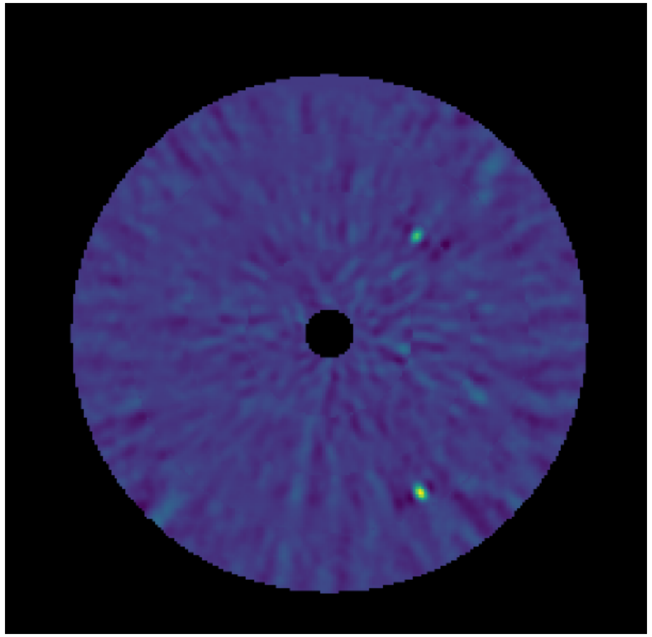
Algorithmic Improvements and Statistical Analysis

Algorithmic Updates



Structural Changes

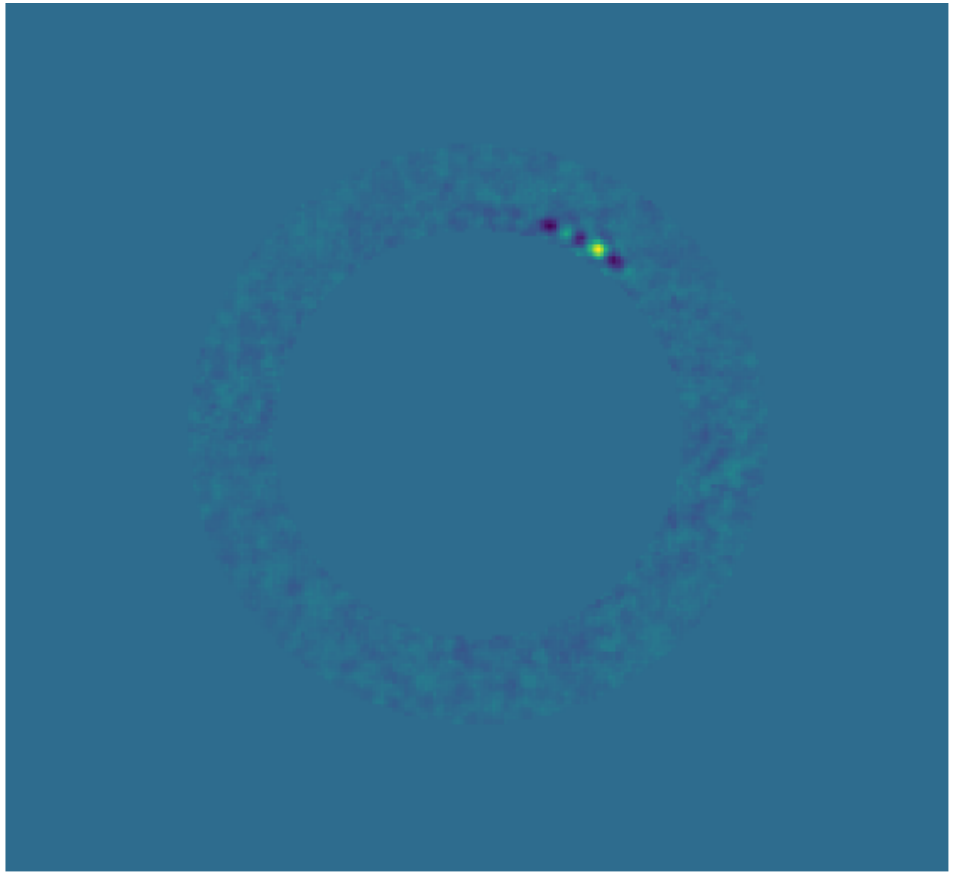
- Full-size datasets
- Rewritten in Python for integration into pyKLIP
- Parallelized



Algorithmic Changes

- Segmentation
- Matched Filter Template Threshold
- SNR Mapping

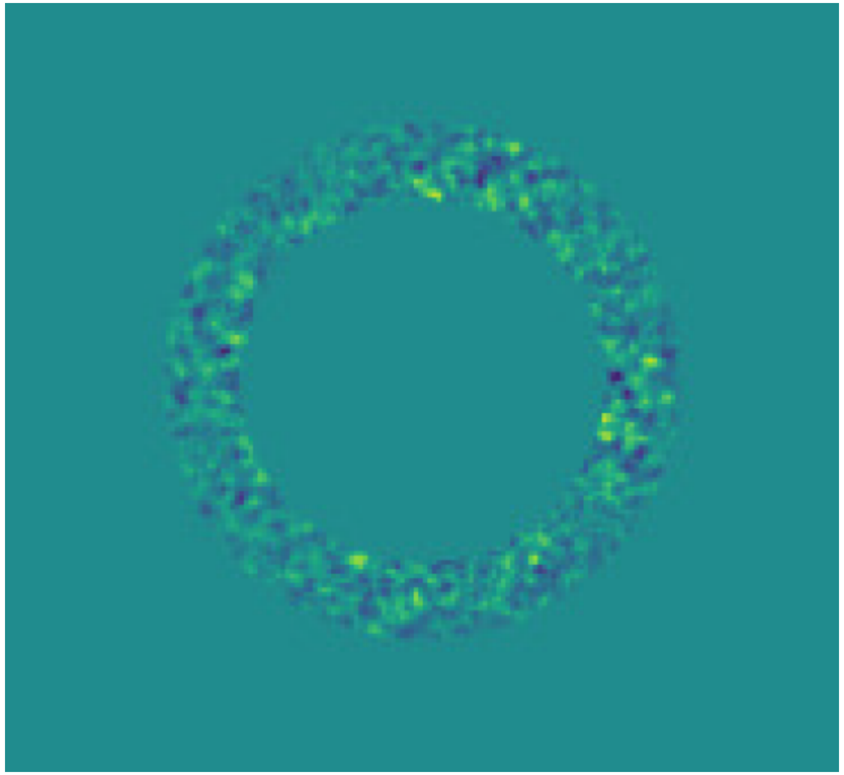
Segmentation



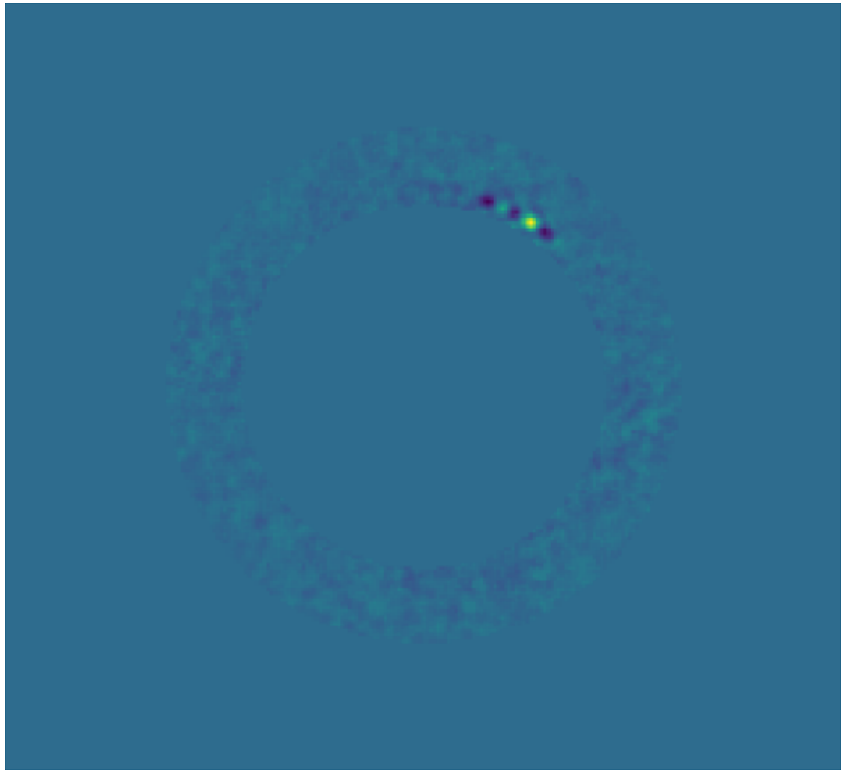
- Matrix sizes scale geometrically
  - Requires subdividing the images
- Noise assumed to be constant at the same separations
- Planet signal dispersed in an arc in unknown locations
- Segment must be full annuli

MF Template Threshold

CSP Result

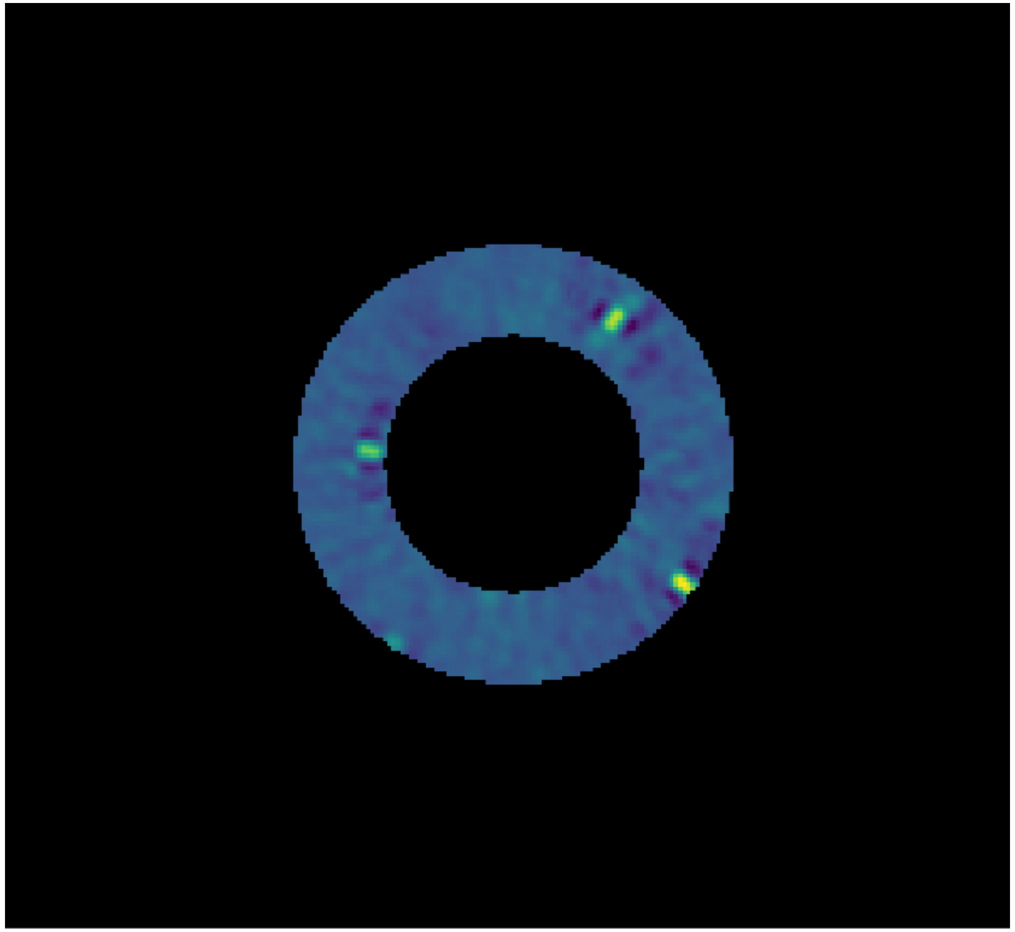


Original Template

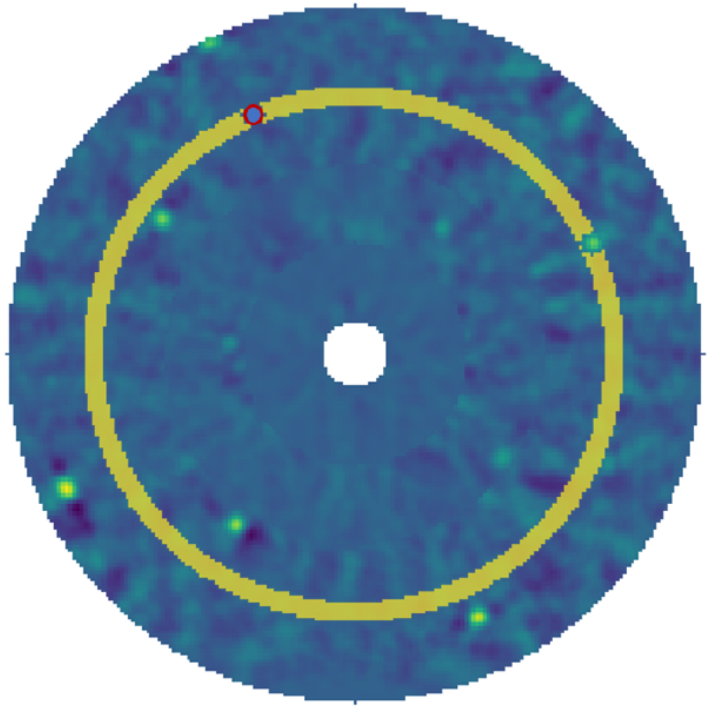


Threshold Template

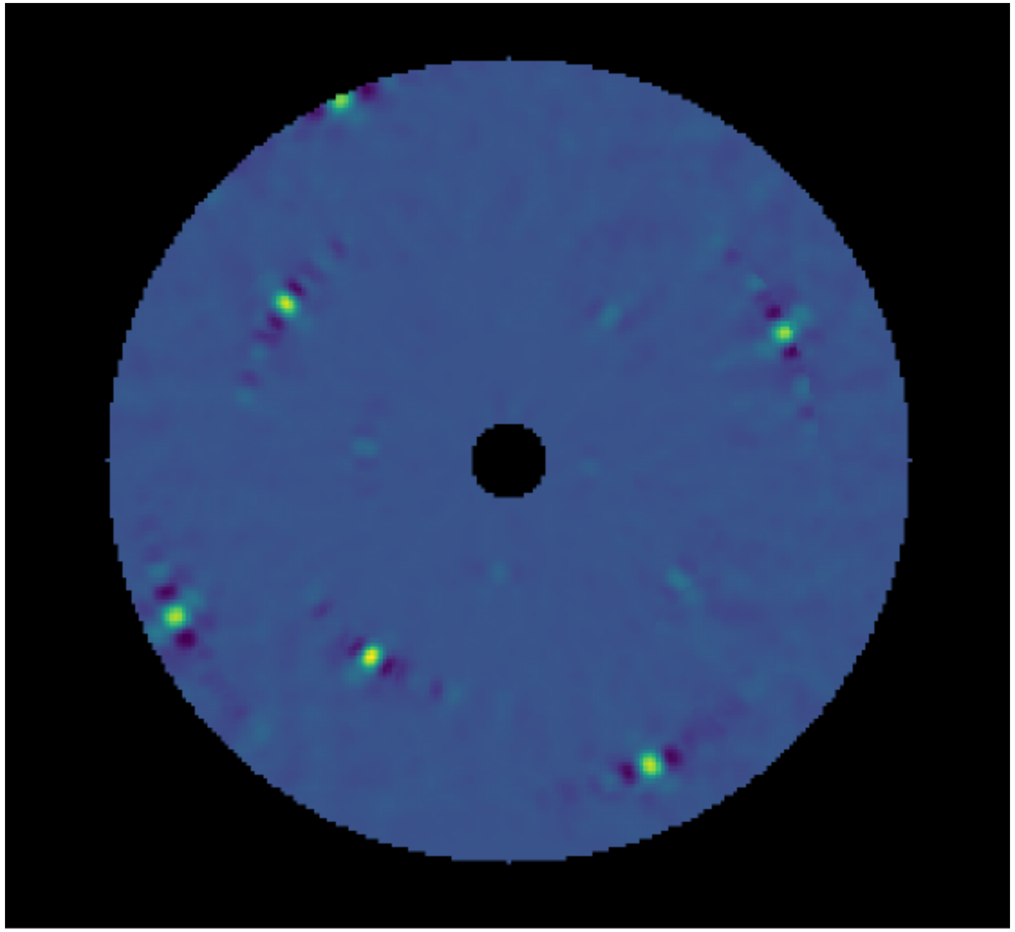
SNR Mapping

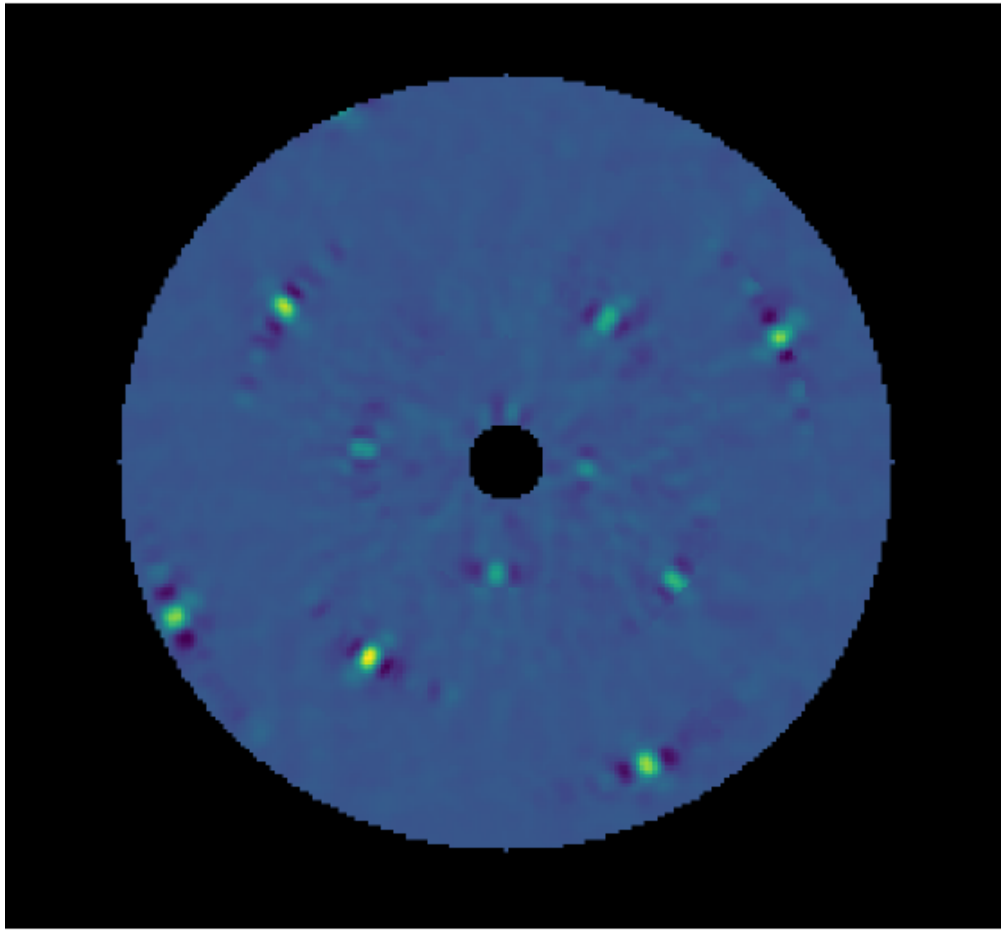


$$SNR = \frac{\hat{\mathbf{s}}(x) - \mu(x)}{\sigma(x)}$$



SNR Mapping

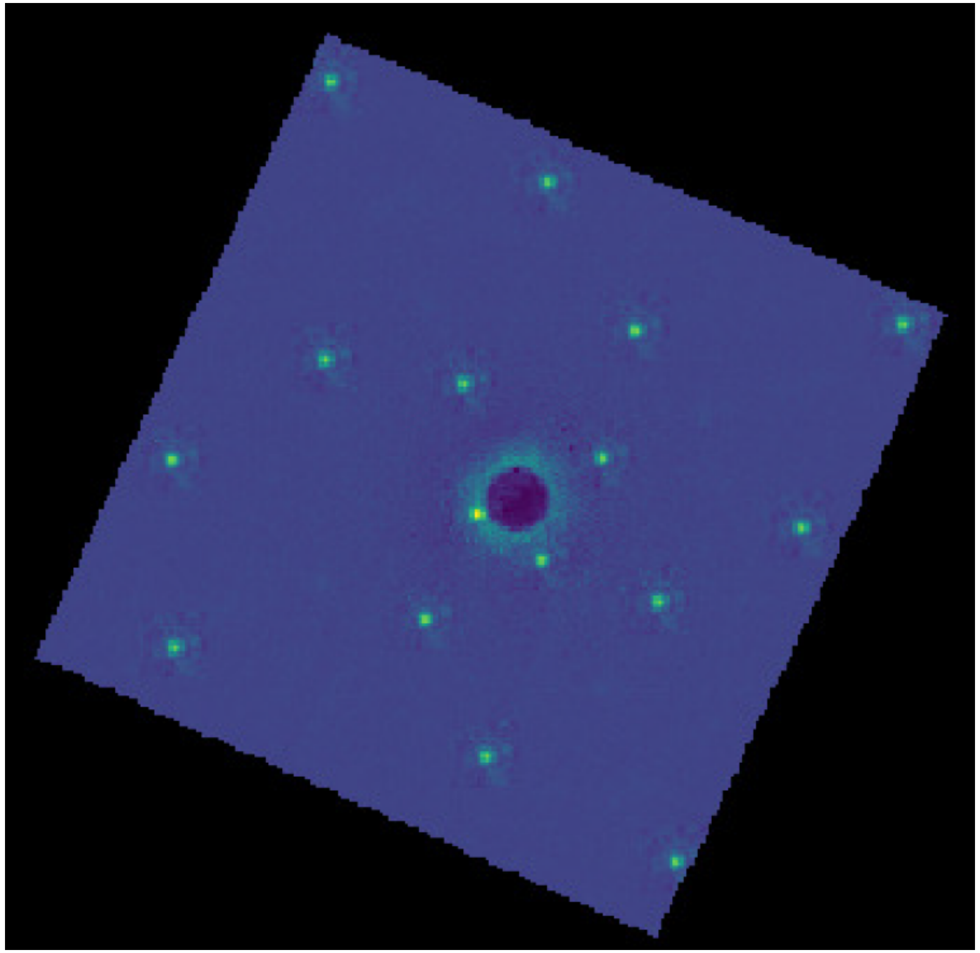




Target Selection and Injection

- GPIES Target
- H-Band
- Between 30 and 45 images
- Integration time between 59 and 60 seconds
- No observing errors
- No debris/dust disks

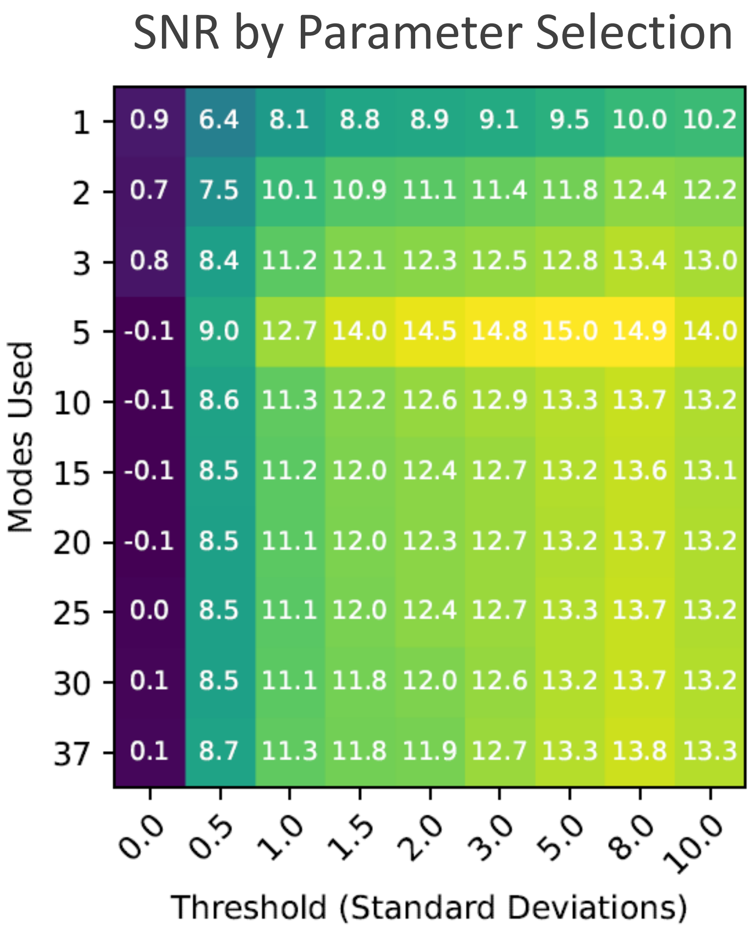
- 337 different observations

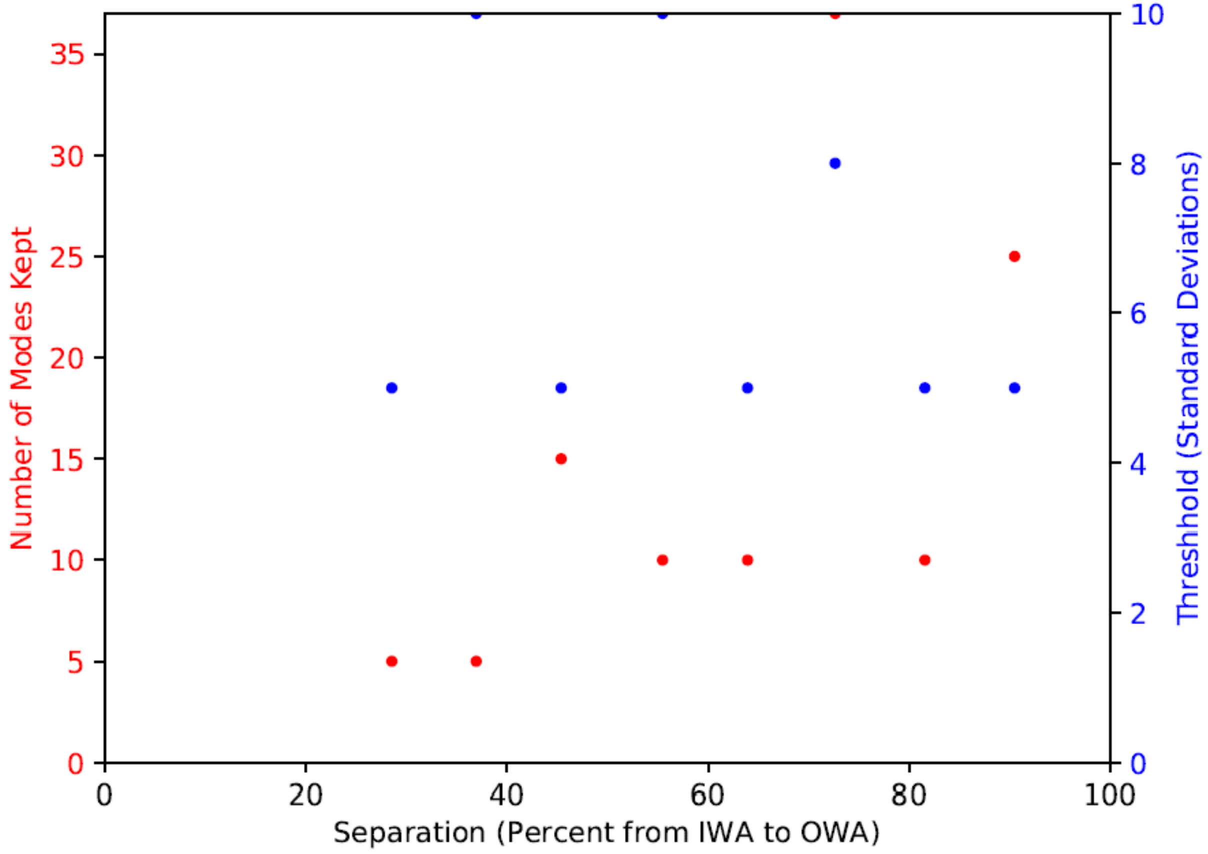


Contrasts:

- 5×10^{-5}
- 5×10^{-6}
- 5×10^{-7}

Global Parameter Selection





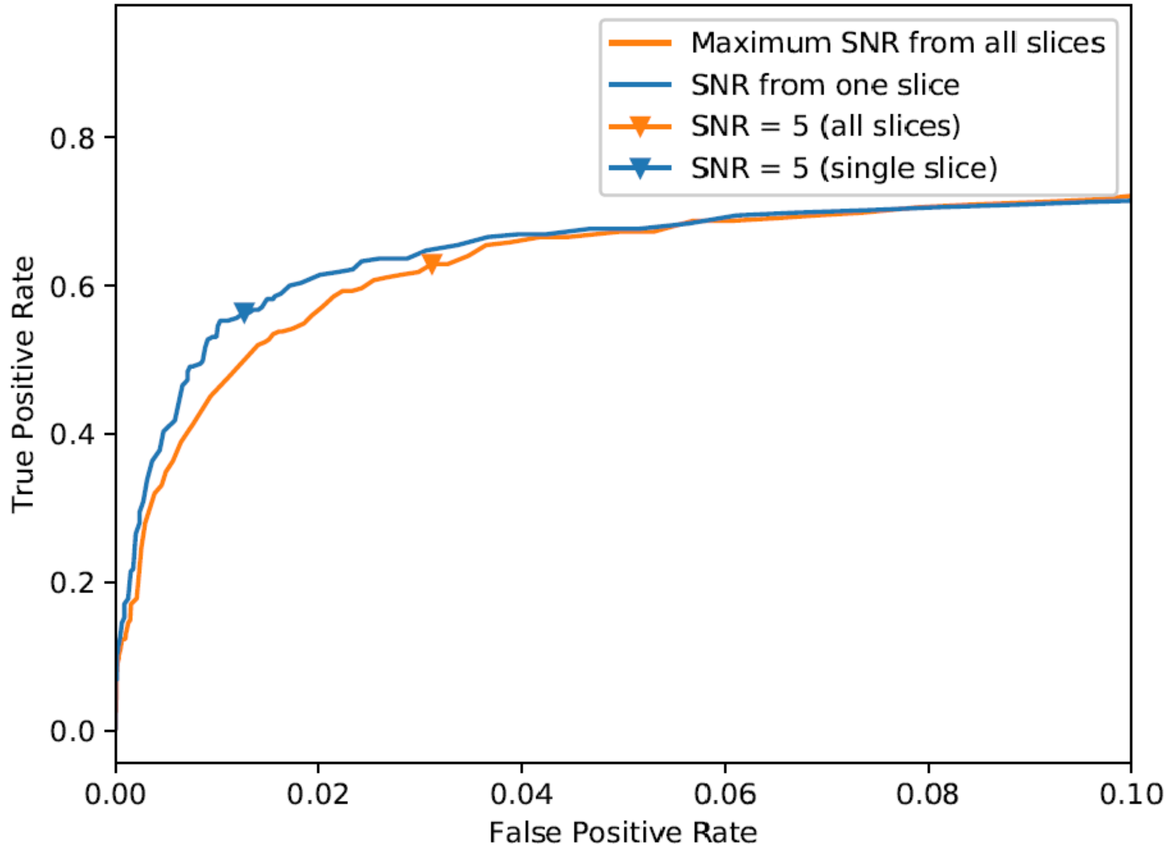
Parameter Selection

Option 1 (most common peak):

- Threshold = 5 sigma
- 10 modes kept

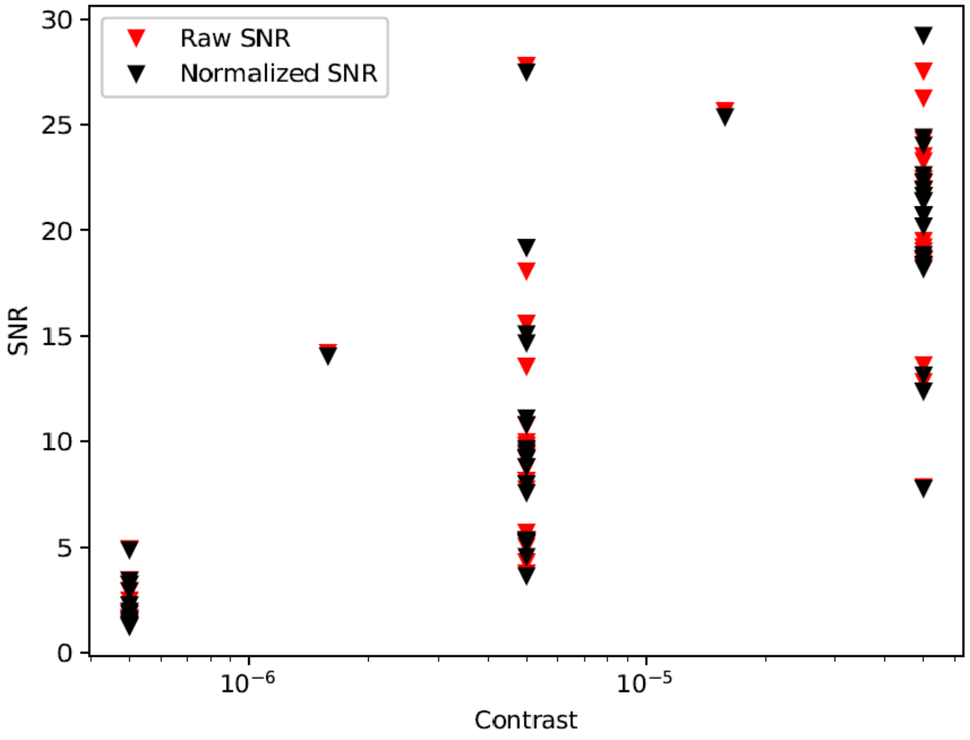
Option 2:

- Analyze data with all combinations of parameters
- Select best combination for every pixel

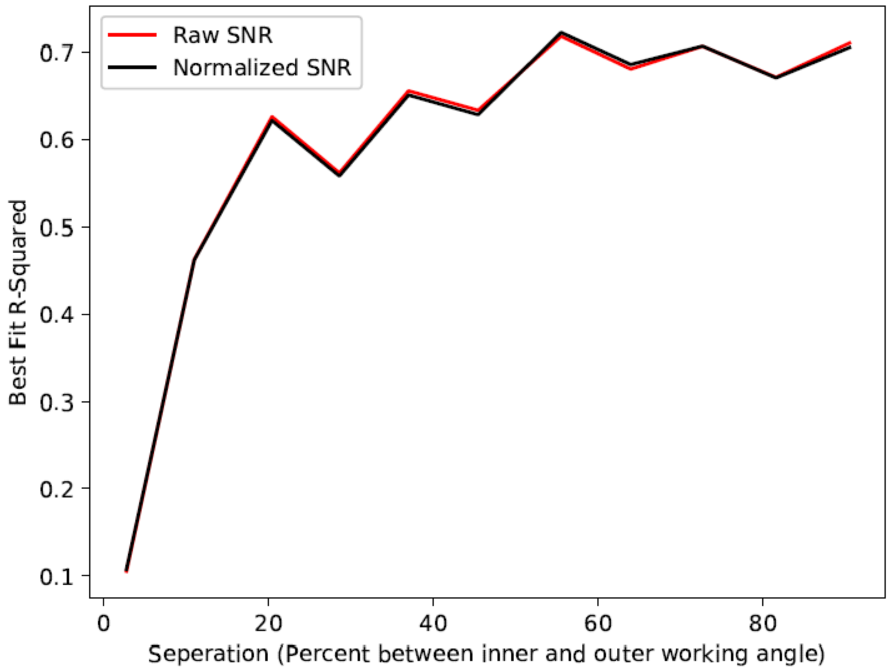


Results: SNR Contours

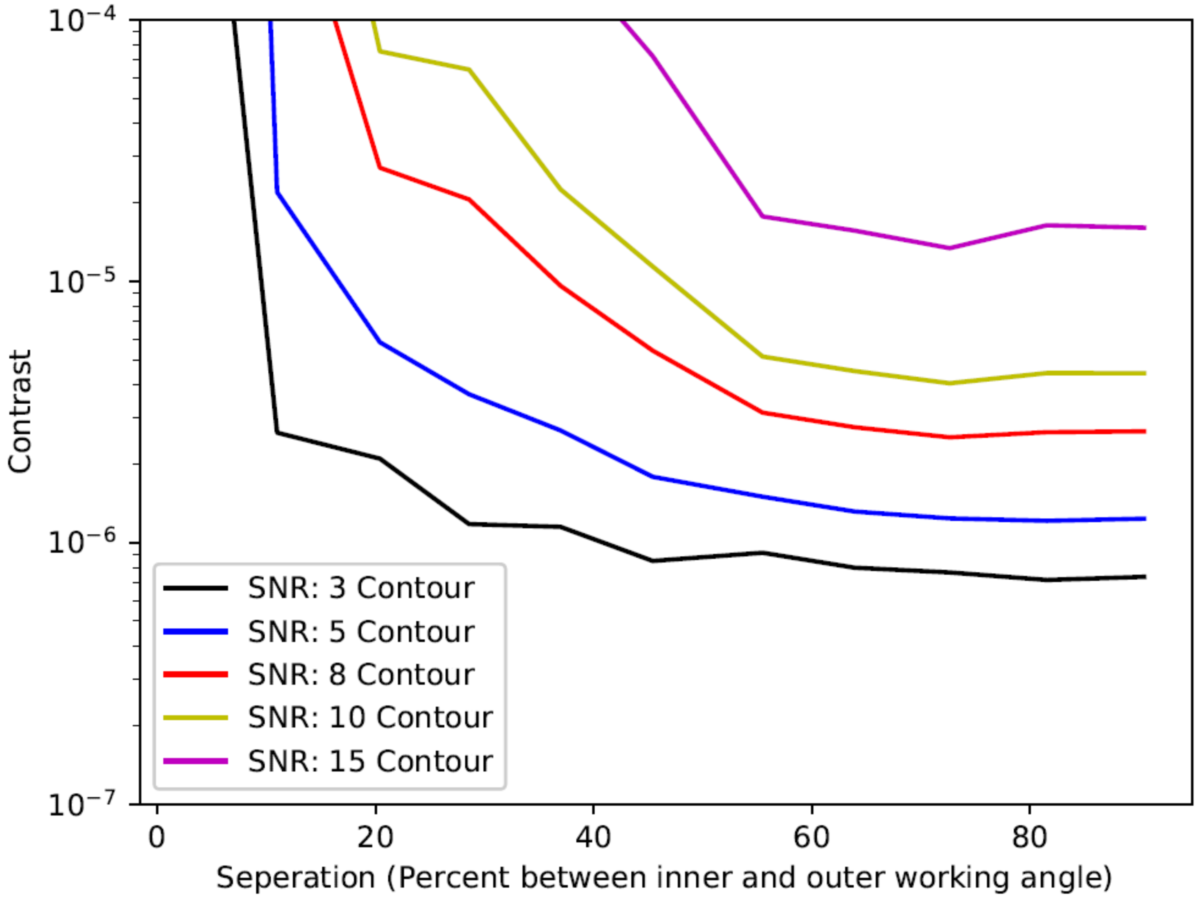
Sample for a single separation, find the best linear fit between contrast and SNR:



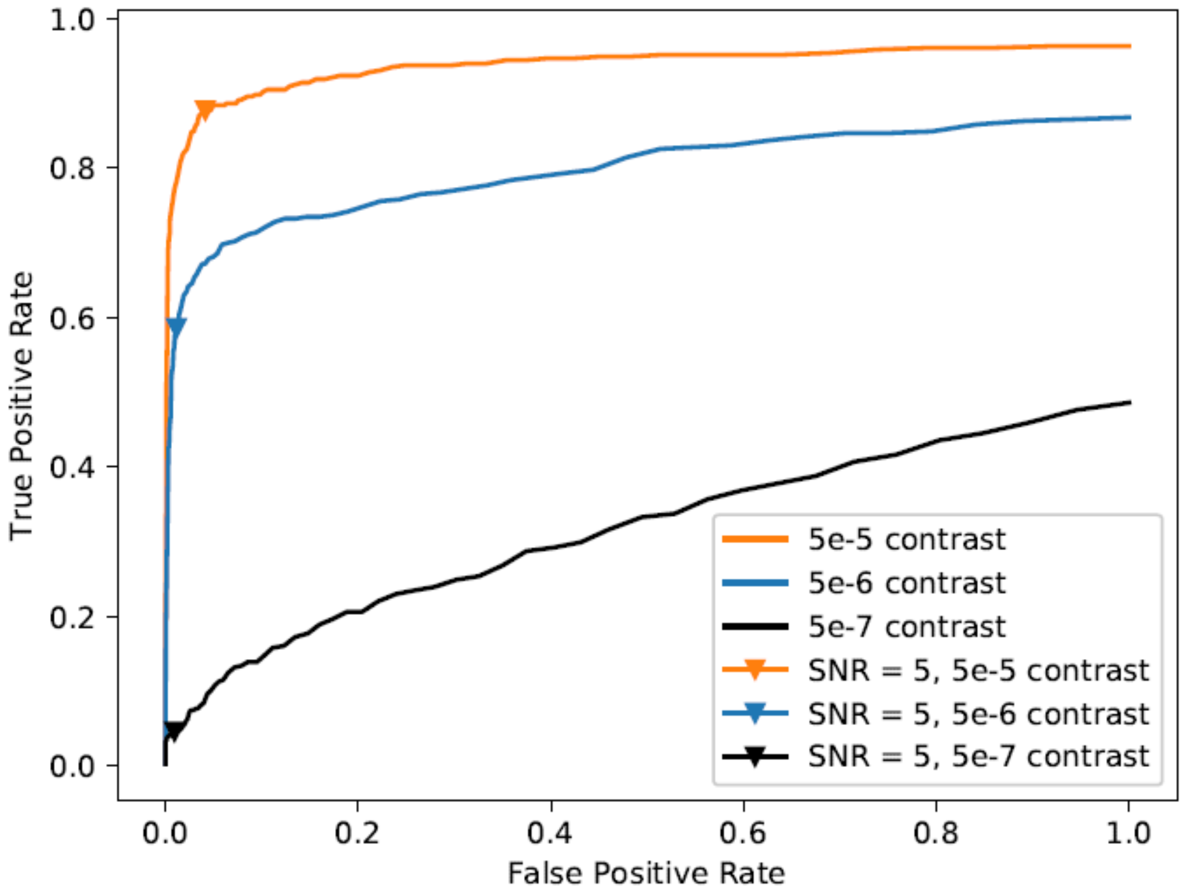
SNR is proportional to the square root of integration time, requiring normalization



Results: SNR Contours



Results: ROC Curve



Optical Design

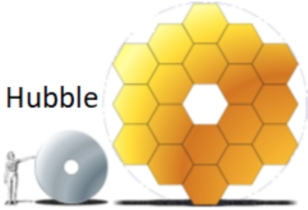
LARGE, SEGMENTED, SELF-ASSEMBLING SPACE TELESCOPE

Mission Concept and Architecture

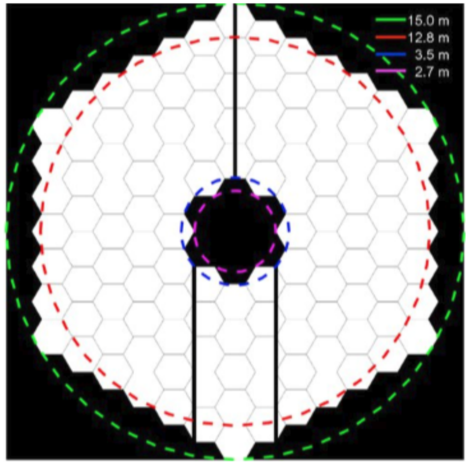
- Approximately 1,000 identical, mass-produced spacecraft
- Spacecraft travel via solar sail to L2
- Each spacecraft combines to form one large telescope via autonomous in-space assembly
- Each flat mirror modulated to control shape
- Completed structure combines with instrument spacecraft and secondary mirrors

JWST

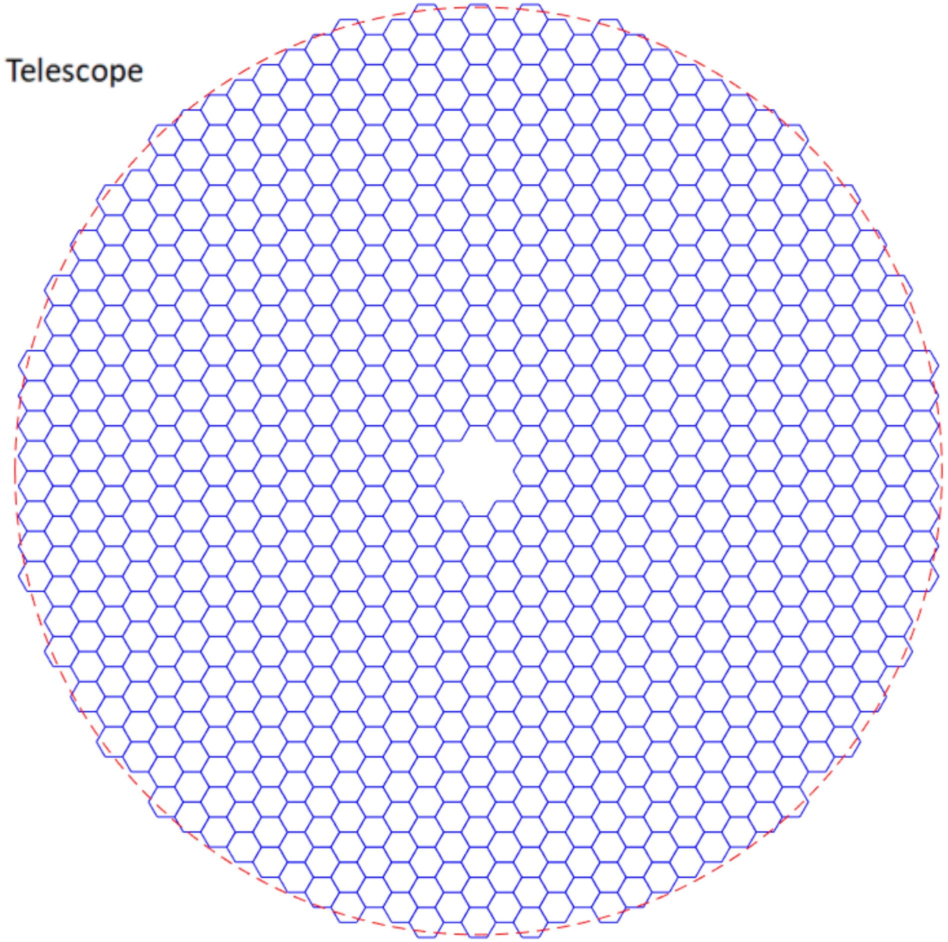
Hubble



LUVOIR



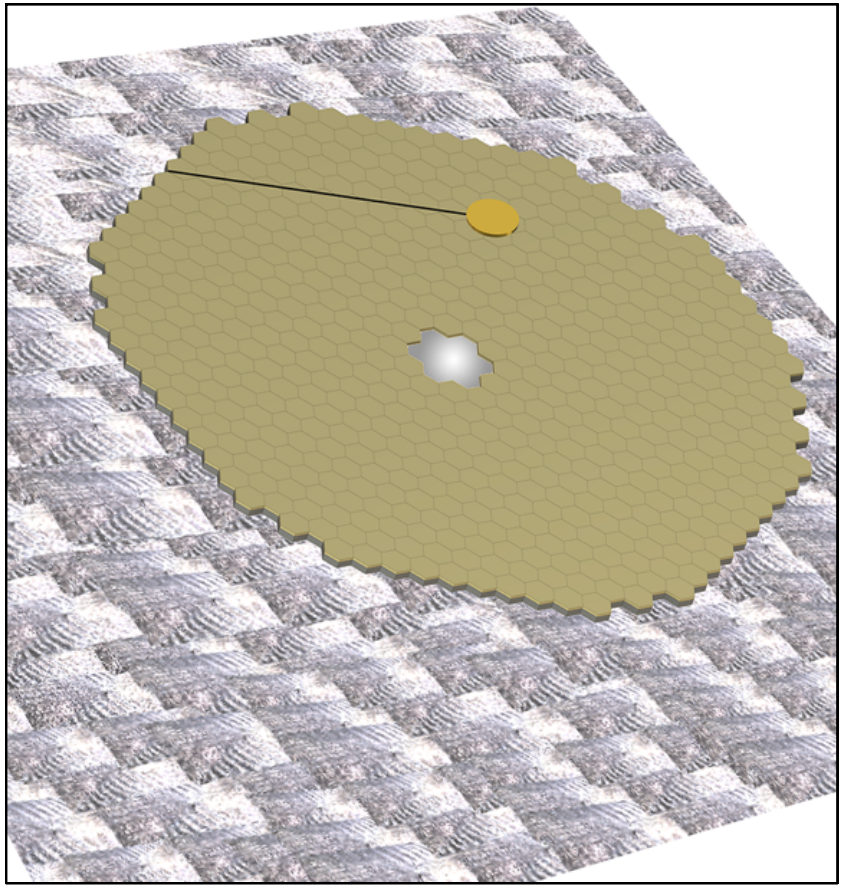
31 Meter Telescope



Optical Design Drivers

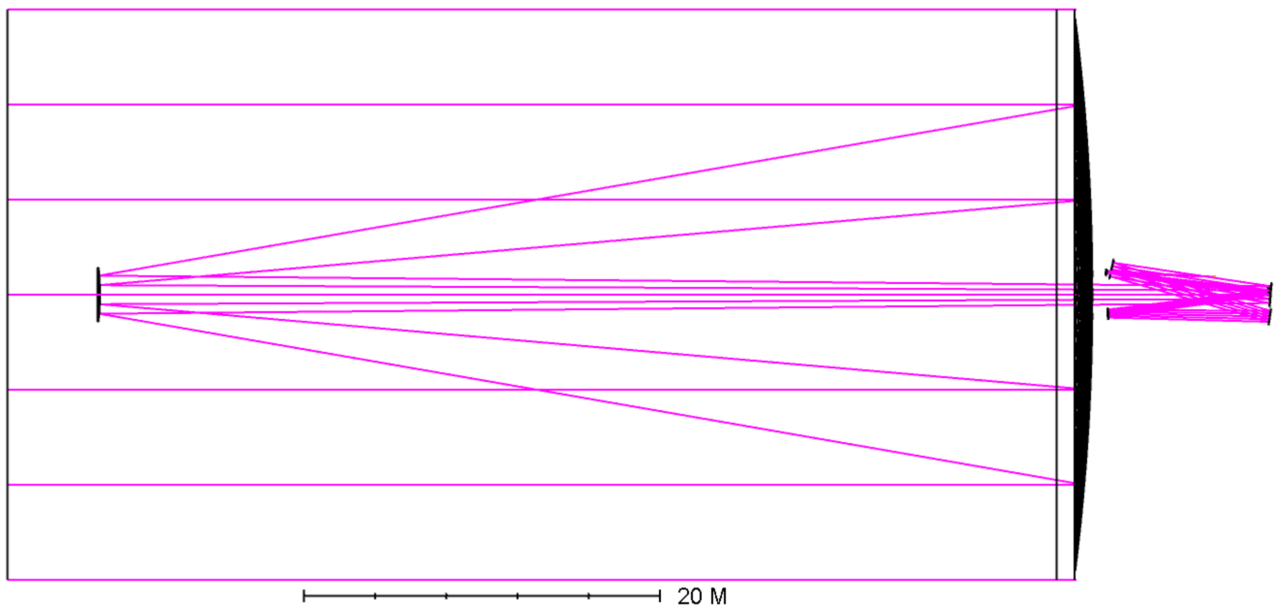
- Identical mirrors
- Feasible actuation scheme
- Monolithic secondary

- Static wavefront error < 9.5 nm RMS
- Focused point spread function

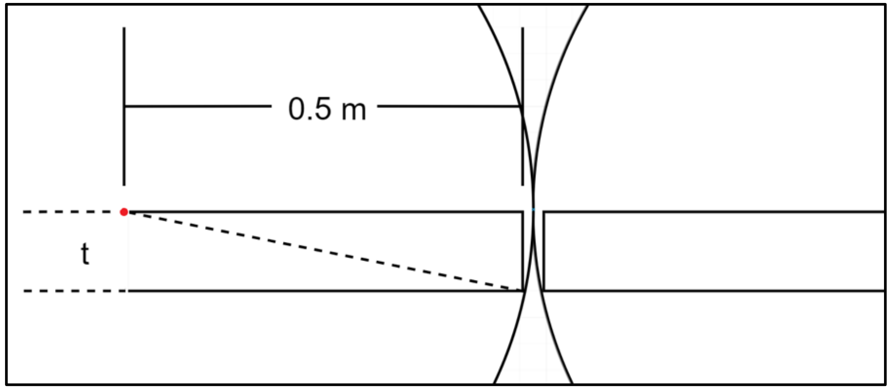


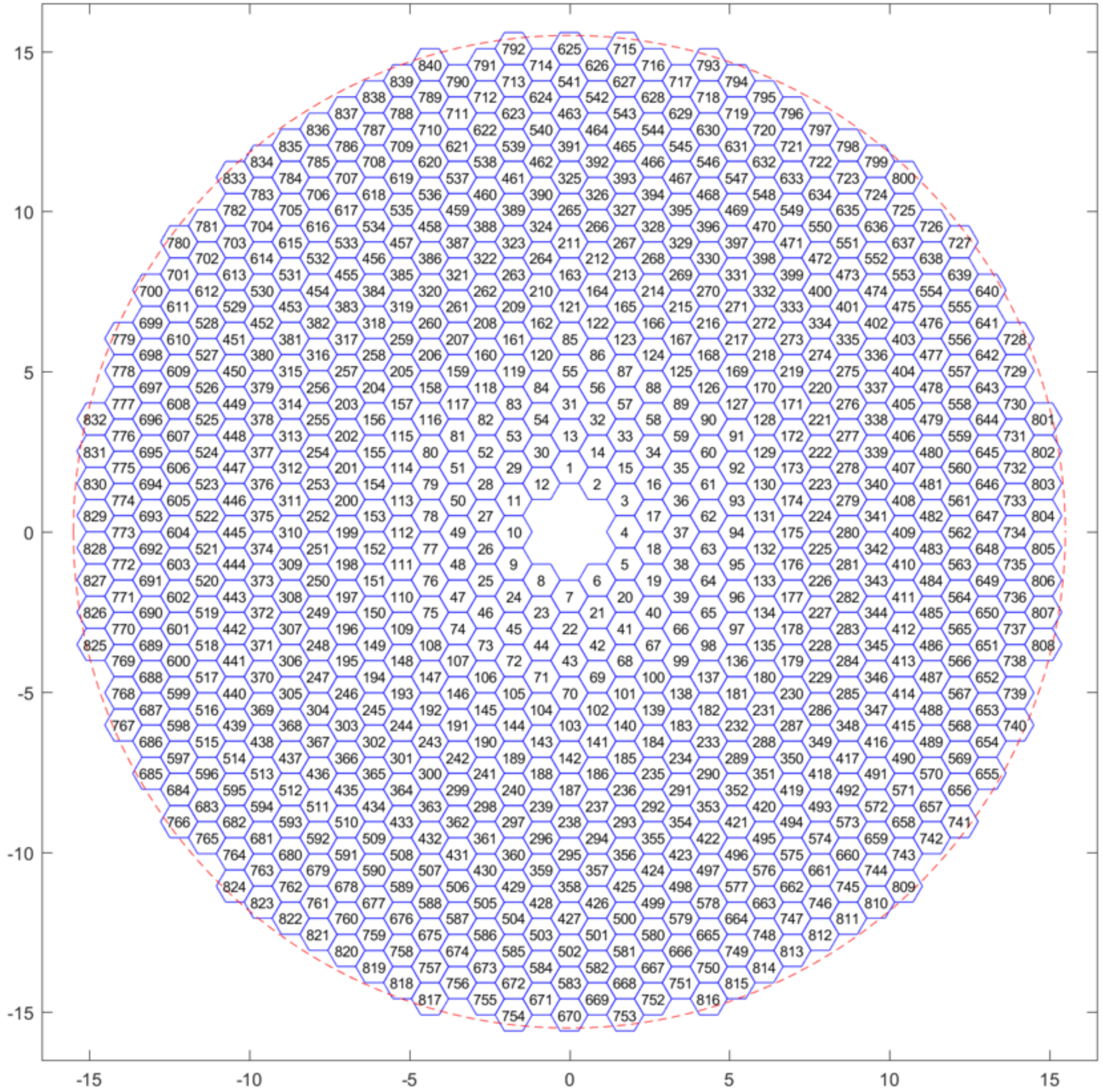
Telescope Design

- Ritchey-Chrétien Cassegrain design
- Primary focal length $^{f}/_{2}$
- Secondary – monolithic 3.06 m
- Each primary segment is 1m, flat-to-flat
- Total effective focal length of $^{f}/_{5.6}$



Segmentation





840 Mirrors

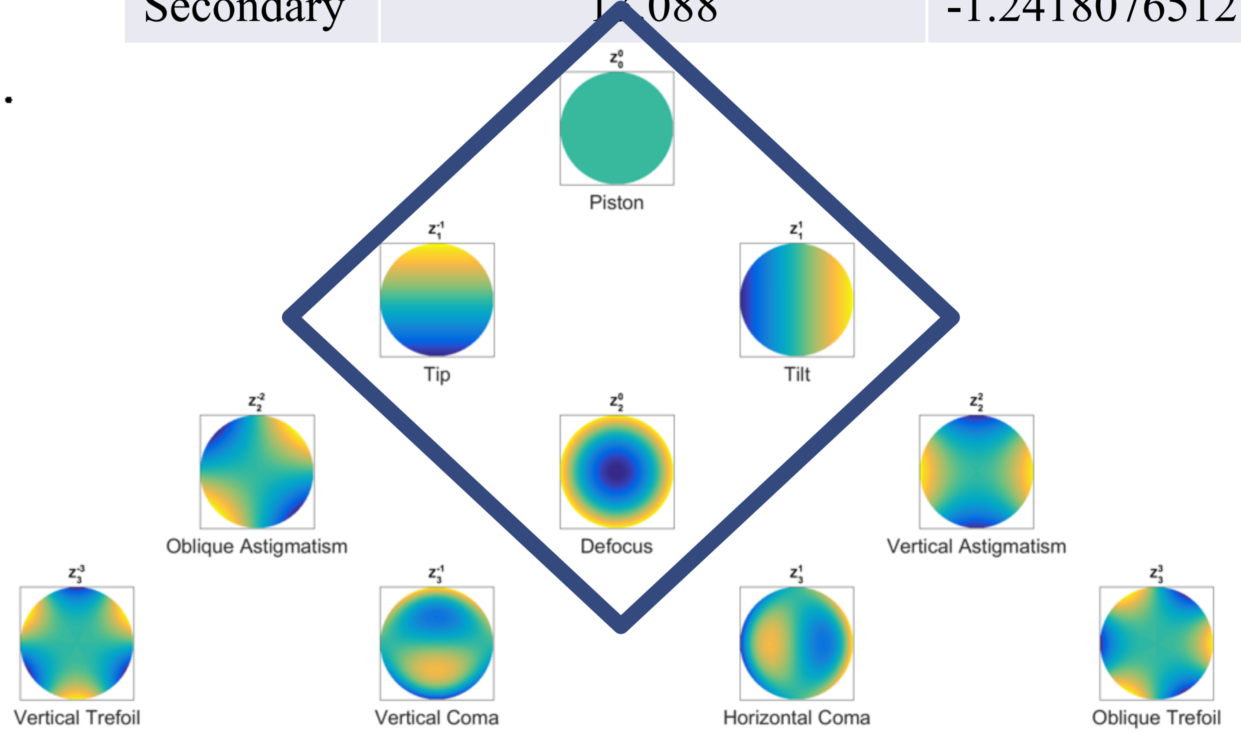
Mirror Modal Decomposition

- Zernike decomposition of each mirror
- Ideal Shape:

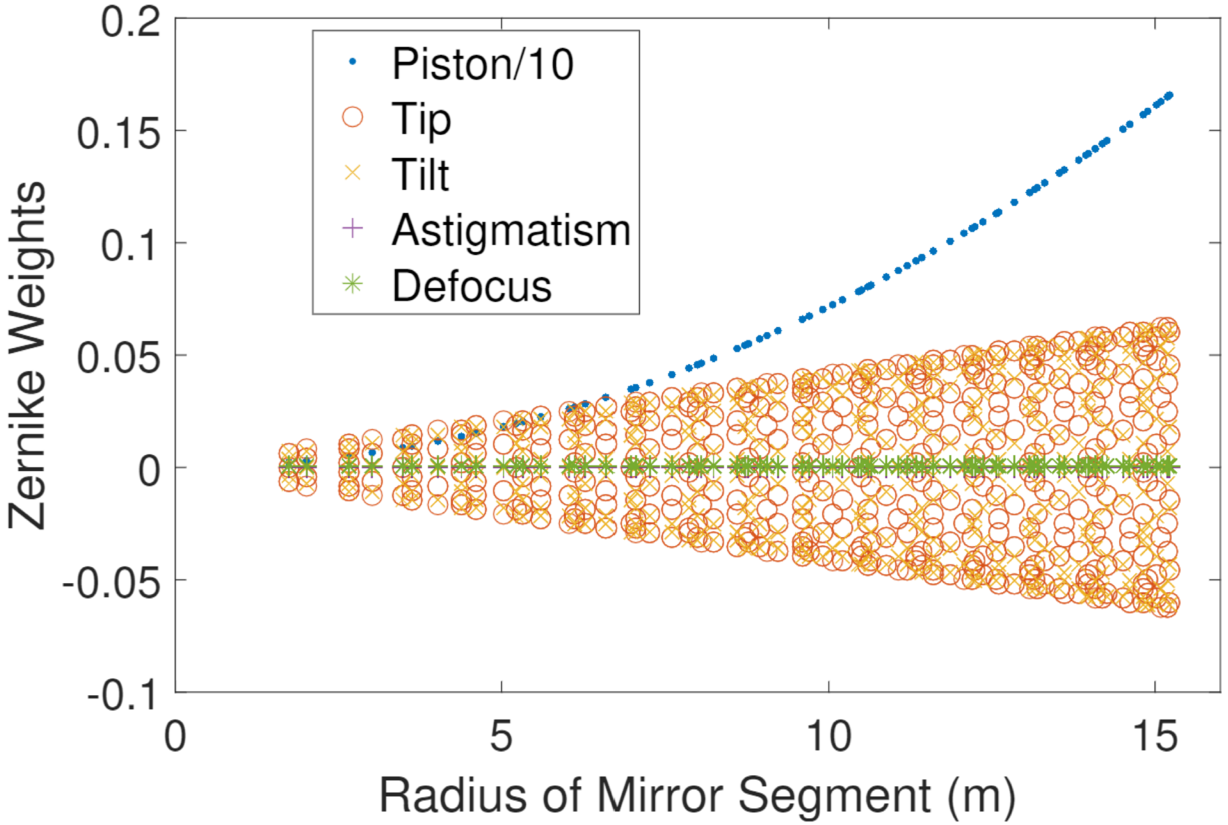
$$z = \frac{R + \sqrt{R^2 - (K+1)r^2}}{K+1}.$$

- JWST actuators can provide:
  - Piston
  - Tip/Tilt
  - Defocus

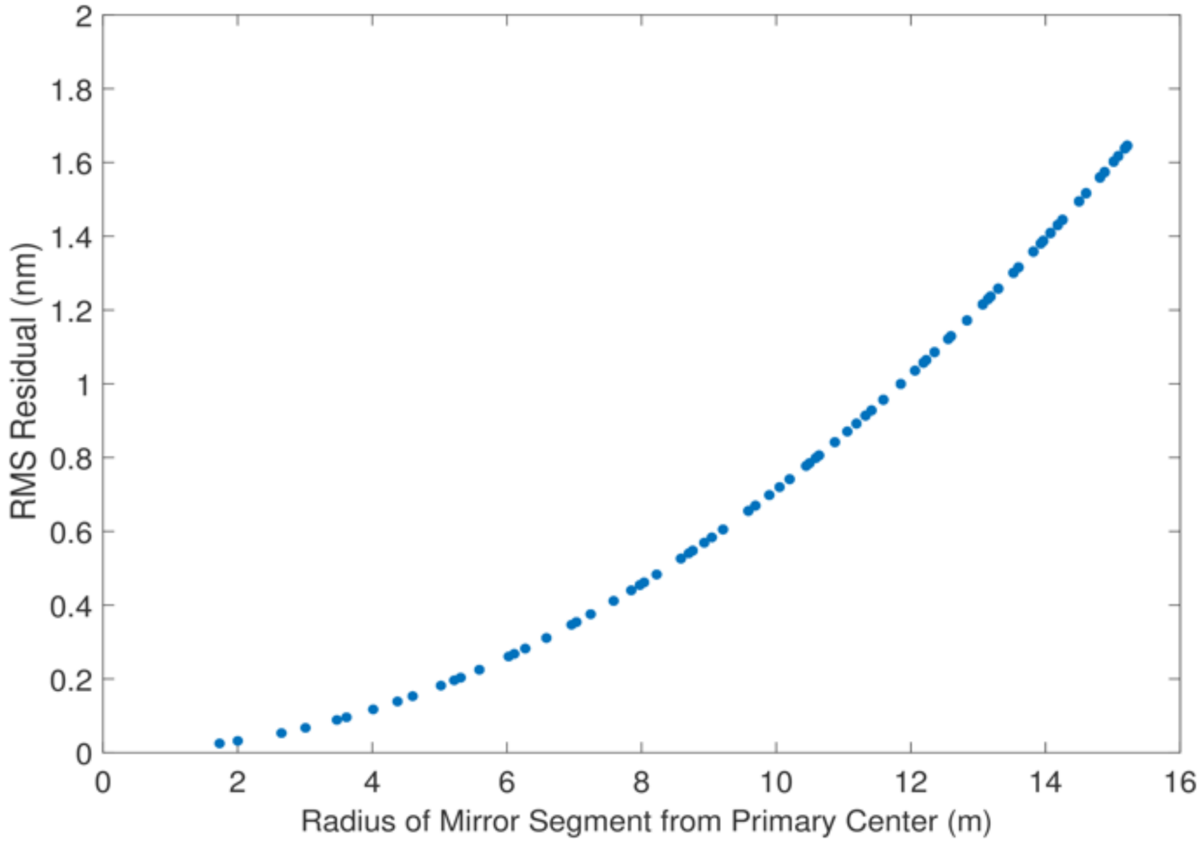
	Radius of Curvature (m)	Conic Constant
Primary	124	-1.000615574022776
Secondary	$13.\overline{088}$	-1.241807651272672

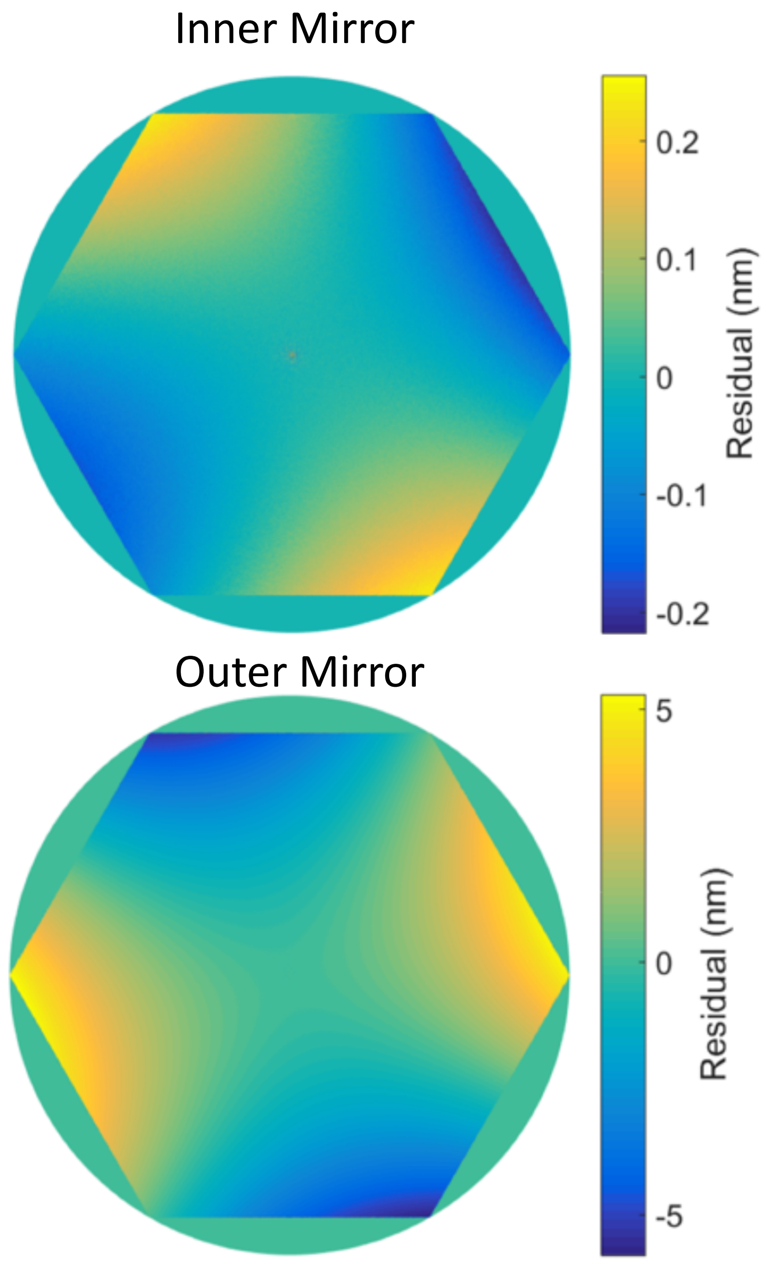


Mirror Modal Decomposition

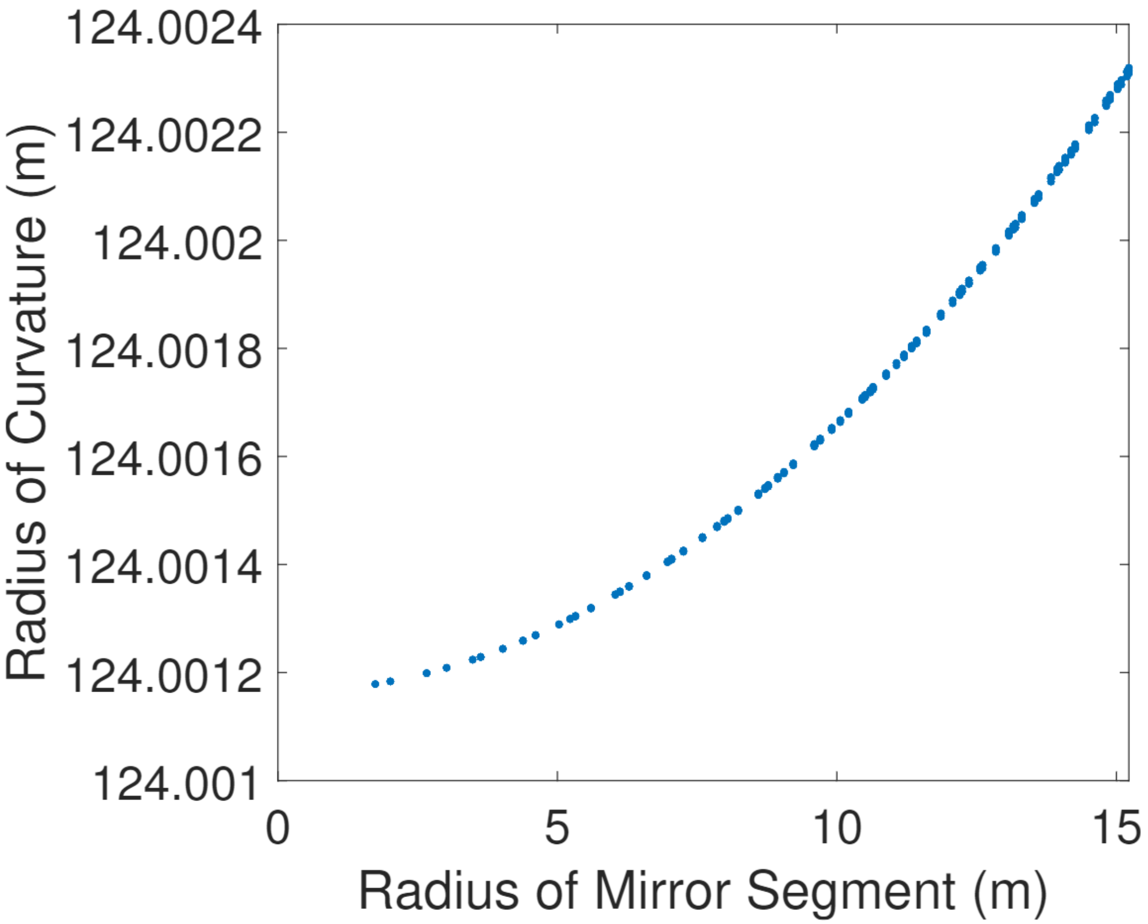


RMS Error

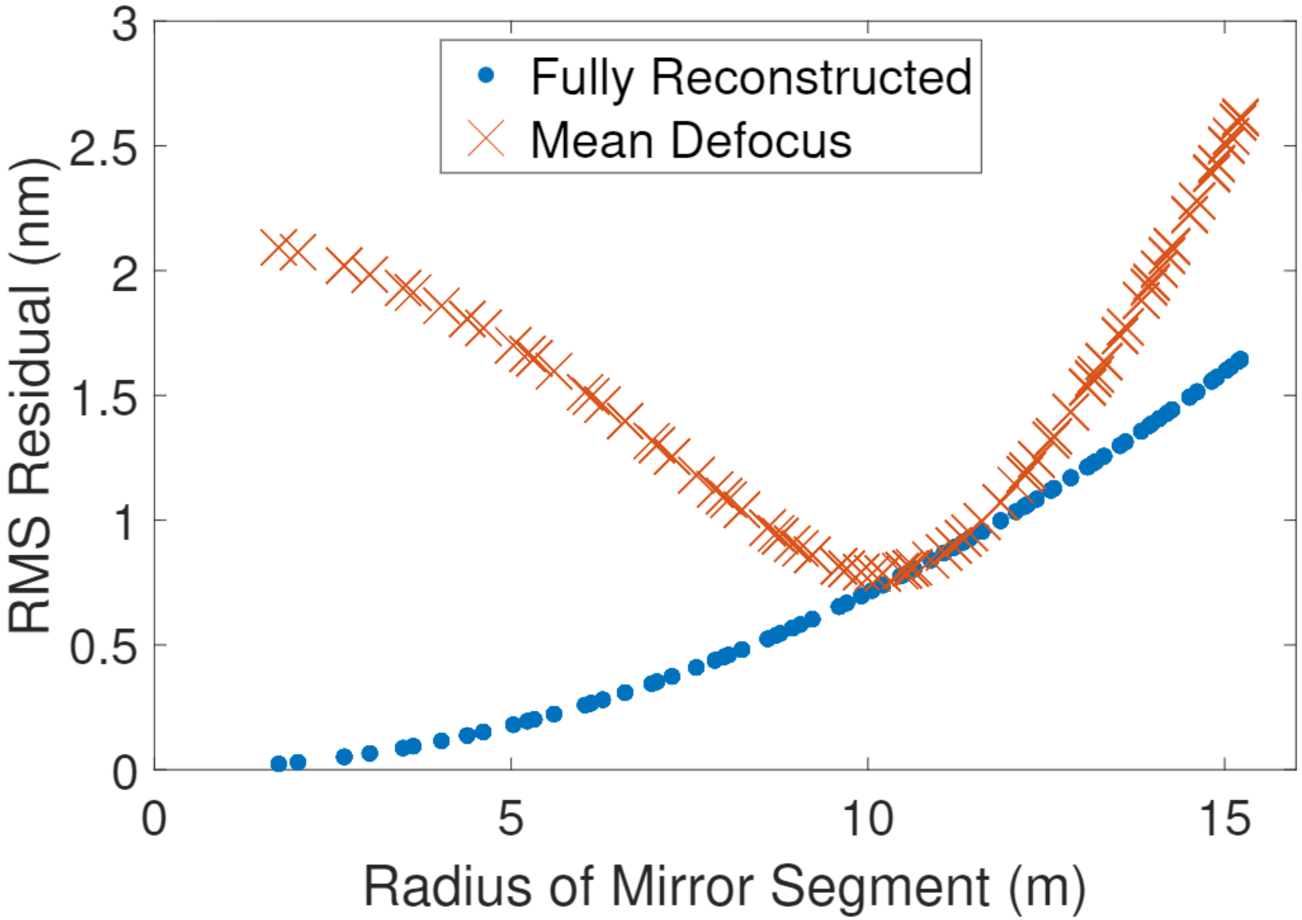

RMS Residual (nm)
Radius of Mirror Segment from Primary Center (m)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16


Inner Mirror
Residual (nm)
0.2
0.1
0
-0.1
-0.2
Outer Mirror
Residual (nm)
5
0
-5

RMS Error

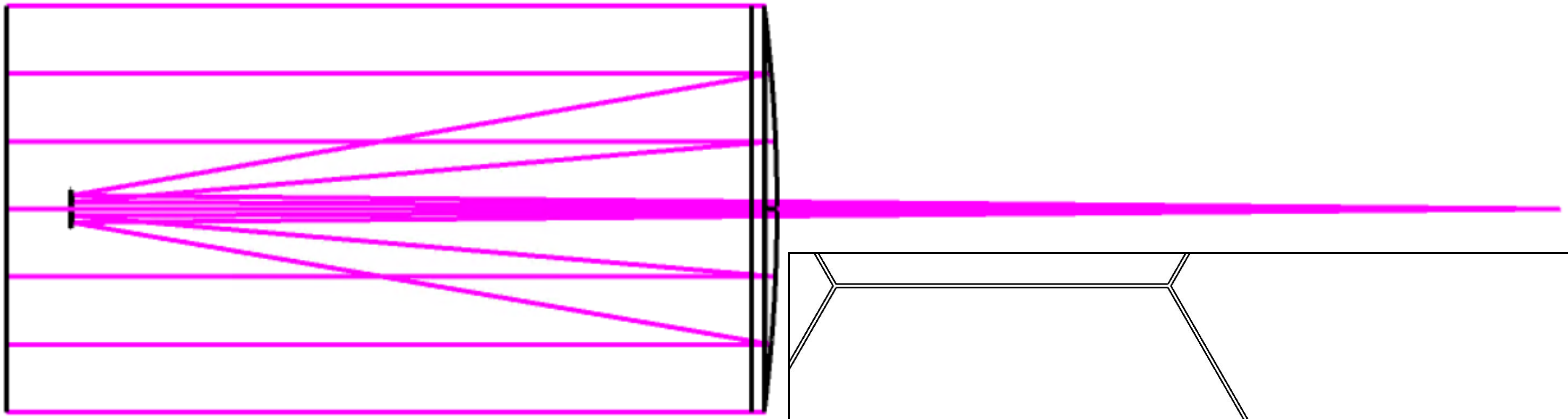


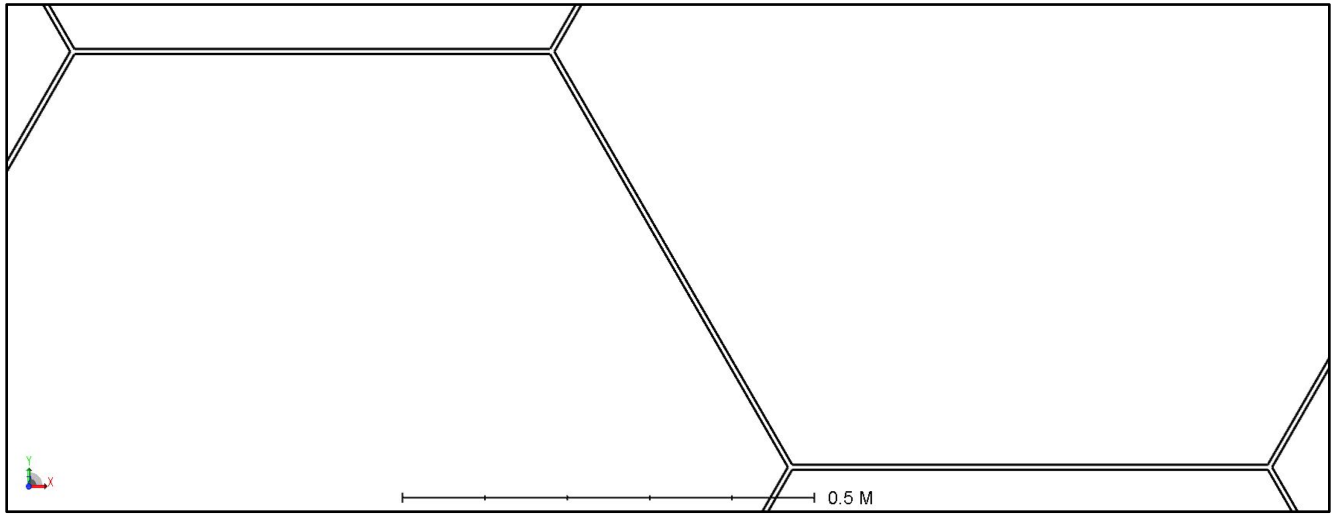
124.0024
124.0022
124.002
124.0018
124.0016
124.0014
124.0012
124.001
Radius of Curvature (m)
0
5
10
15
Radius of Mirror Segment (m)



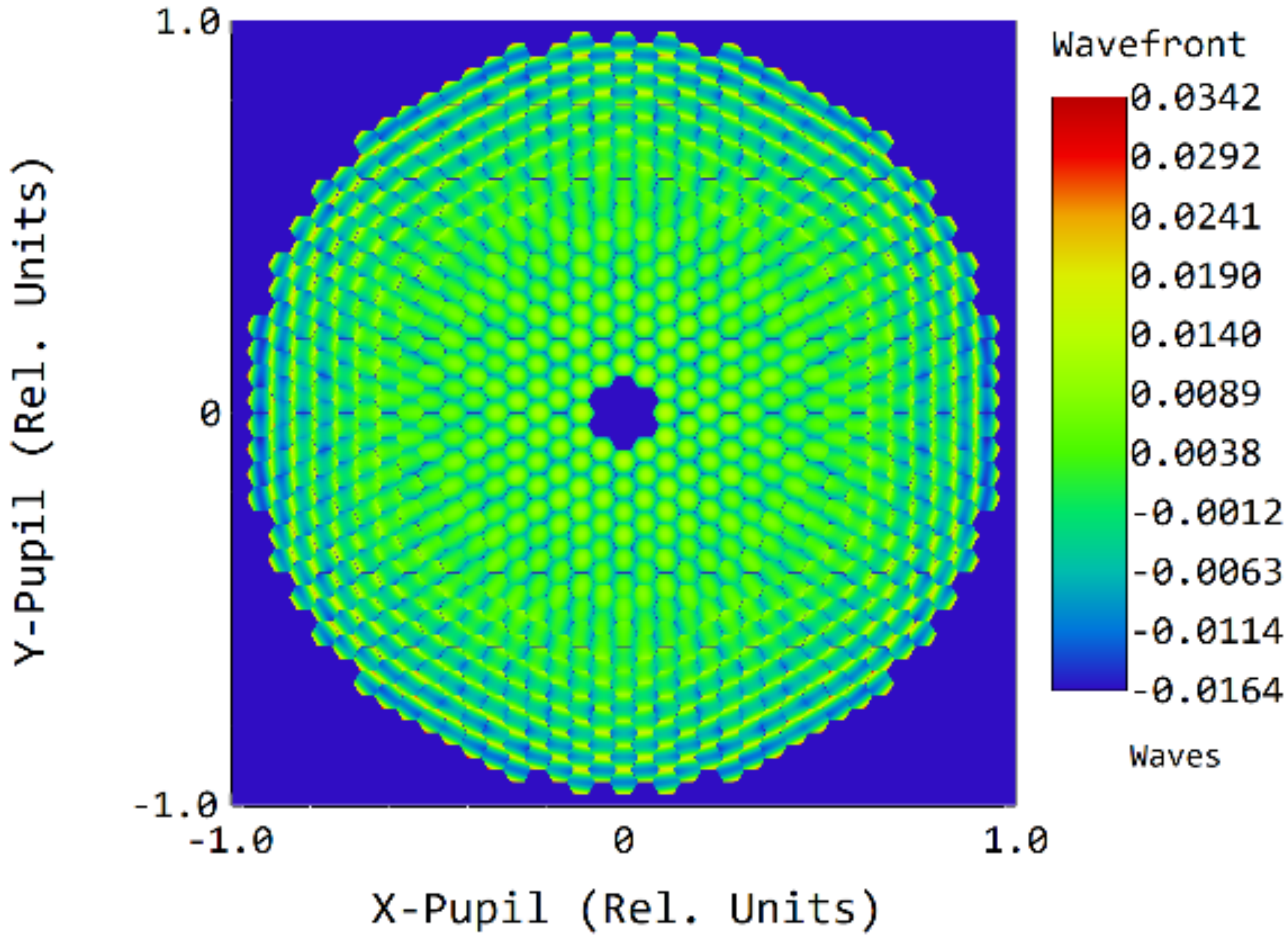
Fully Reconstructed
Mean Defocus
3
2.5
2
1.5
1
0.5
0
RMS Residual (nm)
0
5
10
15
Radius of Mirror Segment (m)

Modelling - OpticStudio

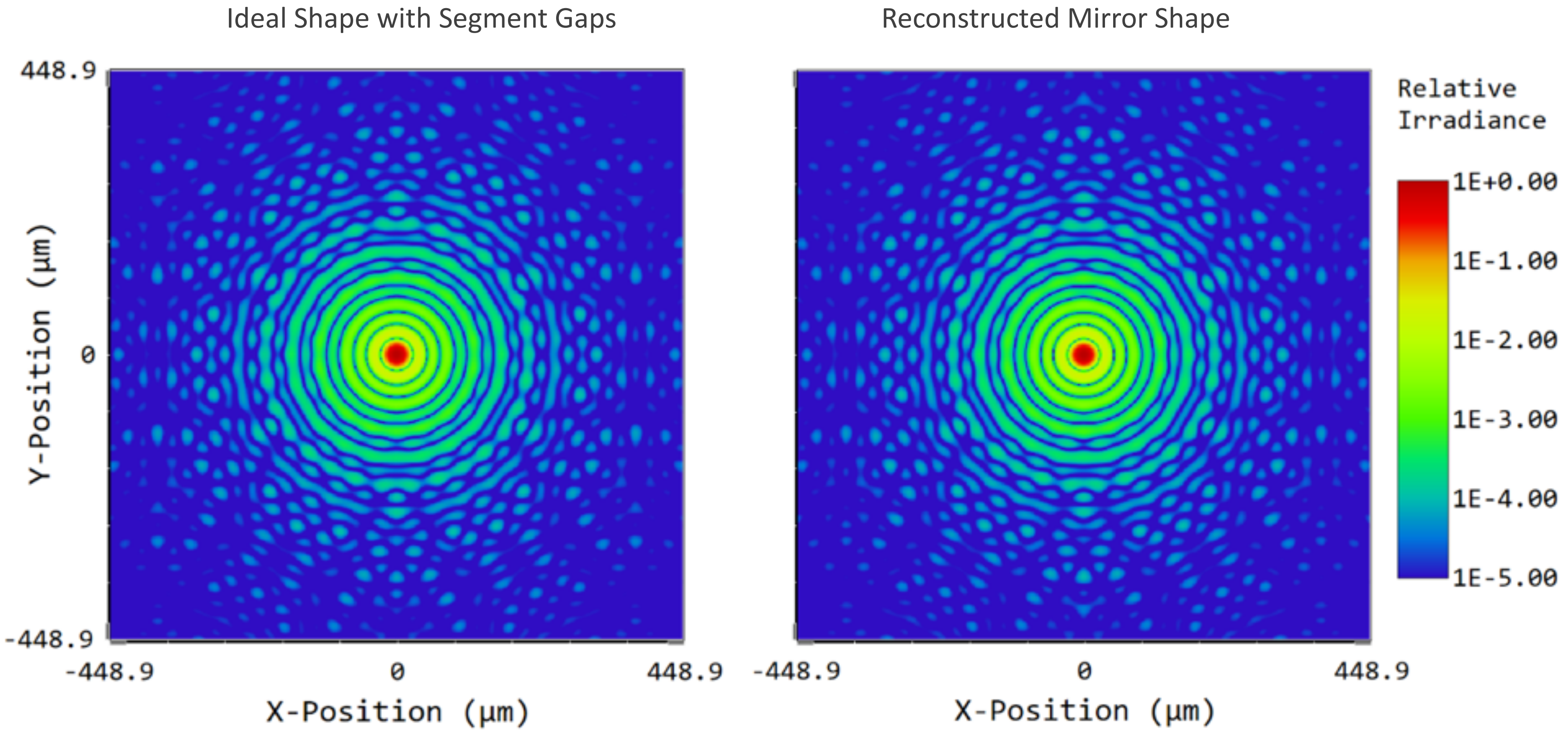




Analysis - Wavefront Error

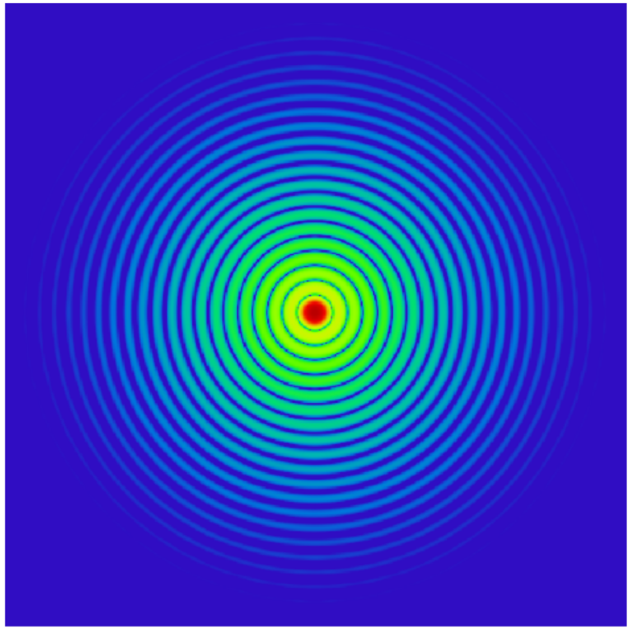


Analysis – Point Spread Function

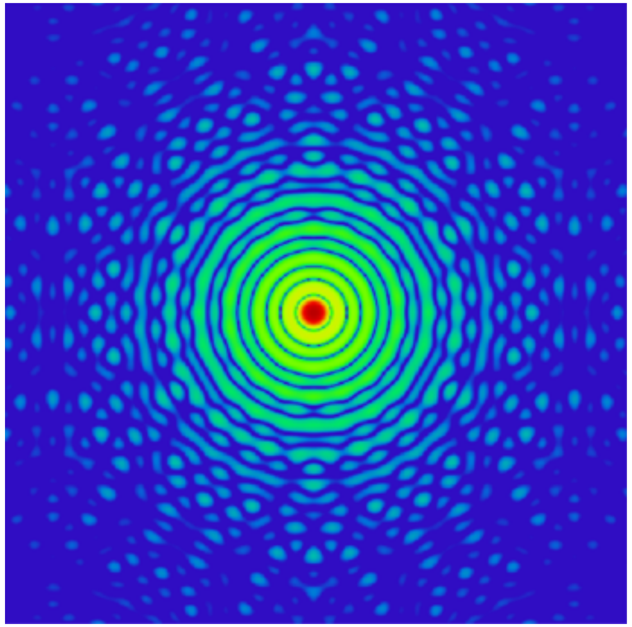


Analysis – Strehl Ratio

Perfectly Ideal/No gaps



Gaps/Reconstructed Mirror



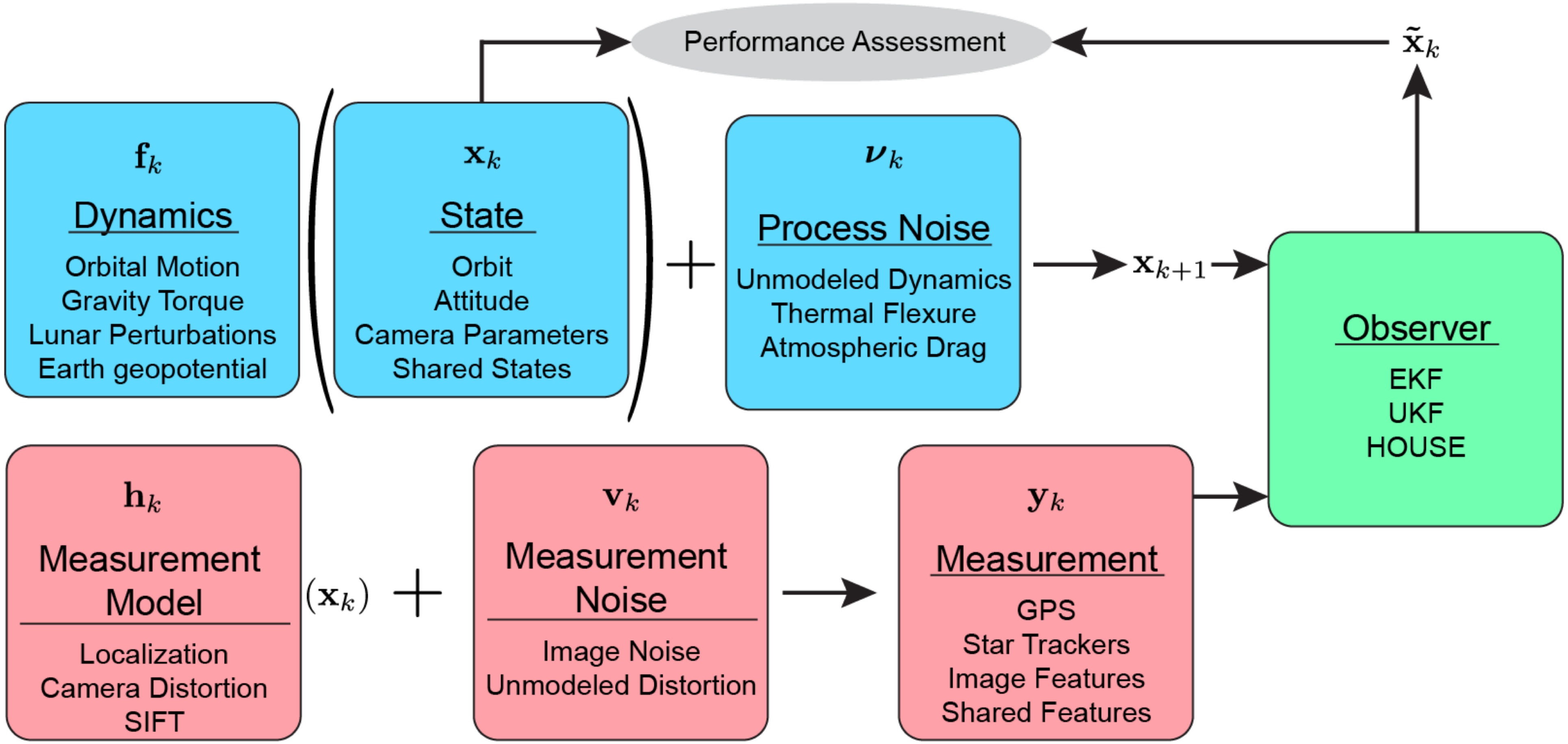
Strehl Ratio: 0.9986

Project Conclusions

- Using Ritchey-Chrétien Cassegrain design
- Modular, random assembly requires uniform design
- Each segment can be approximated with piston, tip, tilt, and defocus
- Given
  - Aligned secondary
  - No manufacturing error
  - No dynamic wavefront control
  - No need for alignment
- A 31-meter self-assembling space telescope is optically **feasible**
  https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20190018062.pdf

Satellite Image Filtering

Technical Overview



Slide courtesy of Dmitry Savransky

Camera Model - Overview

Complete state: $\boldsymbol{x} = [\boldsymbol{q}\ \boldsymbol{r}\ \boldsymbol{c}\ ]^T$

Camera state: $\boldsymbol{c} = [f\ c_1\ c_2\ c_3]^T$

Focal Distance: f

Camera Distortion Model:

$$r_d = r_u(1 - c_1 - c_2 - c_3 + c_1 r_u + c_2 r_u^2 + c_3 r_u^3)$$

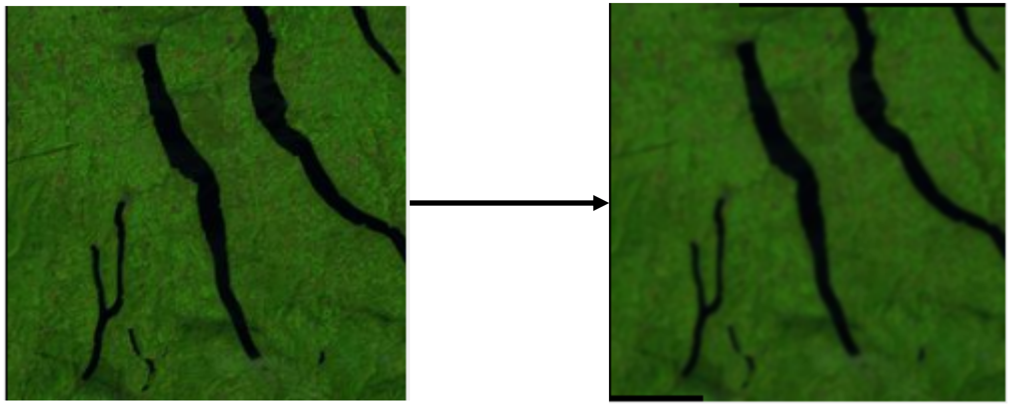
Camera Model - Defocus

Gaussian convolution blurring for defocus

$$\sigma_G = \rho \frac{fs}{2\sqrt{2}N}\left(\frac{1}{f} - \frac{1}{u} - \frac{1}{s}\right)$$ (Mannan and Langer, 2016)

ρ converts physical-space to pixel-space

$$N = \frac{f}{A}$$

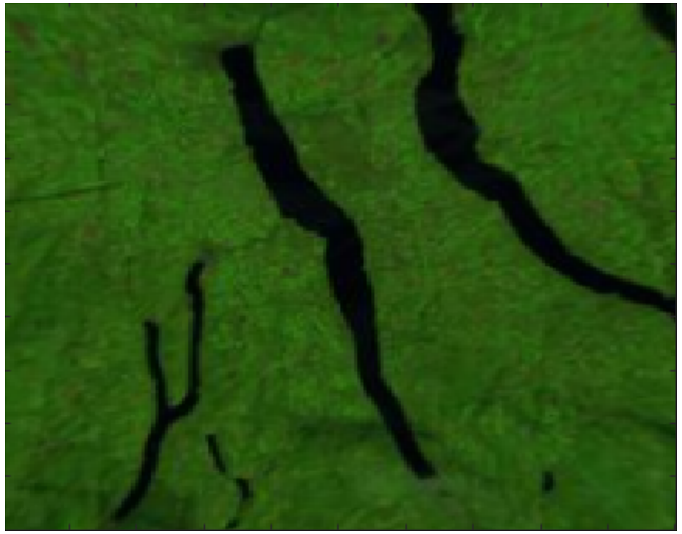


Camera Model – Distortion

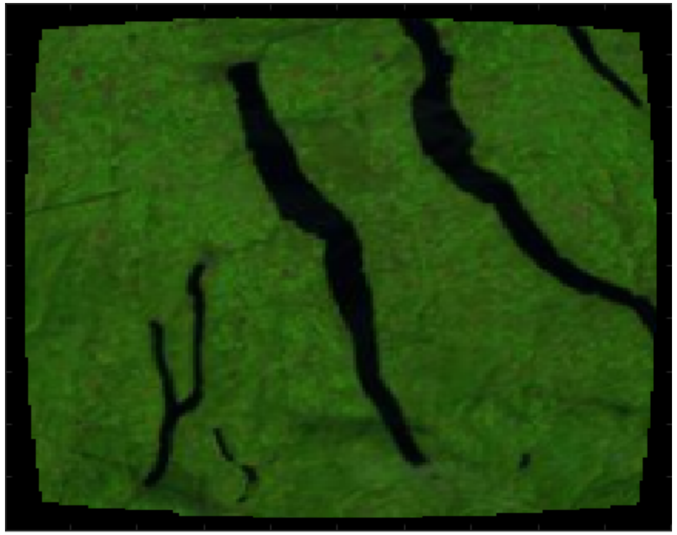
Radially symmetric:

$$r_d = r_u(1 - c_1 - c_2 - c_3 + c_1 r_u + c_2 r_u^2 + c_3 r_u^3)$$

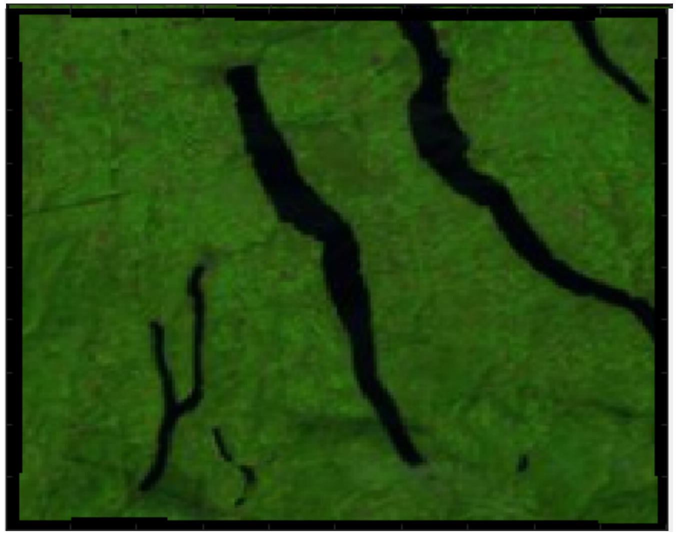
Pincushion



Barrel



Mustache



Scale Invariant Feature Transform (SIFT; Lowe, 2004)

Detect image extrema

Localize keypoints (subpixel)

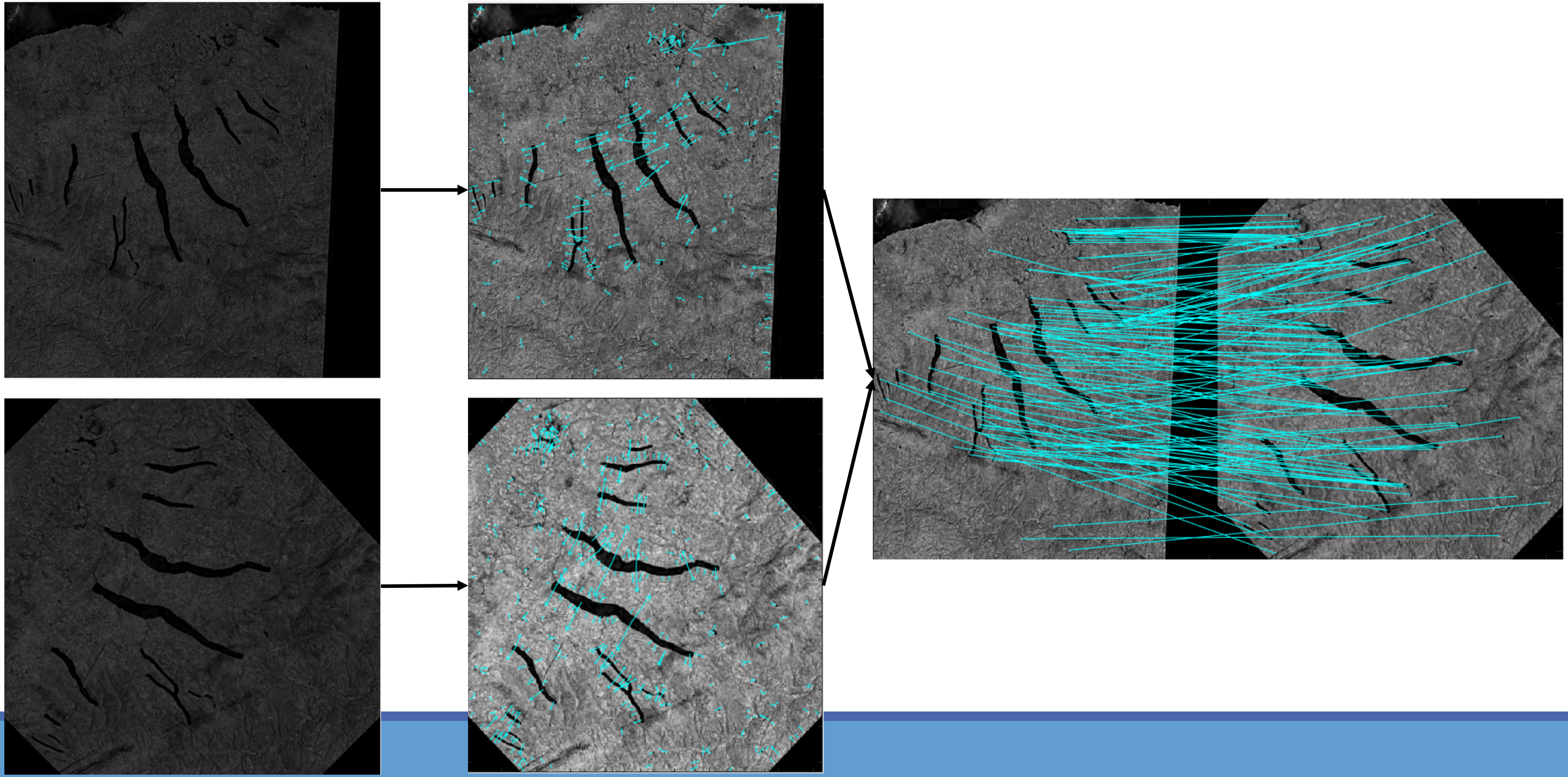
Orientation assignment via local gradient

Generate keypoint descriptors

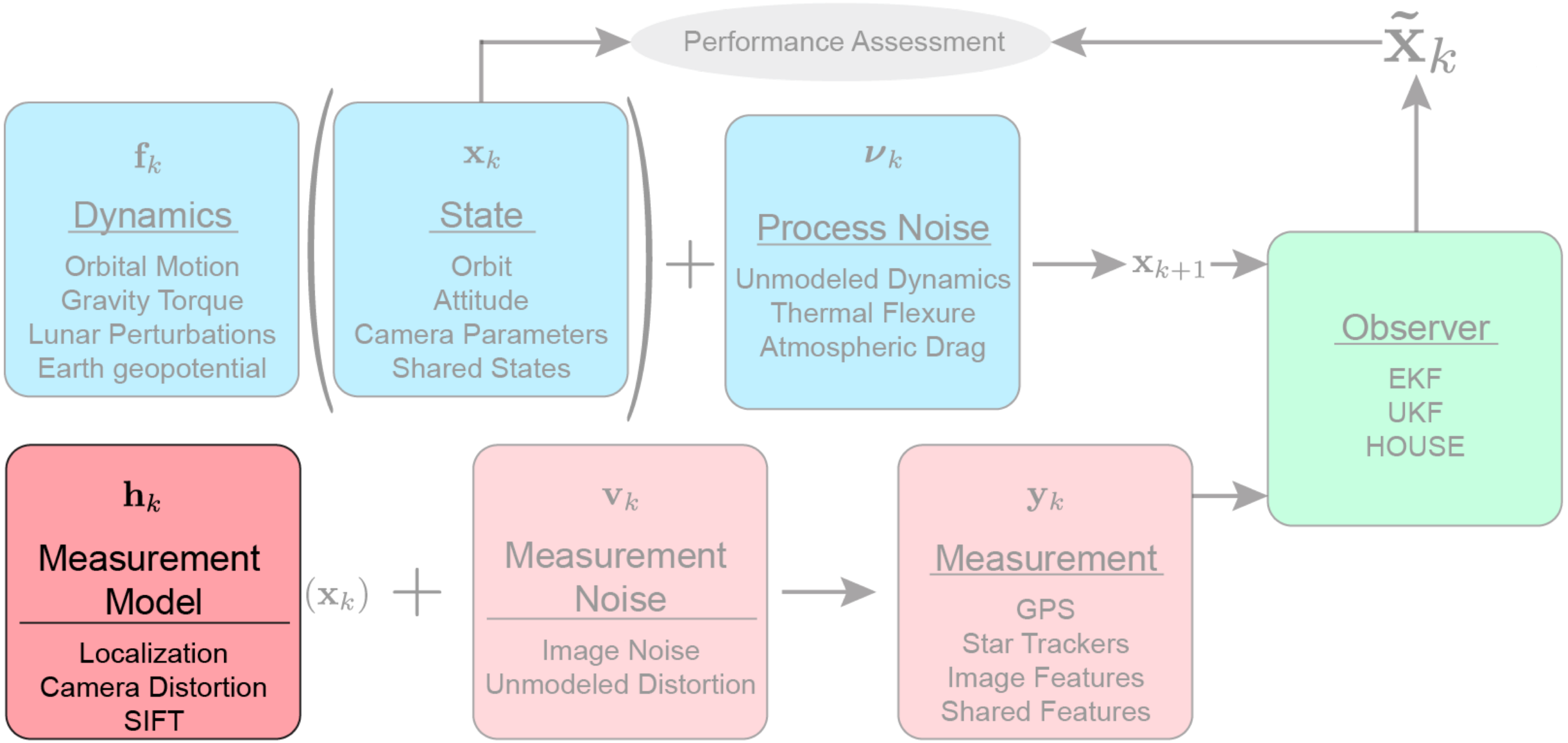
- Each a 128-element vector
- Independent of scale, orientation, illumination

Compare keypoint descriptors from different images

SIFT Example



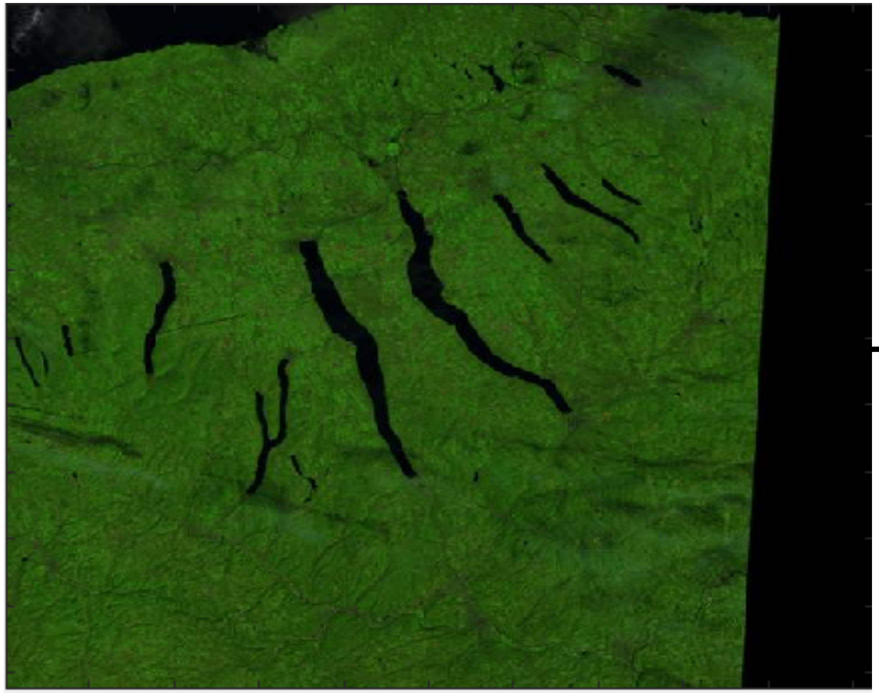
Measurement Model



Slide courtesy of Dmitry Savransky

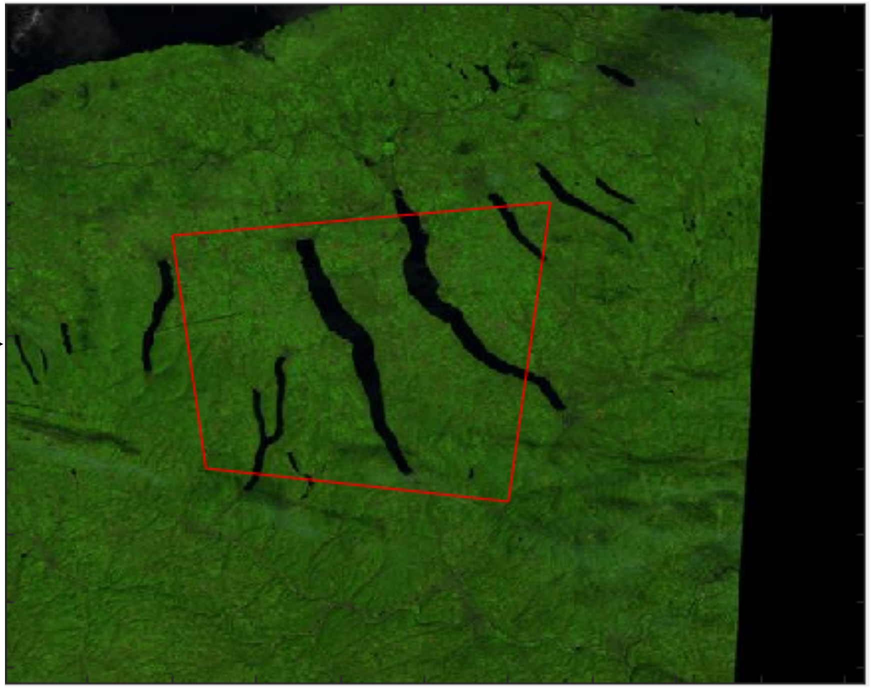
Image Points from Satellite Position

Known Data:
LandSat 8 (simulation)
Previous constellation images (implementation)



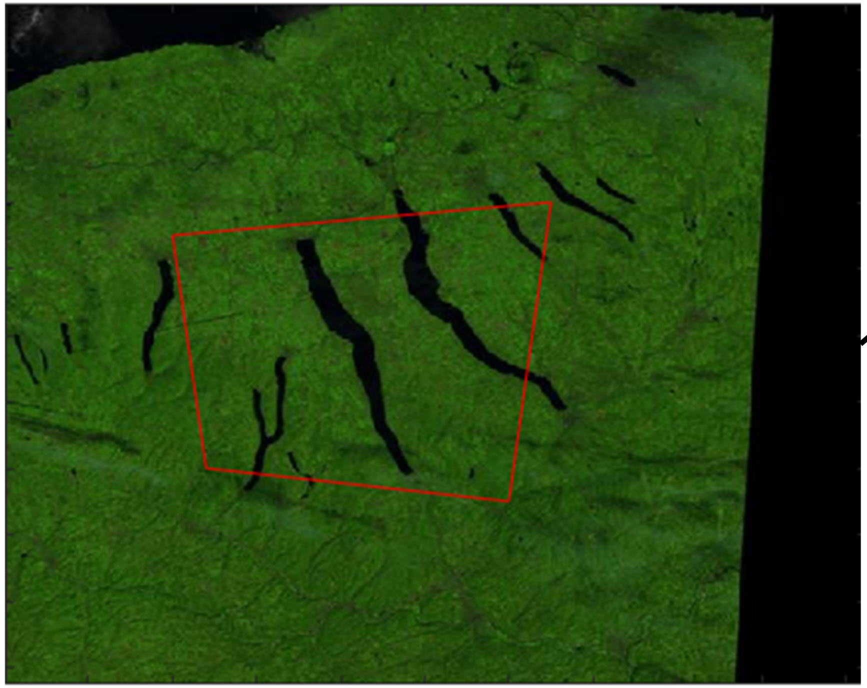
FOV limitations,
$\boldsymbol{q},\ \boldsymbol{r}_{G/O}\ ,\ \boldsymbol{r}_{O/O'}$

Locations of image boundaries

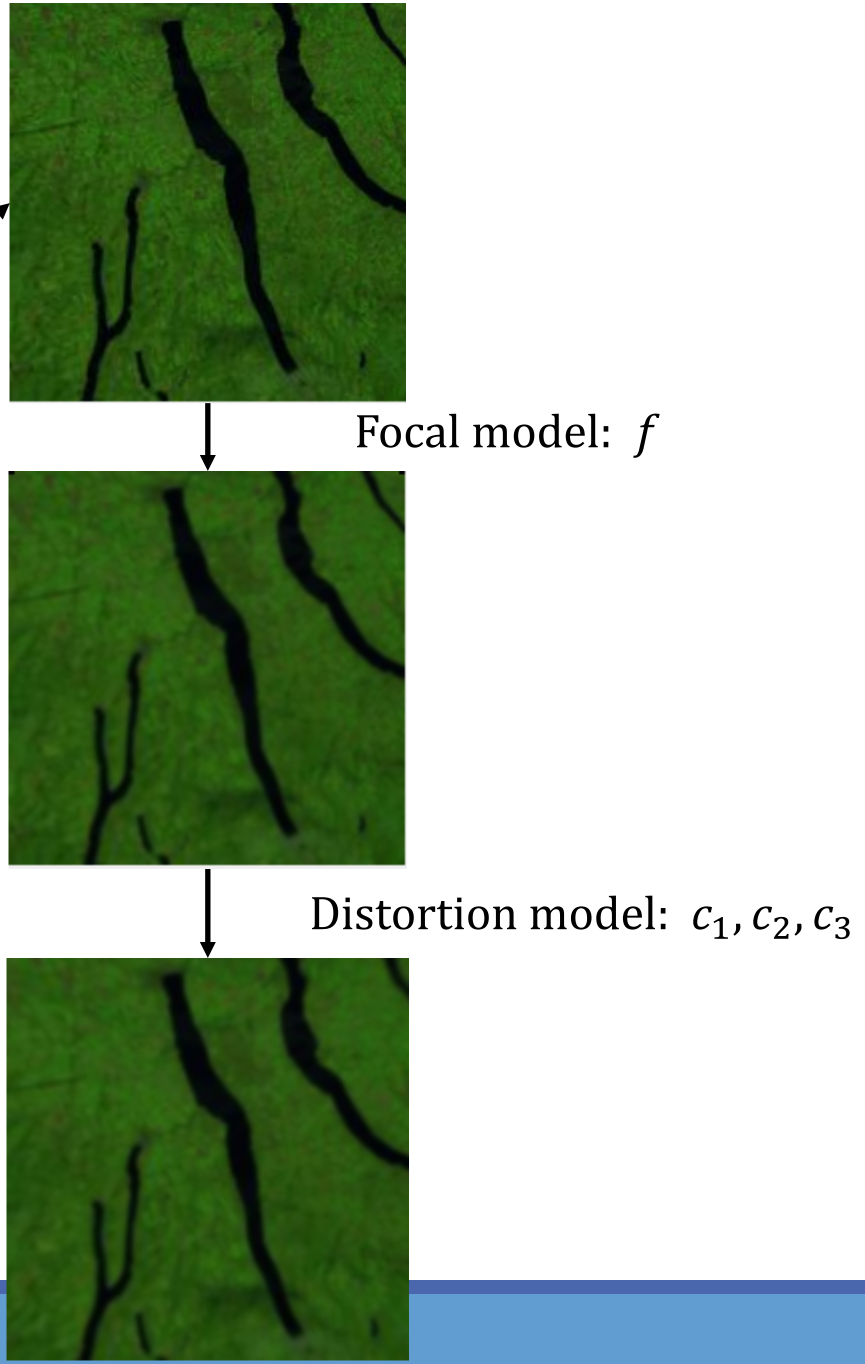


Distorting the Image

Locations of image boundaries

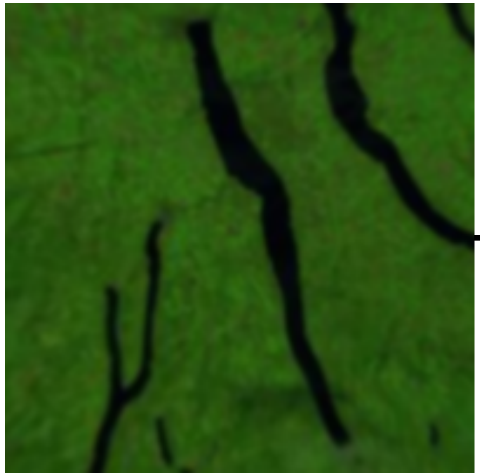


Homogenous
Coordinate transform,
Bicubic
interpolant



Measuring SIFT Keypoints

Predicted Image



SIFT Image

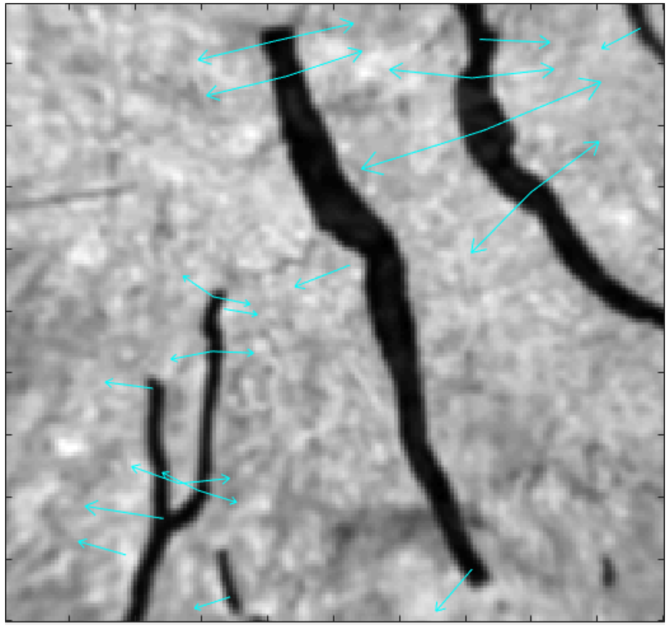


Image Measurement Y

K locations

$Kx128$ descriptor matrix

Measurement Noise

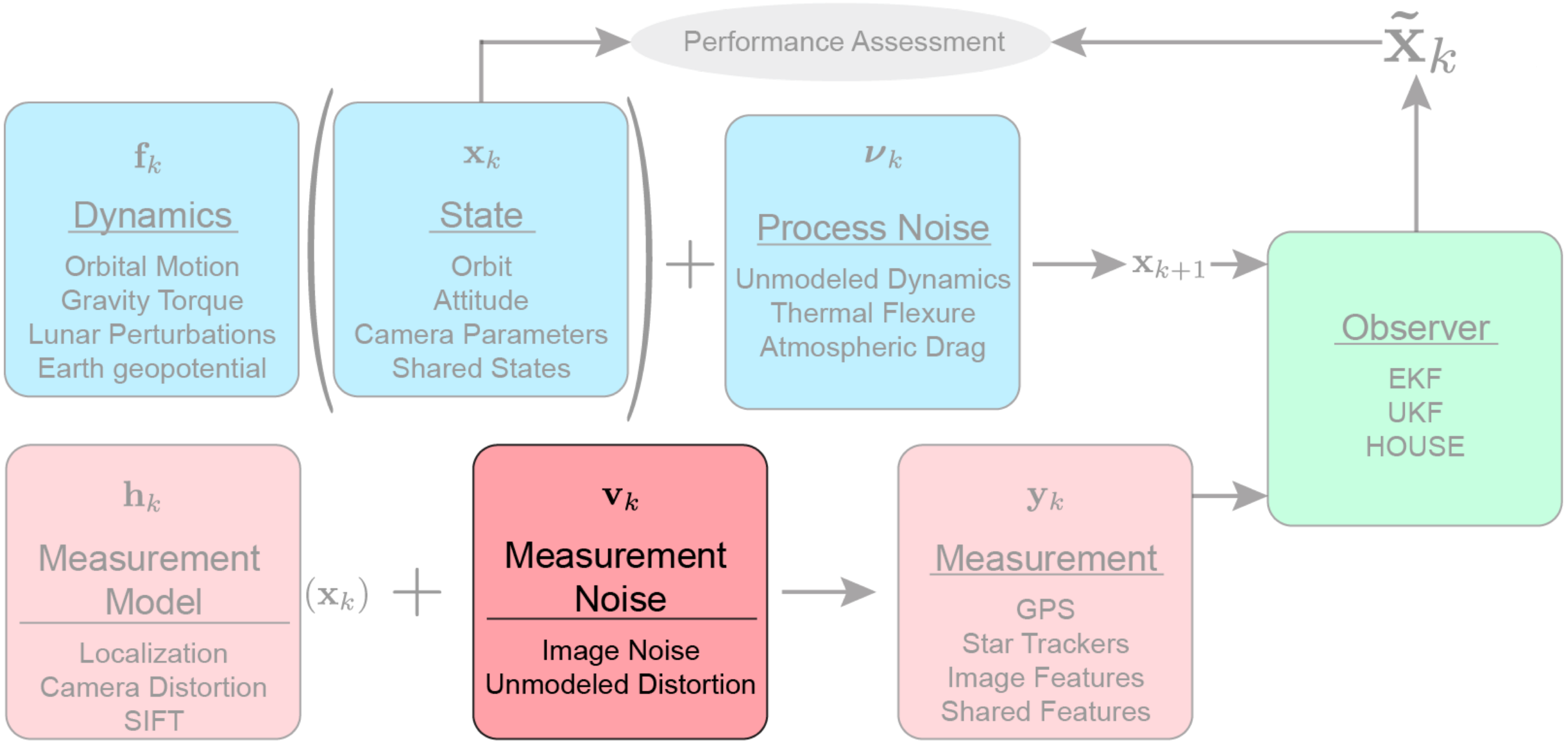
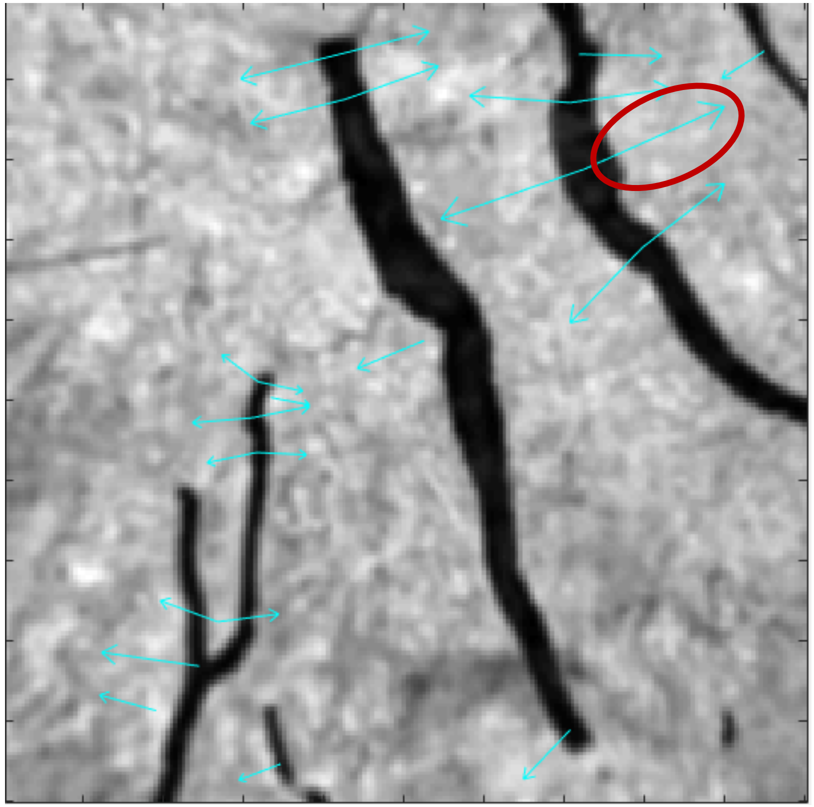


Image Measurement Function Analysis

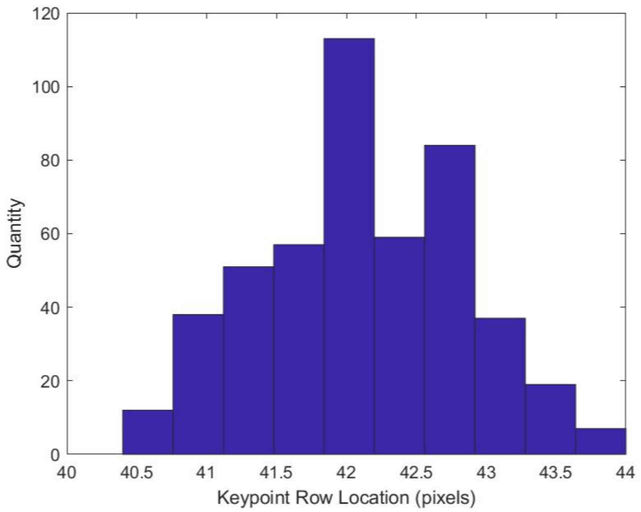
Apply Gaussian noise to state vector 500 times

Compute complete image measurement function

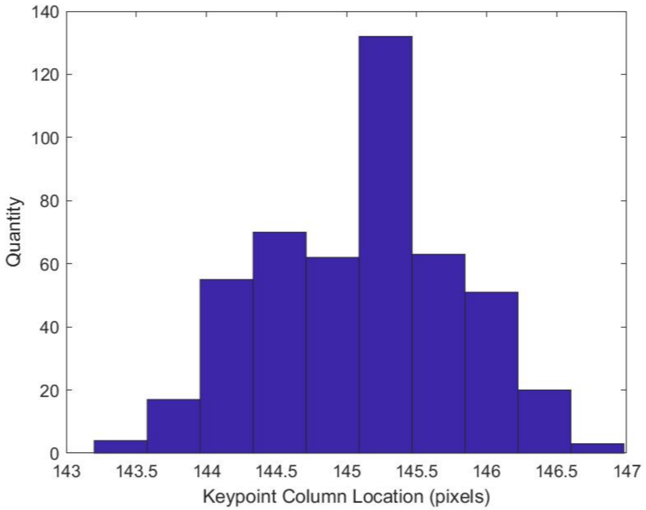
Analyze impact on a given keypoint location and descriptor



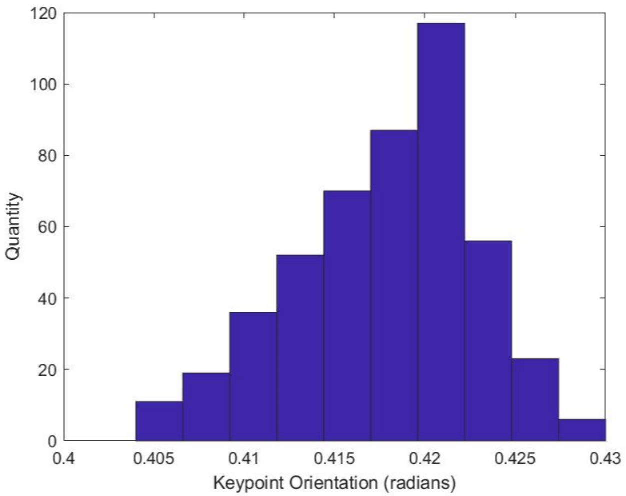
Non-Guassianity: Location



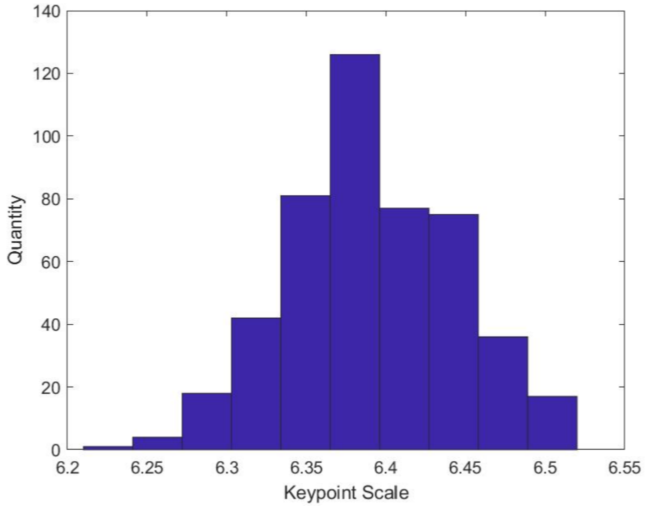
Mean: 42.2   Std. Dev.: 0.73   Skewness: -0.007   Kurtosis: 2.416



Mean: 145.1   Std. Dev.: 0.69   Skewness: -0.048   Kurtosis: 2.50



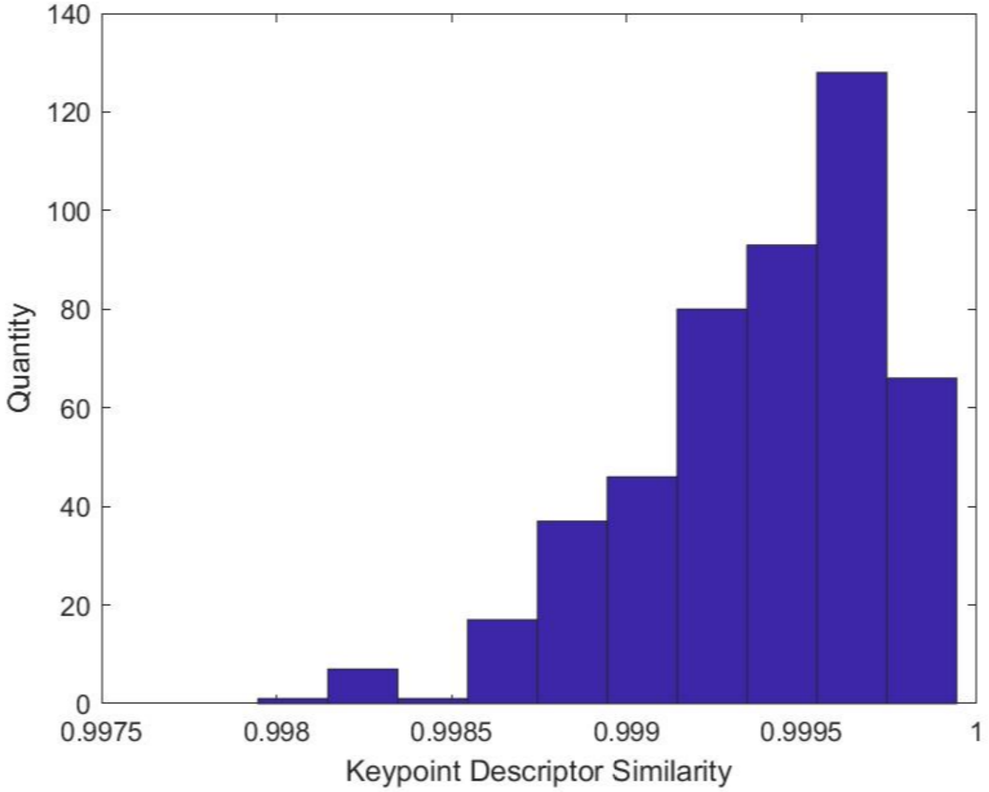
Mean: 0.418   Std. Dev.: 0.005   Skewness: -0.506   Kurtosis: 2.80



Mean: 6.39   Std. Dev.: 0.05   Skewness: -0.041   Kurtosis: 2.80

Non-Guassianity: Descriptors

- Keypoint descriptors are
  - unit vectors
  - location, orientation and scale independent
  - can be compared via dot product
- Because the comparison has a natural upper bound of 1, it is inherently non-Gaussian
- Each noise result was compared to the original
  - Mean: 0.994
  - Standard Dev.: 3.54×10^{-4}
  - Skewness: - 0.094
  - Kurtosis: 3.67



Overview of Presentation

Identifying Exoplanets in Direct Imaging Data with Common Spatial Pattern Filtering

- “Blind source separation algorithms for PSF subtraction from direct imaging.” Poster Presentation. AAS 2017.
- “Planet signal extraction from direct imaging using common spatial pattern filtering.” Oral Presentation. SPIE Optics and Photonics, 2017.
- “Common spatial pattern filtering for detection of circumstellar discs.” Poster Presentation. SPIE Telescopes and Instrumentation, 2018
- Shapiro, J., Savransky, D., Ruffio, J.B., Ranganathan, N., and Macintosh, B. **Detecting Planets from Direct Imaging Observations Using Common Spatial Pattern Filtering.** *The Astronomical Journal*. (2019).
- “Identifying Exoplanets with CSP Filtering and a Forward Model Matched Filter.” Oral Presentation. AAS #235, 2020.
- Shapiro, J., and Savransky, D. **Statistical Properties of the Common Spatial Pattern Filtering with a Forward Model Matched Filter technique for Direct Imaging.** *The Astronomical Journal. (In Prep).*

Optical Design of a Large, Segmented, Space Telescope

- “Optical design of a large segmented space telescope.” Poster Presentation. AAS 2019.
- “Optical design of a modular segmented space telescope.” Oral presentation. SPIE Optics and Photonics 2019
- NASA NIAC Report: https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20190018062.pdf

Satellite Imagery Calibration via Dynamic Filtering

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GPIES Team

THANK YOU

Using Modern Mathematical and Computational Tools for Image Processing

JACOB SHAPIRO
CORNELL UNIVERSITY

B-EXAM

NOV. 3, 2020