Dynamic Filtering for the Analysis of Astrometric and Radial Velocity Data Sets for the Detection of Exoplanets

AIAA Guidance, Navigation and Control - Session 85-GNC-50

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August 12th, 2009

Motivation

- The most successful methods for planet-finding (so far) are indirect detections via observations of stars.
- Ground-based indirect detection techniques are limited in the size of planets they can find.
- Finding Earth-like planets will require the development of both new observational tools and analysis techniques.
- We seek to analyze the effectiveness of well-developed filtering techniques for orbit estimation in finding extra-solar planets and constraining their orbits.

Outline

Known Exoplanets and Discovery Methods

http://exoplanet.eu/catalog.php

- Radial Velocity and Astrometry
  - 332 planets in 282 systems
  - 61 planets confirmed via Transit photometry
- Imaging
  - 11 planets in 9 planetary systems
- Microlensing and Pulsar Timing
  - 15 planets in 11 planetary systems

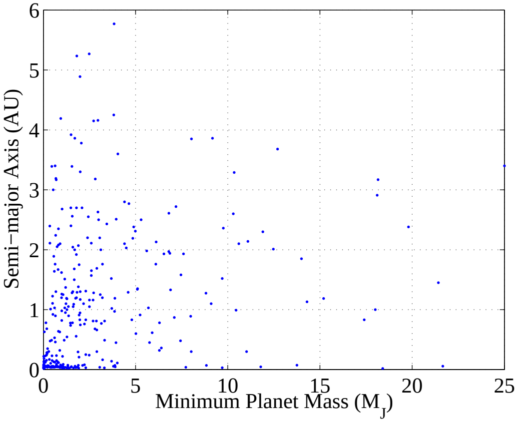

Figure: Semi-major axis vs. Minimum planet mass for known exoplanets.

Astrometry and Radial Velocity

Astrometry

- Uses optical interferometry to find angular distance between two objects
- Use set of reference stars to find position of target star with respect to fixed centroid
- Produces target star's position in plane of the sky

Radial Velocity

- Uses spectroscopy to find wavelengths of target star's emitted light
- After accounting for other effects, remaining changes in wavelength are attributed to doppler effect
- Produces target star's velocity along the line of sight

Both methods require us to infer the presence of planets from motion of the target star - 'Stellar Wobble'

Periodograms and Fitting

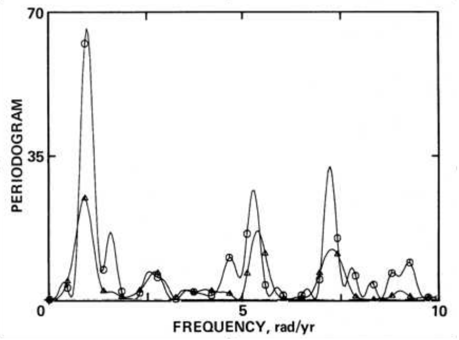

Figure: Periodogram output. [Black and Scargle, 1982]

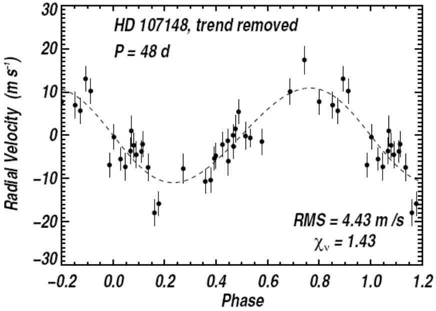

Figure: Orbital fit to radial velocity data. [Butler et al., 2006]

Treat astrometry and radial velocity as data streams containing periodic signals. Find the periodicities and fit orbits. [Sozzetti, 2005]

An Alternate Approach

Treat astrometry and radial velocity as partial observations of an underlying dynamical system.

- System evolution is governed by known laws, but parameters are unknown
- Dynamic filtering can be used to reconstruct the parameters

Advantages:

- Can use exact dynamic model
- No *a priori* assumptions about planetary system
- May be possible to simultaneously fit whole system
- May be possible to detect long period orbits

Possible Issues:

- Sensitive to noise and initial conditions
- Sensitive to nonlinearities
- Model dependent

Dynamic Filtering

- Astrometric (or RV) measurements ($\mathbf{z}$) at time k are a function of a state vector ($\mathbf{x}$) describing the positions of all orbiting planets, and time, with added noise $\mathbf{n}$ of covariance R:

$$\mathbf{z}_k = \mathbf{f}(\mathbf{x}_k, k) + \mathbf{n}$$

- The solution to this problem is a minimization with respect to $\mathbf{x}$ for N observations of the cost function:

$$J = \sum_{k=1}^{N} \left[\mathbf{z}_k - \mathbf{f}(\mathbf{x}_k, k)\right]^T R^{-1} \left[\mathbf{z}_k - \mathbf{f}(\mathbf{x}_k, k)\right]$$

  subject to the constraints of the physical system (i.e. Newtonian dynamics) and any inherent constraints in the formulation of the state (i.e., quaternion definition, eccentricity bounds, etc.).

- We can re-formulate this as a recursive filter, using each observation to update the estimate of the underlying state, and our knowledge of the physical system to propagate the state in time.

Outline

System Model

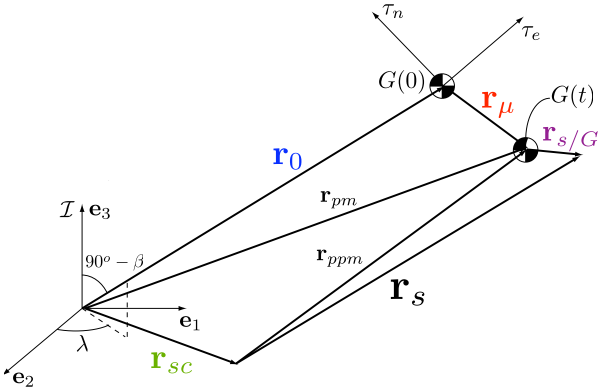

Figure: Schematic of an astrometric observation.

$$\mathbf{r}_s = \mathbf{r}_0 + \mathbf{r}_\mu - \mathbf{r}_{sc} + \mathbf{r}_{s/G}$$

Fundamental astrometric observation is $\hat{\mathbf{r}}_s$ (can be decomposed into two angles)

Choice of State

Form of state vector determines form of dynamic updates, observation, and how easy it is to constrain states from describing non-physical orbits.

Some options:

- Position and velocity: $\mathbf{x} = [\mathbf{r} \quad \dot{\mathbf{r}}]^T$
- Orbital elements: $\mathbf{x} = \begin{bmatrix} a & e & I & \omega & \Omega & \nu \end{bmatrix}^T$
- Quaternions: $\mathbf{x} = \begin{bmatrix} a & e & \epsilon_1 & \epsilon_2 & \epsilon_3 & \epsilon_4 & \nu \end{bmatrix}^T$, $\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \epsilon_4^2 = 1$
- Angular Momentum and Eccentricity vector:
  $\mathbf{x} = \begin{bmatrix} L_1 & L_2 & L_3 & e_1 & e_2 & E \end{bmatrix}^T$, $\mathbf{L} = \mathbf{r} \times \dot{\mathbf{r}}$, $\mathbf{e} = \frac{1}{\mu}(\dot{\mathbf{r}} \times \mathbf{L} - \mu\hat{\mathbf{r}})$

- Position and velocity state proved most effective.
- Fitting orbits and masses simultaneously proved too difficult - split into orbital fitting with assumed masses and line search (or Monte Carlo) for masses.

Choice of State (cont.)

Let the state vector for a system of n planets be:

$$X = \begin{bmatrix} \mathbf{r}_1 & \dot{\mathbf{r}}_1 & \ldots & \mathbf{r}_n & \dot{\mathbf{r}}_n & \mathbf{r}_{s/G} & \dot{\mathbf{r}}_{s/G} \end{bmatrix}^T$$

with the state estimate propagation given by

$$\ddot{\mathbf{r}}_j = -\sum_{k \neq j} \frac{\mu_k \mathbf{r}_{k/j}}{|\mathbf{r}_{k/j}|^3} \qquad j = 1, \ldots, n, s/G \qquad \mathbf{r}_{k/j} = \mathbf{r}_k - \mathbf{r}_j$$

Choice of State (cont.)

Let the state vector for a system of n planets be:

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Augment state with constant parameters to account for unknown proper motion and stellar distance

$$\bar{X} = \begin{bmatrix} \mathbf{r}_1 & \dot{\mathbf{r}}_1 & \ldots & \mathbf{r}_n & \dot{\mathbf{r}}_n & \mathbf{r}_{s/G} & \dot{\mathbf{r}}_{s/G} & \mathbf{r}_\mu & \varpi \end{bmatrix}^T$$

$$\varpi = \frac{a}{\|\mathbf{r}_0\|}$$

Extended Kalman Filter

[Crassidis and Junkins, 2004]

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{f}(\hat{\mathbf{x}}(t), t) \qquad \dot{\mathbf{P}}(t) = \mathbf{F}(t)\mathbf{P}(t) + \mathbf{P}(t)\mathbf{F}^T(t) + \mathbf{Q}$$
$$\hat{\mathbf{x}}_0 = E[\mathbf{x}(0)] \qquad \mathbf{P}_0 = E[(\mathbf{x}(0) - \hat{\mathbf{x}}_0)(\mathbf{x}(0) - \hat{\mathbf{x}}_0)^T]$$
$$\mathbf{F}(t) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(t)} \qquad \mathbf{Q}(t) = E[\mathbf{w}(t)\mathbf{w}^T(\tau)]$$

$$\hat{\mathbf{x}}_{k_i}^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_{k_i}\left(\mathbf{y}_k - \mathbf{h}(\hat{\mathbf{x}}_{k_{i-1}}^+) - \mathbf{H}_{k_i}(\hat{\mathbf{x}}_k^- - \hat{\mathbf{x}}_{k_{i-1}}^+)\right) \qquad \hat{\mathbf{x}}_{k_0}^+ = \hat{\mathbf{x}}_k^-$$
$$\mathbf{K}_{k_i} = \mathbf{P}_k^- \mathbf{H}_{k_i}^T \left(\mathbf{H}_{k_i}\mathbf{P}_k^- \mathbf{H}_{k_i}^T + \mathbf{R}_k\right)^{-1} \qquad \mathbf{H}_{k_i} = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k_i}^+}$$
$$\mathbf{P}_{k_i}^+ = (\mathbf{I} - \mathbf{K}_{k_i}\mathbf{H}_{k_i})\,\mathbf{P}_k^- \qquad \mathbf{R}(t) = E[\mathbf{v}(t)\mathbf{v}^T(\tau)]$$

To simplify observation equation, we expand $\hat{\mathbf{r}}_s$ to second order in ϖ:

$$\hat{\mathbf{r}}_s \approx \hat{\mathbf{r}}_0 + \varpi\left(\tilde{\mathbf{r}}_\mu - \tilde{\mathbf{r}}_{sc} + \tilde{\mathbf{r}}_{s/G} - (\hat{\mathbf{r}}_0 \cdot \tilde{\mathbf{r}}_\mu)\hat{\mathbf{r}}_0 + (\hat{\mathbf{r}}_0 \cdot \tilde{\mathbf{r}}_{sc})\hat{\mathbf{r}}_0 - (\hat{\mathbf{r}}_0 \cdot \tilde{\mathbf{r}}_{s/G})\hat{\mathbf{r}}_0\right)$$

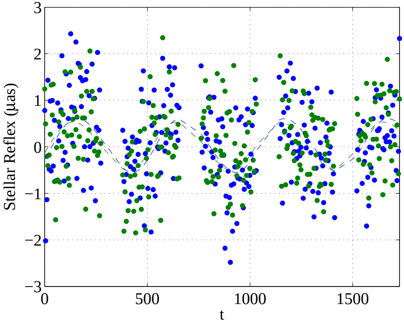
Some Modifications

- Position and Velocity state makes it easy to describe open orbits
  - Introduce inequality constraints of the form $D\bar{X} \leq d$ to constrain orbital specific energy
  - At each time step, solve quadratic programming problem of the form $\min_{\tilde{x}} \left(\tilde{x}^T W \tilde{x} - 2\bar{X}^T W \tilde{x} \right)$ s.t. $D\tilde{x} \leq d$ [Simon and Simon, 2006]
- Nonlinearities in state propagation make filter very sensitive to initial conditions
  - Attempt to constraint initial conditions via periodograms and other coarse analysis of data
  - Introduce random restarts when state or covariance diverges
- Covariance estimate extrapolation is potential source of problems for nonlinear state update
  - Evaluated particle filter-like approach
  - Generate set of random states distributed according to current covariance estimate with mean of current state
  - Propagate random states and find covariance

Outline

Simulated Data

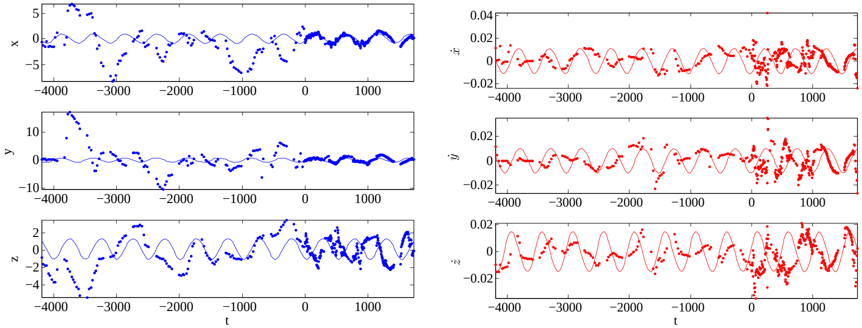
- 1 $M_\odot$ star at 10pc
- 15 years of data
  - 10 years of radial velocity data
  - 5 years of astrometric and radial velocity data
- Data spaced to simulate spacecraft operation constraints
- Radial velocity noise of up to 1 m/s
- Astrometry noise of up to 0.82 μas



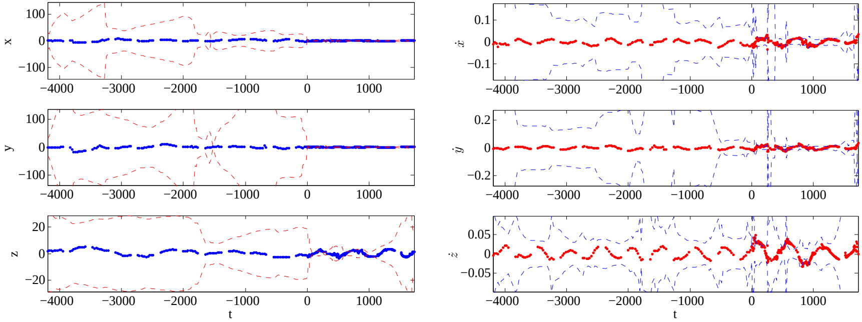
- Integrate equations of motion with Runge-Kutta-Nyström 8-6 variable time step scheme [Papakostas and Tsitouras, 2000]

Single Jupiter Mass Planet with Noise

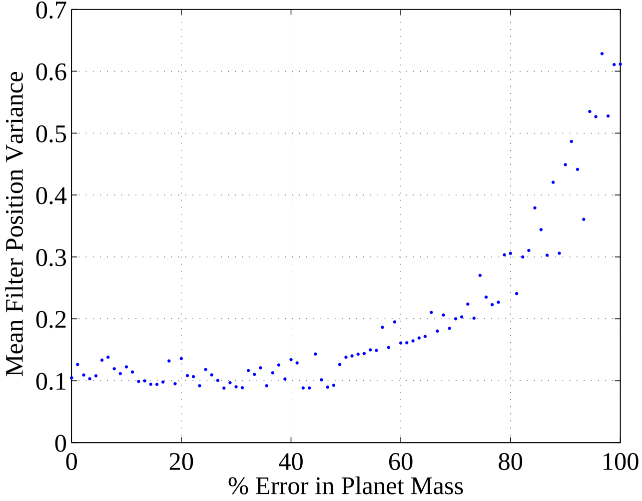
Single Earth Mass Planet with Noise



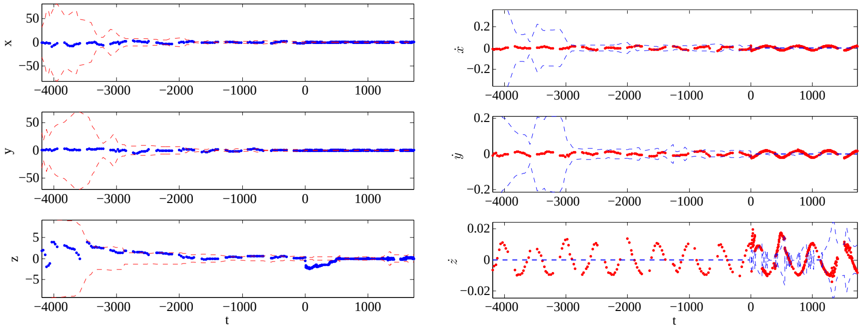
Single Earth Mass Planet with Noise (cont.)



Ambiguity in Planet Mass



Jupiter Mass Planet in multi-body system with Noise



Conclusions

- The EKF implementation can fit orbits for Earth-sized planets
- The EKF implementation cannot (so far) constrain masses to better than $\pm$ 50%.
- The EKF implementation cannot (so far) fit Jupiter mass and Earth mass planets simultaneously.
- The large noise magnitude is the hardest part of this problem.
- Better constraints on initial conditions would significantly reduce processing time and improve filter efficiency.
- Other filter designs should be investigated.

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