Return Visit Optimization for Planet Finding Missions

Dmitry Savransky and N. Jeremy Kasdin, Princeton University



Abstract

We present and expand a rigorous formulation of an existing framework for the simulation and assessment of the capabilities of spaceborne instruments for the direct detection of extra-solar planets. We further use this framework to explore the problem of optimal timing for return visits to systems with prior detections, so as to maximize the probability of verification and orbital characterization. A number of return timing strategies are presented, and it is shown that the best choice of strategy depends on both the target star's distance and bolometric luminosity. Simulation results are presented for probable detection candidates within a sphere of 30 parsecs of the sun.

Simulation of Planetary Detection

In order to simulate the performance of an instrument, it is first necessary to model an arbitrary solar system as observed from Earth. We adopt the method pioneered by Brown ([1],[2],[3]), which consists of two steps: First, a Keplerian orbit is placed in the *y*-*z* plane of a Cartesian coordinate system whose origin is located at the system's primary star, with the observation vector lying along the negative *z* axis. The orbit is then arbitrarily rotated using the Euler angles ψ , θ , and ϕ . The instantaneous position of a planet on such an orbit is given by the current true anomaly (v), the orbit's eccentricity (*e*), and semi-major axis (*a*). The position of the planet in our reference frame at the time of observation ($r_{n/s}$) is given by the parameter set (ψ , θ , ϕ , a, e, v):

 $r_{p/s} = \begin{bmatrix} r(\cos v \sin \theta \sin \phi + \sin v (\cos \theta \cos \psi \sin \phi + \cos \phi \sin \psi)) \\ r(\cos v \sin \theta \cos \phi + \sin v (\cos \theta \cos \psi \cos \phi - \sin \phi \sin \psi)) \\ r(\cos \theta \cos v - \sin \theta \sin v \cos \psi) \end{bmatrix}$

where r is the distance between star and planet, given by the usual conic equation.

We can expect to measure two values from a planetary detection - the apparent separation of the planet and star (*s*) and the difference in brightness between the planet and star (Δ mag). The apparent separation is the projection of the star-planet vector into the plane of the sky, and so is given by the first two coordinates of $r_{p/s}$. Assuming a Lambert phase function for the planet, and a constant albedo (p_E) and planetary radius (R_E) equal to that of the Earth:

Long Range Return Visit Simulation

A sample of 1 million orbits was created with planets placed on them at points where they would be detected. The planets were then propagated forward on their orbits, with checks to see whether they were detectable at every 1/10th of a year. The target star was taken to be a twin of the sun, located 10 pc from Earth. Detection limits on the instrument were set to $\Delta mag < 25$ and *s* between 0.57 and 2.205AU (the lower limit is a currently achievable IWA for a space-borne coronagraph, while the upper limit marks the maximum separation for 'habitable' orbits in this system). The initial population of orbits was drawn from uniform distributions of *a* in [0.4, 30] and *e* in [0, 0.8] (the range of these parameters for all known solar and extra-solar planets). [4]



Figure 4. Return visit detection rates for Sun-twin at 10 pc. Blue curves represent results for all of the planets simulated, while red curves describe only Earth-like planets (a in [0.7, 1.5], e in [0, 0.35]). The solid curves show the percentage of the initially detected planets that are found a second time as a function of re-visit time. The dashed lines are the percentage of planets found a second time when the return time is calculated as 1/2 of the estimated orbital period (these lines represent one specific return time per planet and are *not* functions of time).

The overall best strategy from this simulation appears to be to return as quickly as possible after initial detection. Unfortunately, this will often be impossible in a real mission, and orbital characterization requires detection at various separated points on the orbit. The second strategy is to return at a later time which maximizes detection. In this case, that would be 1.5 years for Earth-like planets and 2.2 years for all planets. However, this corresponds to the mean orbital periods of the populations in question, and is problematic for the same reasons as the first strategy. Using the orbital period approximation yields nearly the same detection rates as this approach, and would improve the chances of accurate orbital characterization.

Figure 1. Schematic of arbitrary orbit model: The red ellipse represents the original orbit, the blue ellipse represents the rotated orbit, and the shaded grey ellipse shows the projection of the orbit onto the plane of the sky (the view of the orbit from Earth). The black and green dotted lines are the Earth-star and Earth-planet observation vectors, respectively.

$$\Delta \text{mag} = -2.5 \log \left[p_E \left(\frac{R_E}{|r_{p/s}|} \right)^2 \frac{\sin(\beta) + (\pi - \beta)\cos(\beta)}{\pi} \right]$$

where β is the star-planet-observer angle. Because the distance to even the nearest stars is so large, the planet-observer vector can be approximated as nearly parallel to the star-observer vector (the green and black dashed lines in Figure 1), so this angle can be calculated as:



red ipse ipse ipse sampled instead, and the eccentric and true anomalies are found via Newton-Raphson iteration of Kepler's equation. These six parameters are enough to estimate the distributions of all other quantities of interest via Monte Carlo simulation.



Figure 2. Schematic of system as observed from Earth. The blue ellipse shows the projection of the orbit onto the plane of the sky. The green portion of the orbit is where the Δ mag would be low enough to allow detection. The red circle is the projected inner working angle (IWA) of the observing instrument.

The probability of detection is a function of the position of the planet on its orbit at the time of observation. For example, in the sample orbit in Figures 1 and 2, detection can occur on less than 1/3 of the orbit, and only on specific intervals. If the orbit lies partially within the IWA of our instrument, then the highest probability of detection occurs in the volume around apoapsis, followed by the volume around periapsis. If we are viewing an orbit directly head on, unobscured portions of the orbit will still lie on opposite sides. The best chance for repeating a detection will come either one or one-half orbital periods after an initial detection. Since the observed illumination of a planet depends on the orientation of the system, there is no guarantee that the contrast between planet and star will be greater or less at any point in the orbit. The best we can do is to return one orbital period later, to ensure the same levels of illumination as during the initial detection.

Return Visit Simulation Using Real Stars

In order to test these return timing strategies, a candidate pool was constructed consisting of 245 stars within 30 pc of Earth. All of these are main sequence stars, with no close, bright companions, no indications of intrinsic variability of low metallicity, and whose luminosities and distances would allow for detection of Earth-like planets. [6] For each star, 100,000 initially found planets were created and detection rates were calculated using absolute return times of 0.5, 1 and 1.5 years, 0.5 and 1 estimated orbital period, and 0.5 and 1 Earth-like period (the period of a 1 AU orbit scaled by the square root of the star's luminosity and using the star's mass estimate).



Planning Return Visits

0.2

0.4

0.6

0.8

1.2

1.4

1.6

1.8

The problem of return visits breaks down into two basic questions: what is the optimal re-visit timing if a planet detection occurred and what is the optimal timing if a detection did not occur during the first visit? Both of these questions can be answered in full only with exact characterization of the planet's orbit, which, of course, is unavailable. These questions can be answered from a global optimization standpoint - picking the best revisit times based on the type of planet (and orbit) we would like to see. However, if a detection does occur, there is the potential to approximately characterize the planet's orbit from available information, which is why we will focus on the first question.

The orbital period is a function of the star's mass and the semi-major axis of the orbit, both of which we can roughly estimate from known data. The mass of the star can be calculated from its luminosity via an empirically or theoretically derived Mass-Luminosity Relation (MLR). We use the empirical MLR by Henry and McCarthy, which is accurate to within 7% for mass ranges corresponding to visual magnitudes between 1.45 and 10.25:

$$\log\left(\frac{M}{M_{sun}}\right) = 0.002456V^2 - 0.09711V + 0.4365$$

where *M* is the mass of the star and *V* is the visual magnitude. [5]

In order to estimate *a*, we define a new parameter equal to the ratio between apparent separation and the length of the semi-major axis:

$$E = \frac{s}{a} = \frac{1 - e^2}{e \cos v + 1} \sqrt{\left(\cos v \sin v + \cos \theta \cos \psi \sin v\right)^2 + \left(\sin v \sin \psi\right)^2}$$

While the distribution of Ξ depends on the initial distribution of the eccentricity, the maximum value of this parameter is always one. *The* observed apparent separation is the most likely value for the semi-major axis!



Figure 3. Probability Density Functions of Ξ (= s/a) for different distributions of e. The solid blue line shows Ξ for all possible orbits - e uniformly distributed between 0 and 1. The dashed blue line is for e in [0, 0.35]. The red curve also has e in [0, 0.35], but only 'detectable' planets are sampled: samples are forced to have Δ mag <25 and s>0.6 AU. In all cases, the apparent separation is the most likely estimate for semi-major axis.



Figure 5. Return visit detection rates for 245 stars within 30 pc of Earth using multiple re-visit timing strategies. Each data point represents the percent of initially detected planets found during the re-visit. The results are plotted as a function of the star's visual magnitude.



Figure 6. Comparison of the most successful re-visit timing strategies from Figure 5.

Conclusions and Future Work

We can draw several important conclusions from the last simulation:

- There is no single optimal return strategy: a combination of these (and other) strategies must be employed, along with all available information about the candidate stars, to produce an optimal revisit schedule.
- In most cases, there is a well defined region, describable by apparent magnitude, where the various strategies cross and one is clearly better than another. For example, at V = 5 better results are consistently achieved when returning after 1 estimated orbital period than after 1/2 period.
- While in some cases returning a fixed time after initial detection produces better results than basing the re-visit time on the approximated orbital period, the latter produces much more consistent results (the variance between results for stars of similar *V* is much smaller). The best performance in the absolute time cases occurs on the largest orbits, where the planets would not have moved very far in the given time (and the resulting orbital characterization would be much worse).
- On average, using the orbital period approximation provides more information from two visits than the fixed time strategy. If the planet is detected in the re-visit, the second measurement of apparent separation significantly improves the semi-major axis estimate. In cases where the planet is not found again, the period estimate can be updated, providing tighter bounds on the times when the planet is likely to be

For any distribution of eccentricity, Ξ will range between 0 and $(1+\max(e))$, but low *s* values can correspond to any *a* values, which flattens the distribution of Ξ . As we restrict eccentricities to what we consider to be 'Earth-like', apparent separation continually becomes a better estimator for semimajor axis. Furthermore, the instrument's IWA, which prevents detection much of the time, actually helps here by removing detections with low apparent separations. Using $\Xi = 1$ yields an average 24% error in the estimation of *a* in simulation. detected. The fixed time strategy provides little or no additional information in cases of a failed second detection.

Future work in this area will include the incorporation of other known data about the candidate pool of stars to improve the fidelity of our estimates, as well as the application of the basic simulation framework to other questions, including return timing when no initial detection has occurred, and overall mission planning.

References

[1] Brown, R. A. Obscurational completeness, Astrophysical Journal, Vol. 607, p. 1003-1017, 2004.
[2] Brown, R. A. New information from radial velocity data sets, Astrophysical Journal, Vol. 610, p. 1079-1092, 2004.
[3] Brown, R. A. Single-visit photometric and obscurational completeness, Astrophysical Journal, Vol. 624, p. 1010-1024, 2005.
[4] Butler, R. P., et al. Catalog of Nearby Exoplanets, Astrophysical Journal, Vol. 646, Pg. 505, 2006.
[5] Henry, T. J. and McCarthy, D. W. Jr. The Mass-Luminosity Relation for stars of mass 1.0 to 0.08M ○, Astronomical Journal, Vol.106, No. 2, p.773-789, 1993.
[6] Turnbull, M. http://sco.stsci.edu/tpf_tldb/