Solar Sail Trajectories and Orbit Phasing of Modular Spacecraft for Segmented Telescope Assembly about Sun-Earth L2

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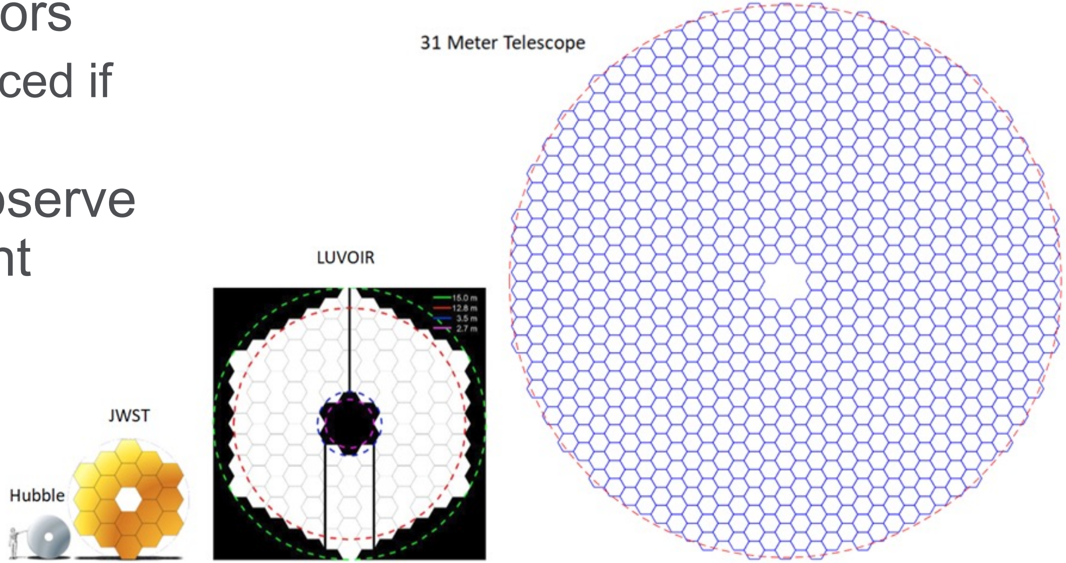
Motivation

- LUVOIR and future space telescopes require bigger primary mirrors
- Easier to segment the mirrors
  - Manufacturing costs reduced if fabricated in bulk
- >15m mirror required to observe main sequence turnoff point past the local group[1]
  - 31m primary mirror, needs 840 mirrors[2]



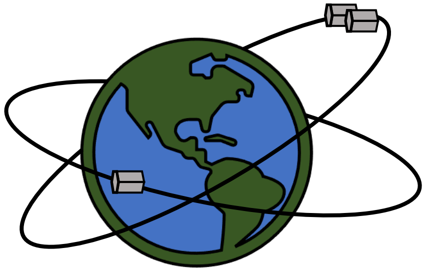
[1]D. A. Fischer, B. Peterson, LUVOIR Team, et al., “The LUVOIR Mission Concept Study Interim Report,” arXiv preprint arXiv:1809.09668, 2018.
[2]J. Shapiro, D. Keithly, G. Soto, D. Savransky, and E. Gustafson, “Optical design of a modular segmented telescope,” Proc. SPIE, Vol. 11116-12, 2019

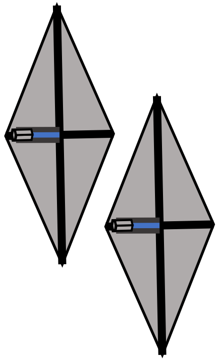
Mission Concept

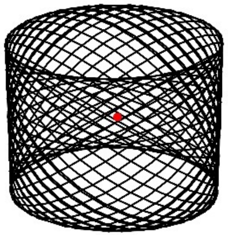
Modular spacecraft start on Earth orbits with mirror as payload.

Solar sails unfurl and propel the mirrors to L2.

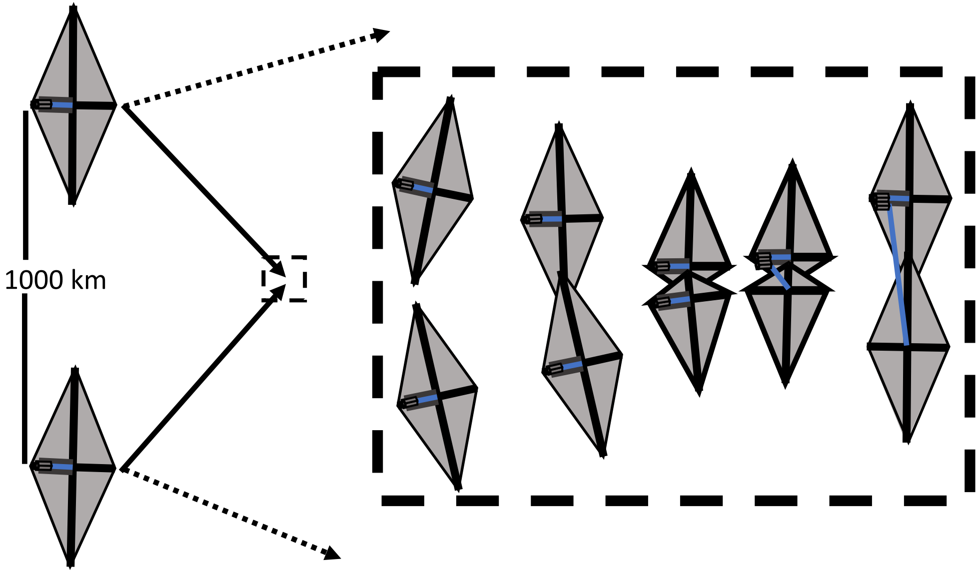
Spacecraft are assembled on a Lissajous orbit.

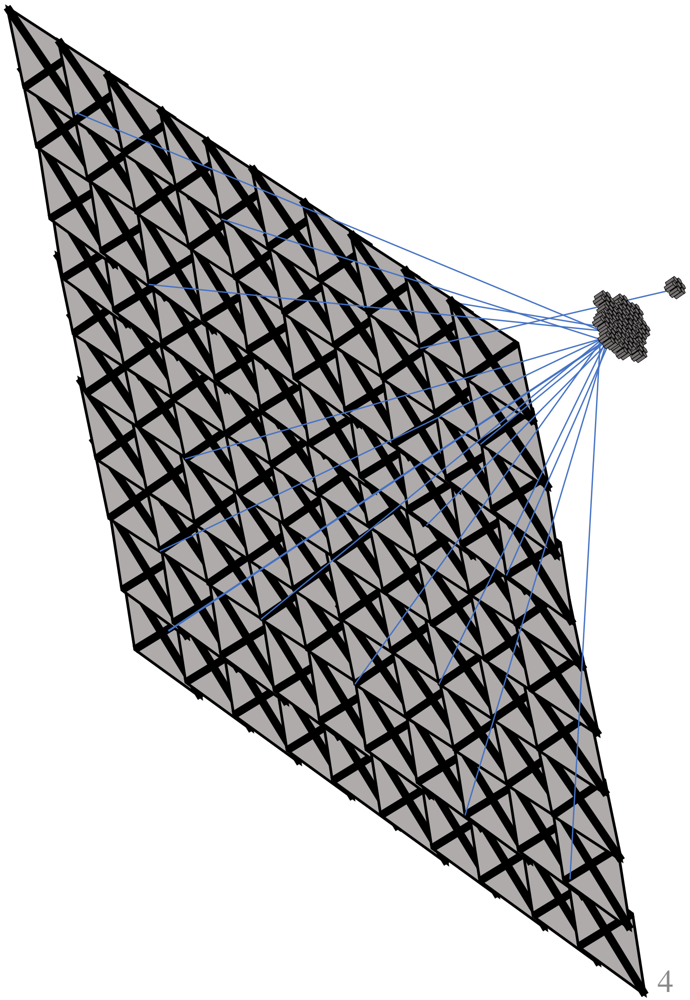






Mission Concept





Dynamical Model

CR3BP equations of motion

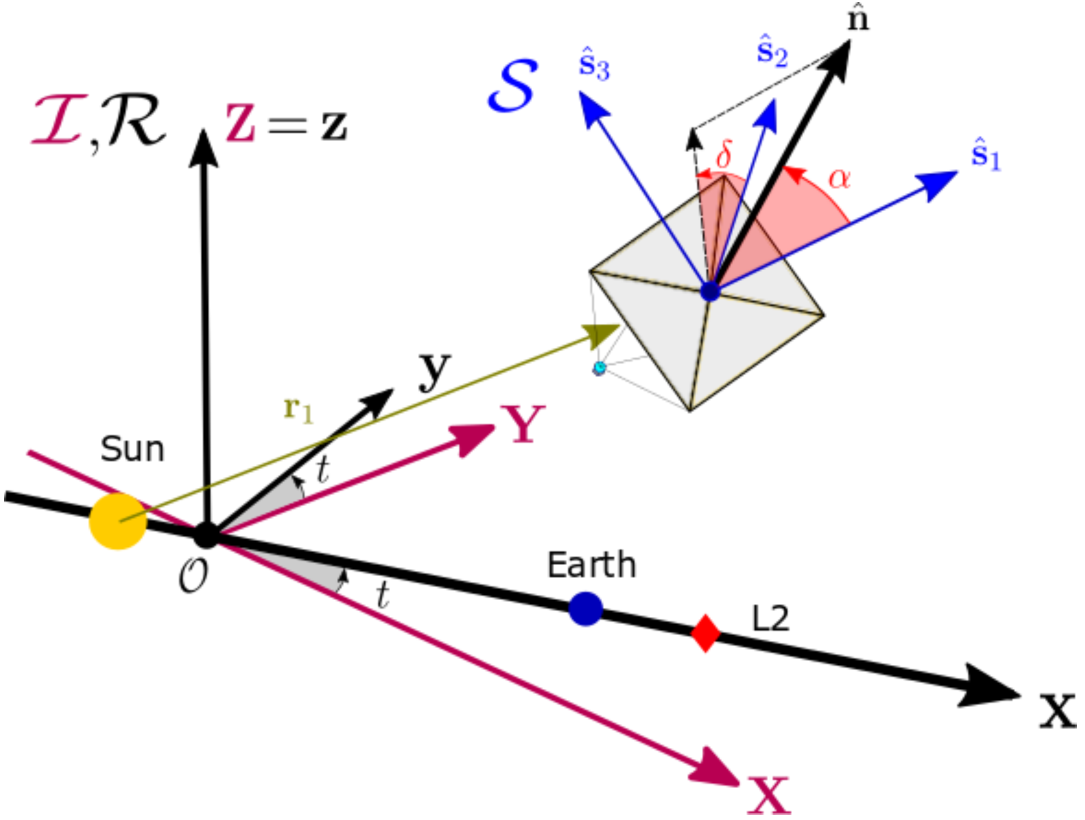
$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}, \mathbf{u}) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ 2\dot{y} + \frac{\partial \Omega}{\partial x} + \mathbf{a}_S \cdot \hat{\mathbf{x}} \\ -2\dot{x} + \frac{\partial \Omega}{\partial y} + \mathbf{a}_S \cdot \hat{\mathbf{y}} \\ \frac{\partial \Omega}{\partial z} + \mathbf{a}_S \cdot \hat{\mathbf{z}} \end{bmatrix}$$

Effective potential and relative distances:

$$\Omega(x, y, z) = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$

$$r_1 = \sqrt{(\mu + x)^2 + y^2 + z^2}$$

$$r_2 = \sqrt{(1 - \mu - x)^2 + y^2 + z^2}$$



Dynamical Model

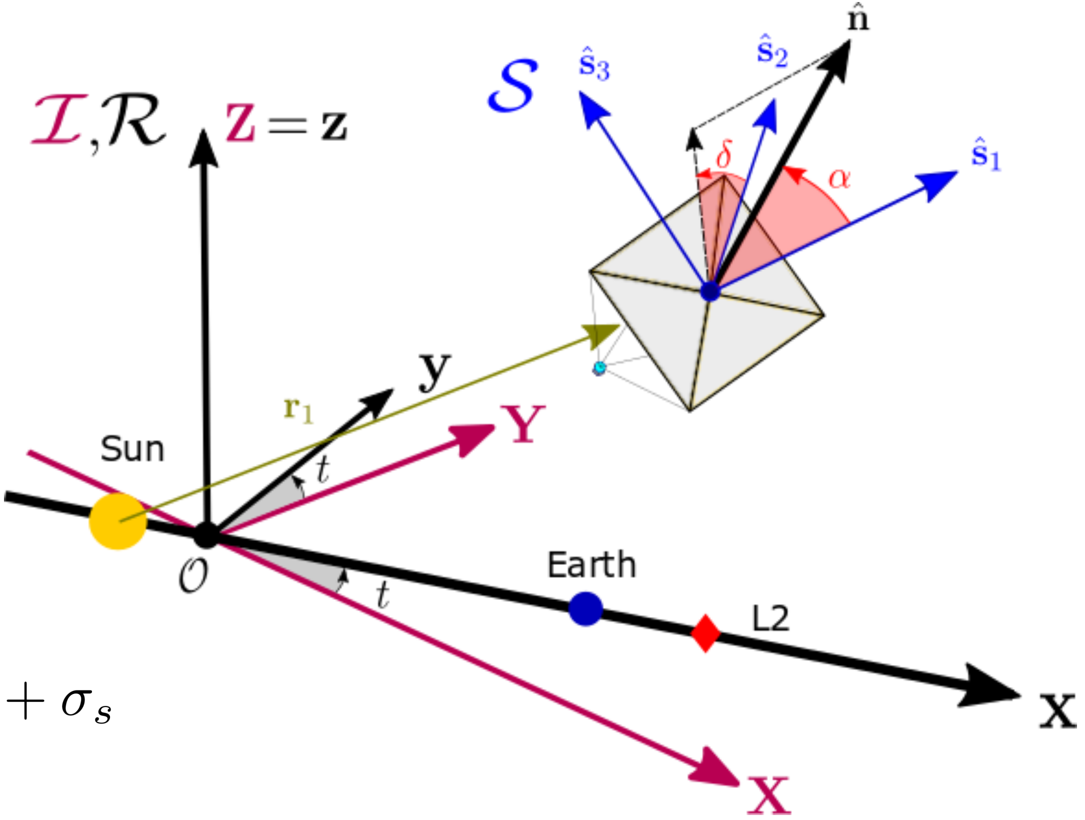
Solar Sail acceleration term[2]

$$\mathbf{a}_S = \beta \frac{1-\mu}{r_1^2} (\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{n}})^2 \hat{\mathbf{n}}$$

Solar Sail performance factor

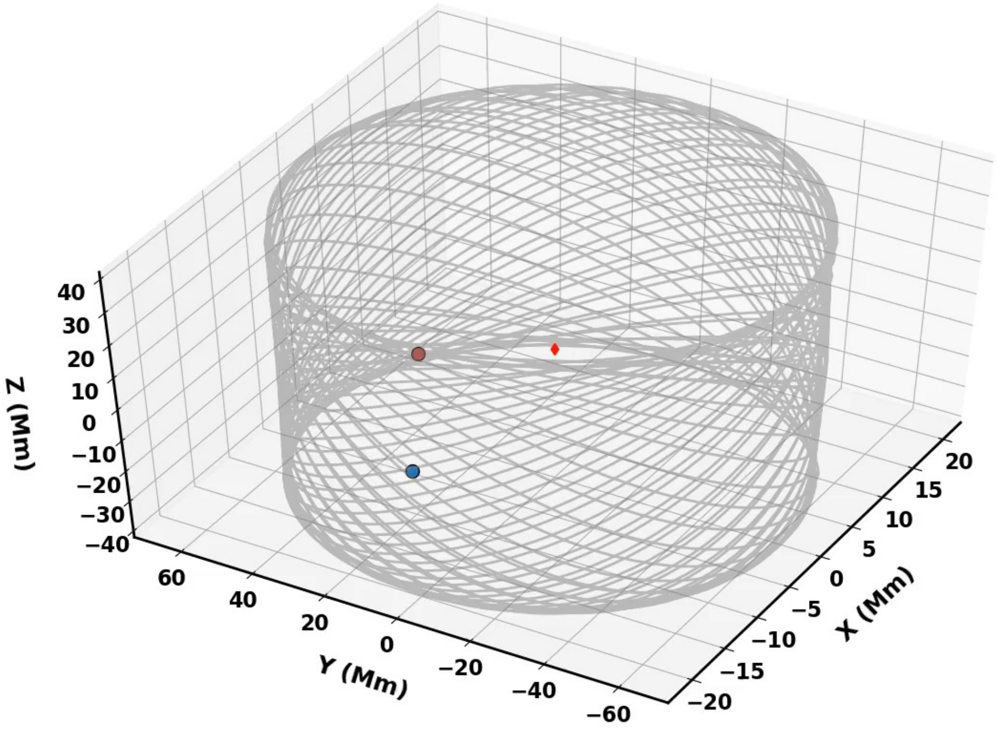
$$\beta = \frac{L_\odot}{2\pi G M_\odot \sigma} = \frac{\sigma^*}{\sigma}$$

$$\sigma = \frac{m_T}{A_s} = \frac{m_p + m_s}{A_s} = \frac{m_p}{A_s} + \sigma_s$$



[2]C. McInnes, Solar Sailing: Technology, Dynamics and Mission Applications. Springer-Praxis Books, 1999.

Final Orbit - Lissajous



- Found through 2-step differential correction process[3]
- 177-day period, about Sun-Earth L2 ecliptic

[3]K. C. Howell and H. Pernicka, "Numerical Determination of Lissajous Trajectories in the Restricted Three-Body Problem," Celestial Mechanics, Vol. 41, 1988

Invariant Manifold Analysis

- State transition matrix found for periodic orbit
  - Vertical Lyapunov
- Monodromy Matrix

$$\mathbf{\Phi}(T, t_0) \begin{cases} \lambda_1 > 1 \\ \lambda_2 = \dfrac{1}{\lambda_1} \\ \lambda_3 = \lambda_4 = 1 \\ \lambda_5 = \bar{\lambda}_6 \ , \ |\lambda_5| = 1. \end{cases}$$

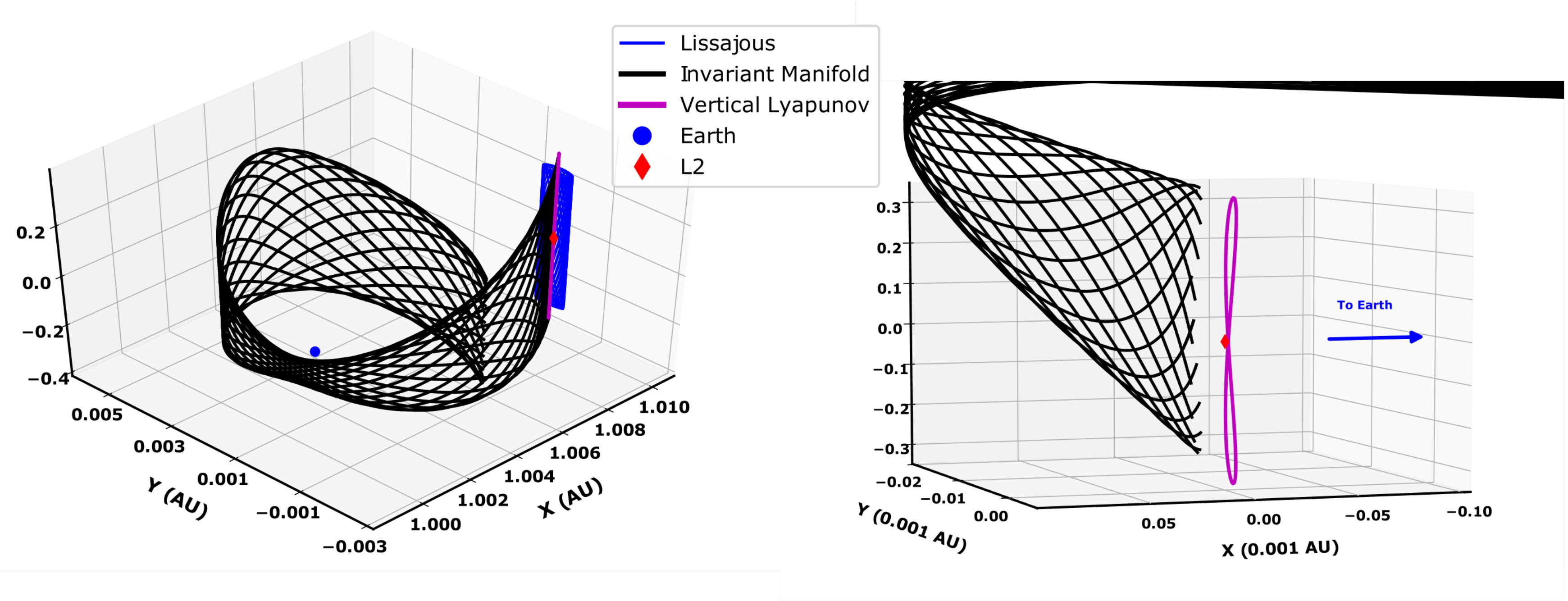
Point on Vertical Lyapunov

$$\mathbf{x}_0^{\mathcal{U}}(\mathbf{x}_0^P) = \underline{\mathbf{x}_0^P} + \epsilon \underline{\mathbf{Y}^{\mathcal{U}}(\mathbf{x}_0^P)}$$

Unstable eigenvector

[4]W. S. Koon, M. W. Lo, J. E. Marsden, and S. D. Ross, Dynamical Systems, the Three-Body Problem and Space Mission Design (2011)

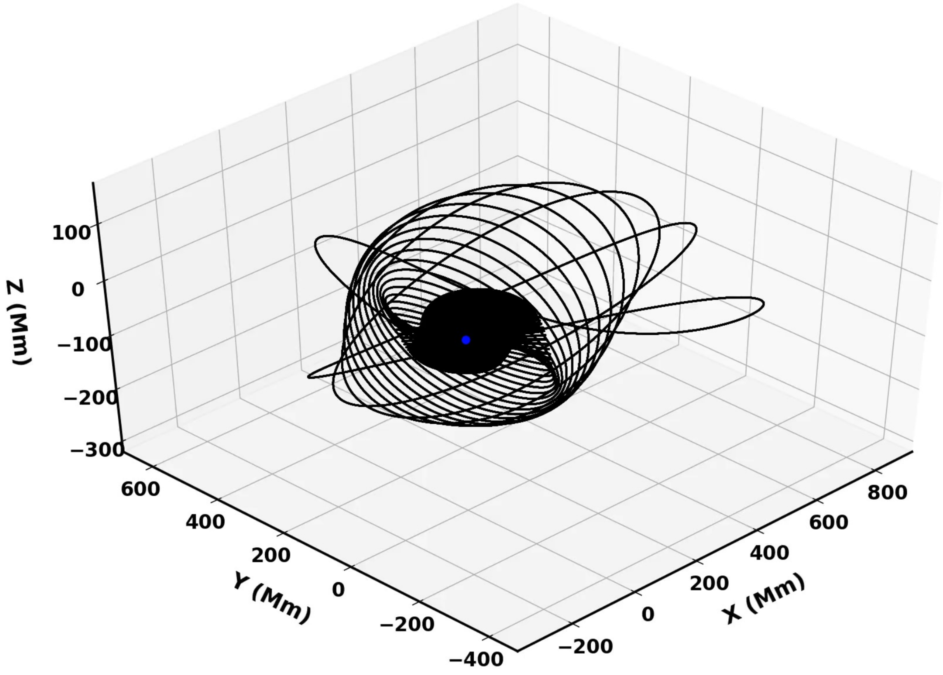
Invariant Manifold Analysis



Earth Escape Trajectories

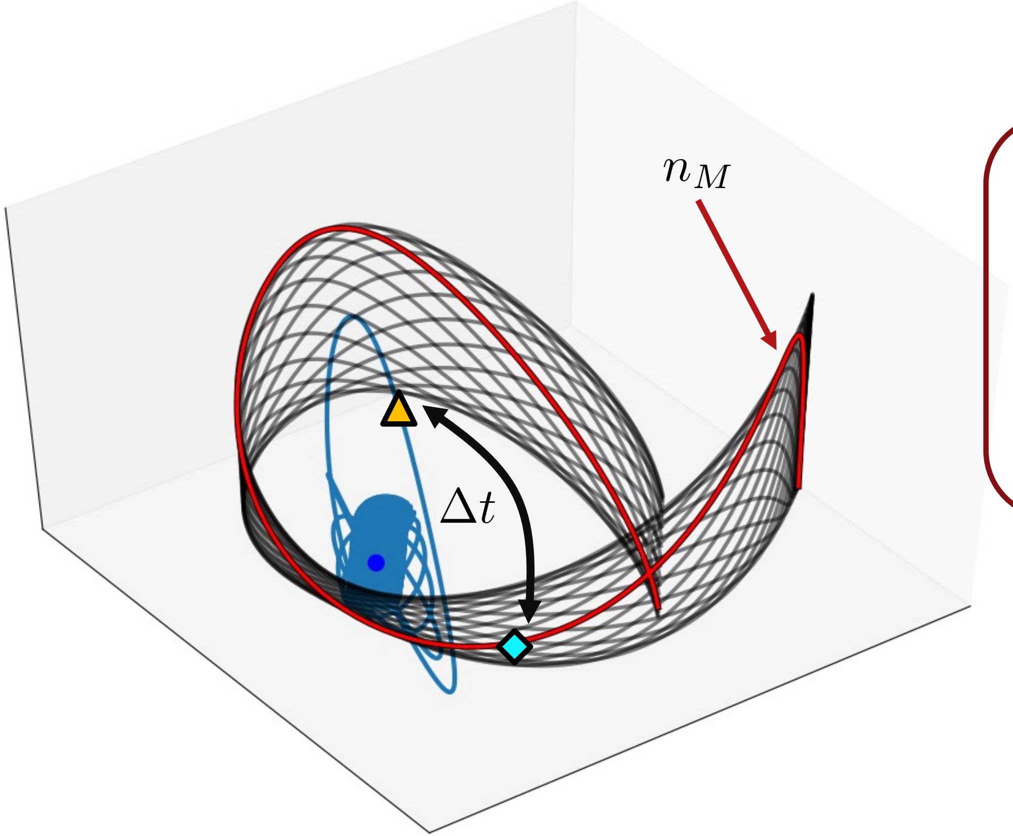
- Energy maximization control law[5] in rotating frame

$$\max_{\hat{\mathbf{n}}} \ \mathbf{a}_S(\hat{\mathbf{n}}) \cdot {}^{\mathcal{R}}\mathbf{v}$$



[5]Coverstone, "Technique for Escape from Geosynchronous Transfer Orbit Using a Solar Sail," JGCD, 2003

Transfer to the Manifold



Best initial guess for solar sail: trajectory with smallest average thrust

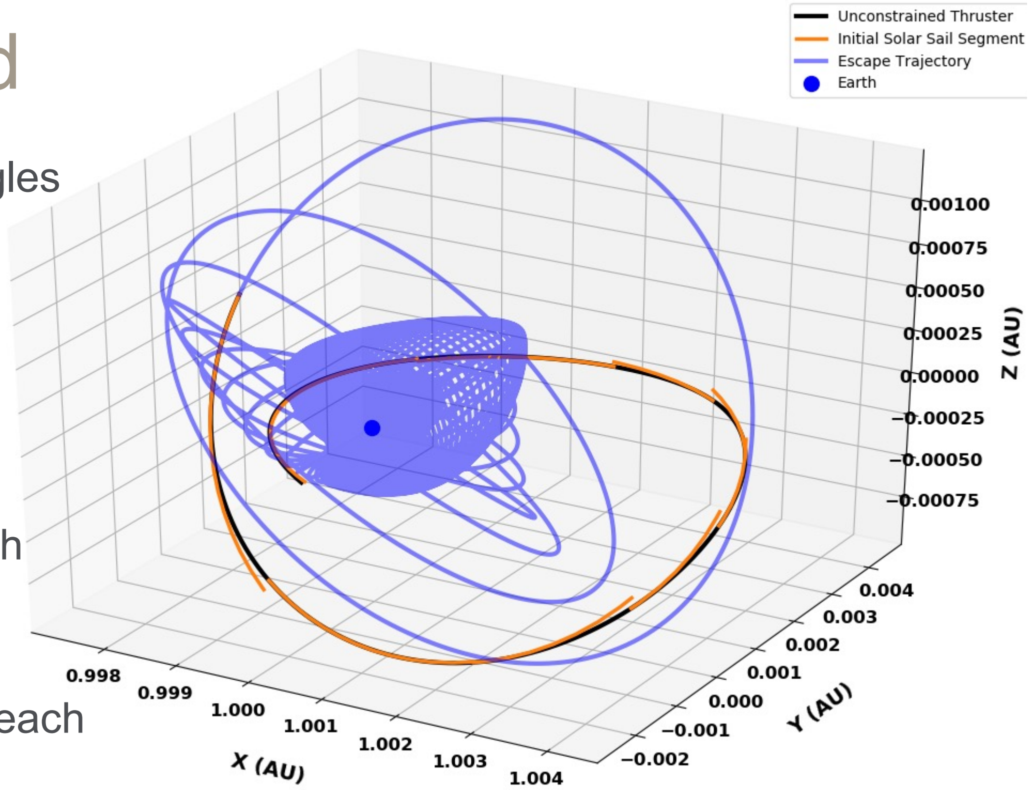
$$\min_{t_E, n_M, t_M, \Delta t} \frac{1}{\Delta t} \int_{t_E}^{t_E+\Delta t} |\boldsymbol{\lambda}_v| \, dt$$

Transfer to the Manifold

- Convert "best" velocity costates to angles

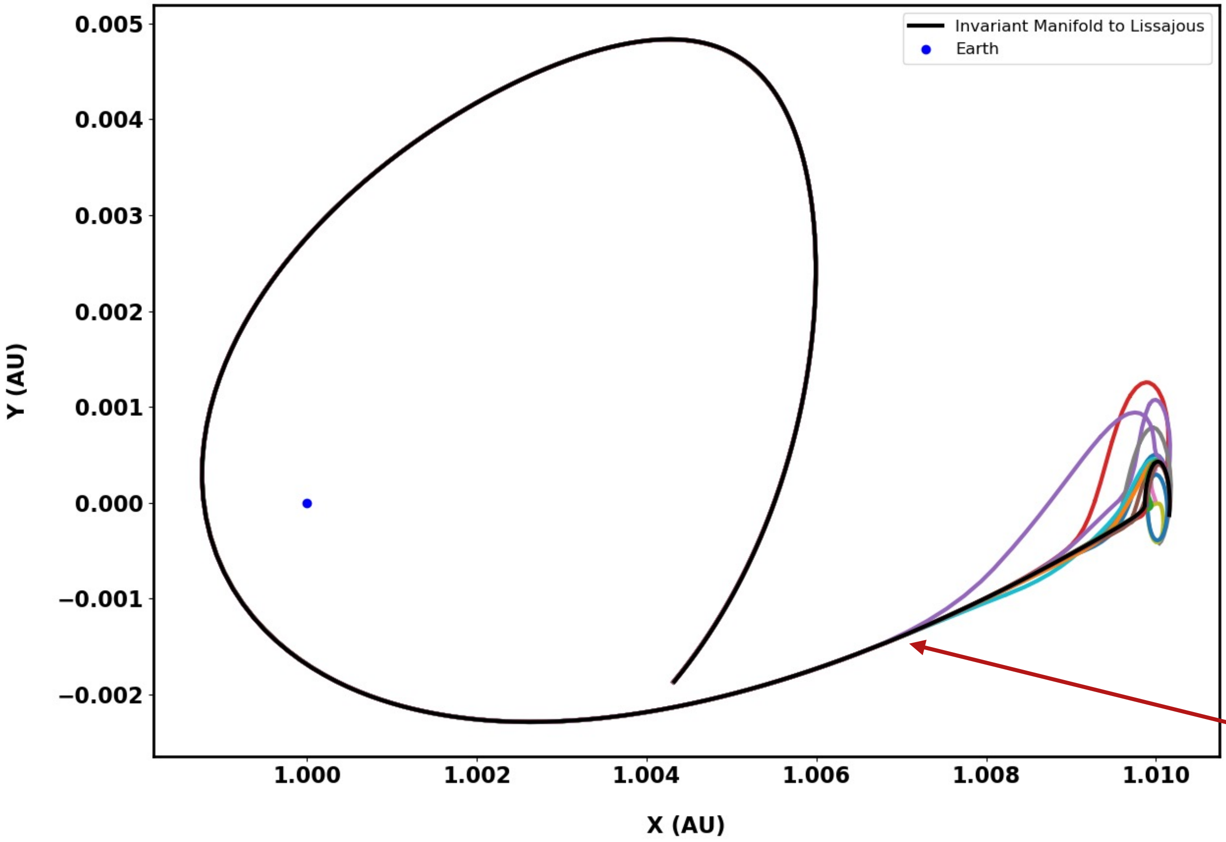
$$ {}^{\mathcal{S}}\hat{\boldsymbol{\lambda}}_v = \begin{bmatrix} \lambda_{4,\mathcal{S}} \\ \lambda_{5,\mathcal{S}} \\ \lambda_{6,\mathcal{S}} \end{bmatrix} = \begin{bmatrix} \cos\bar{\alpha} \\ \sin\bar{\alpha}\cos\bar{\delta} \\ \sin\bar{\alpha}\sin\bar{\delta} \end{bmatrix} $$

- Solve a multiple shooting problem[6] with N = 10 segments
- Start with large β and decrease after each convergent optimization
  - `scipy.minimize` with SLSQP method



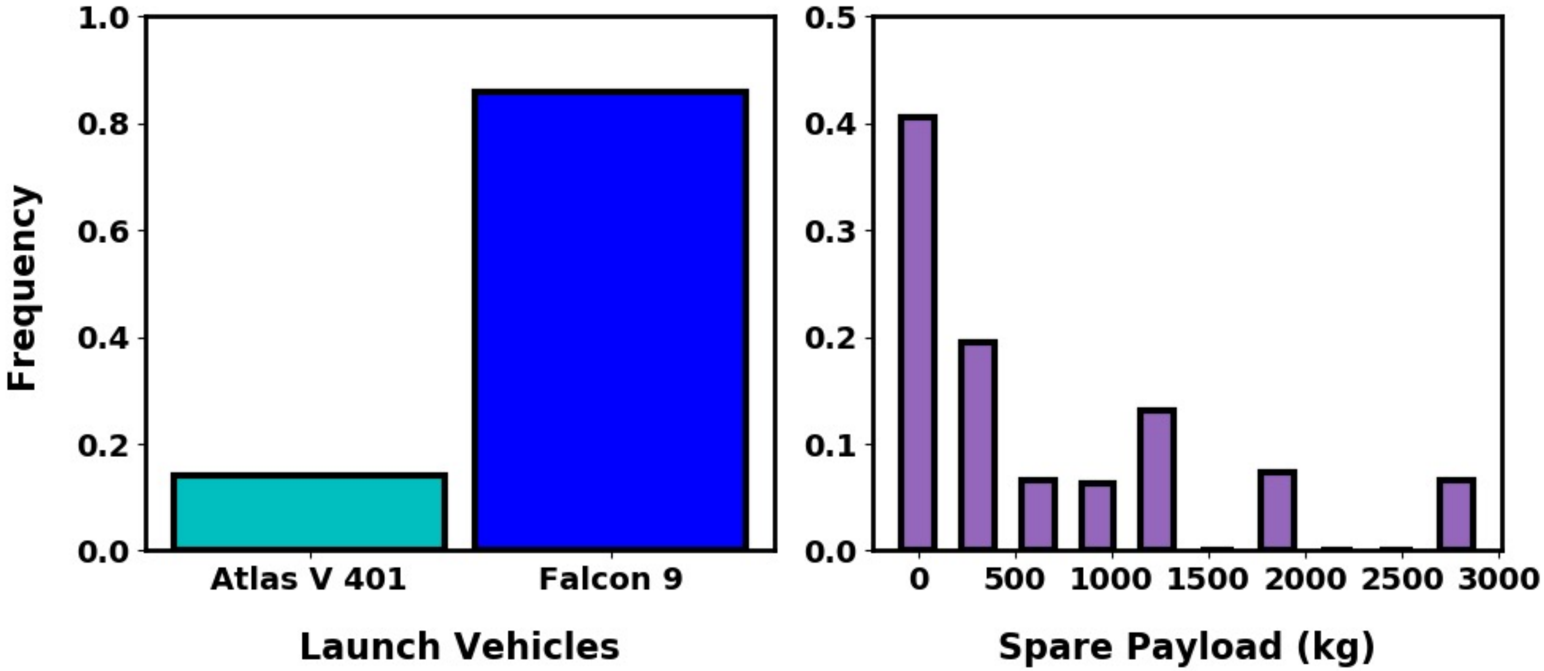
[6] M. Diehl, H. G. Bock, H. Diedam, and P.-B. Wieber, "Fast Direct Multiple Shooting Algorithms for Optimal Robot Control," Fast Motions in Biomechanics and Robotics, 2005.

Transfer to the Lissajous



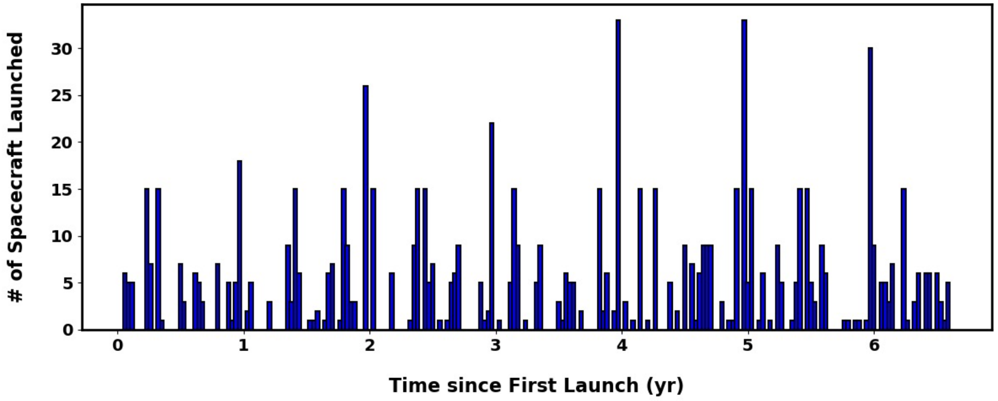
Branch-off point from Manifold

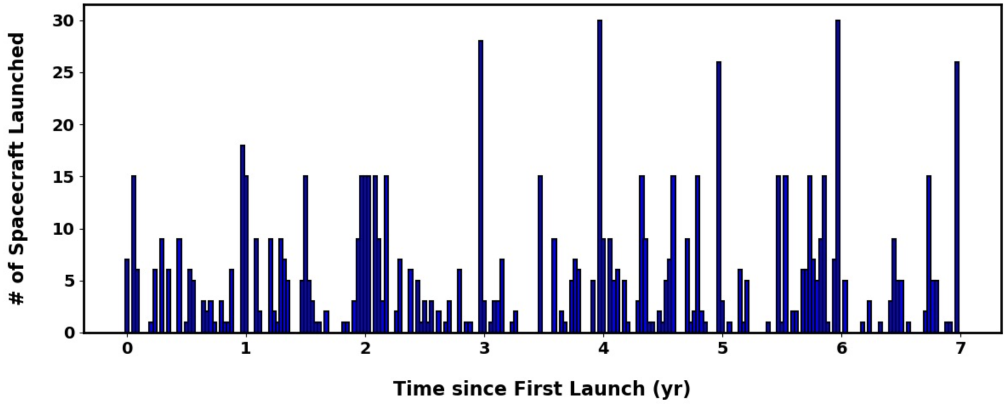
Launch Analysis



Data taken from Launch Log in: http://www.planet4589.org/space/log/launch.html

Design Reference Mission

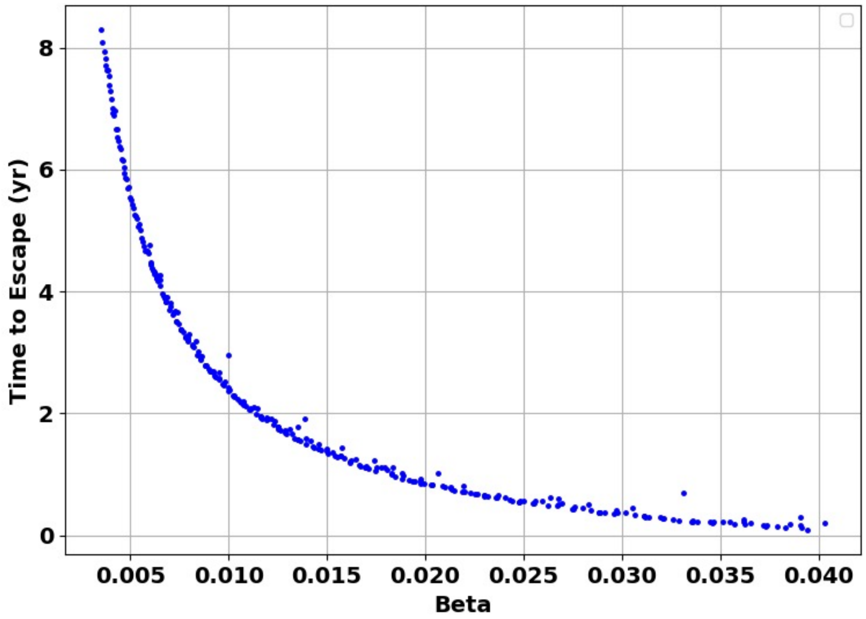


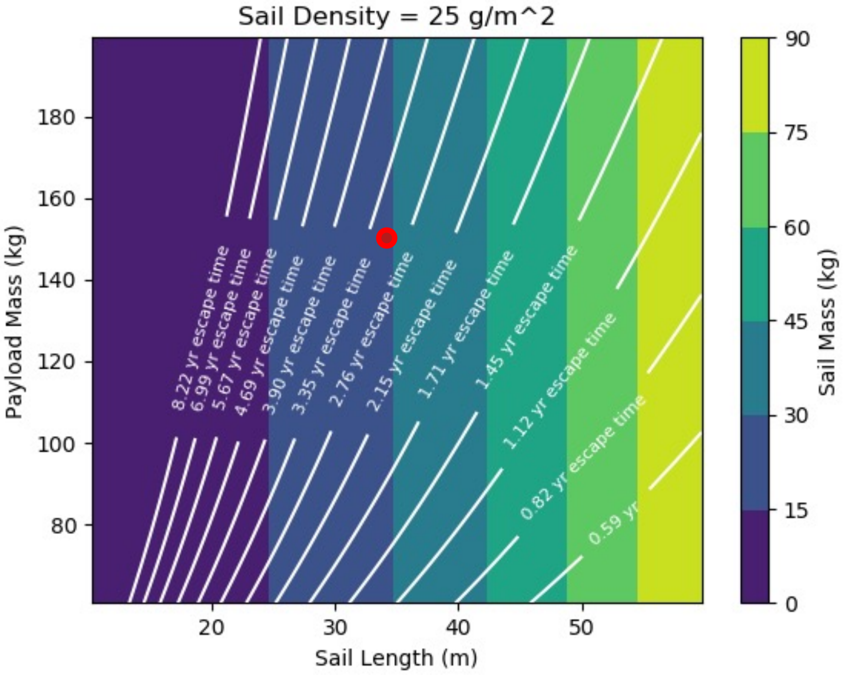


Designing the Sail

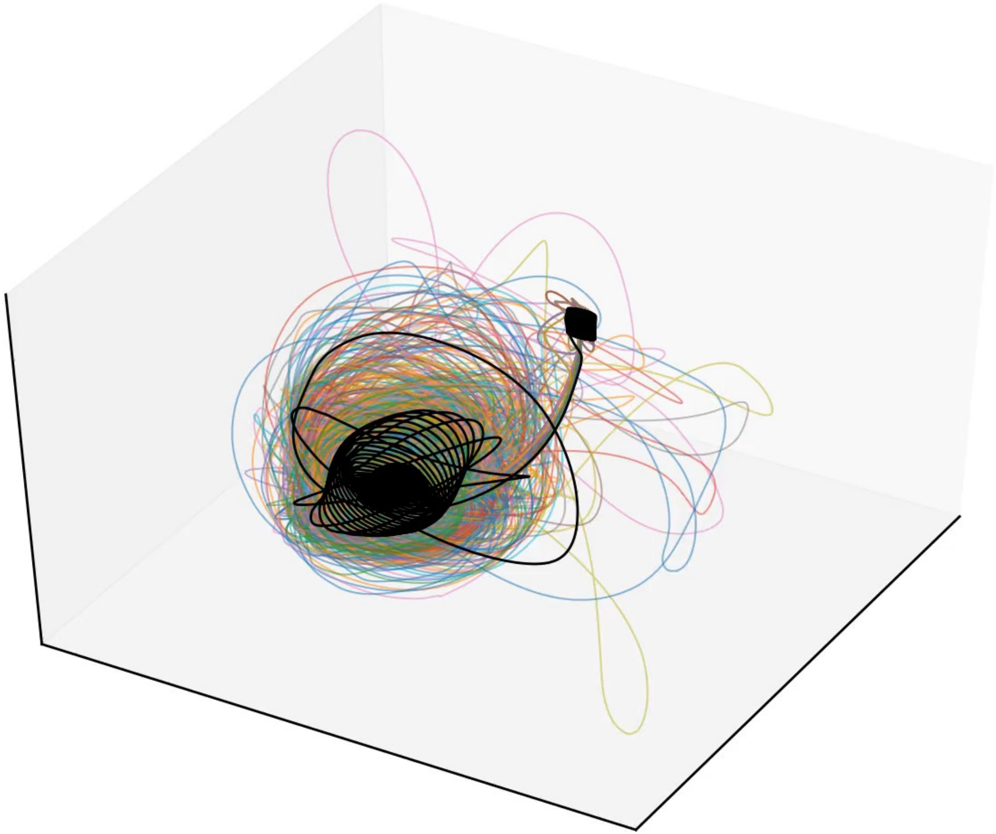
$$\beta = \frac{L_\odot}{2\pi G M_\odot \sigma} = \frac{\sigma^*}{\sigma}$$

$$\sigma = \frac{m_T}{A_s} = \frac{m_p + m_s}{A_s} = \frac{m_p}{A_s} + \sigma_s$$

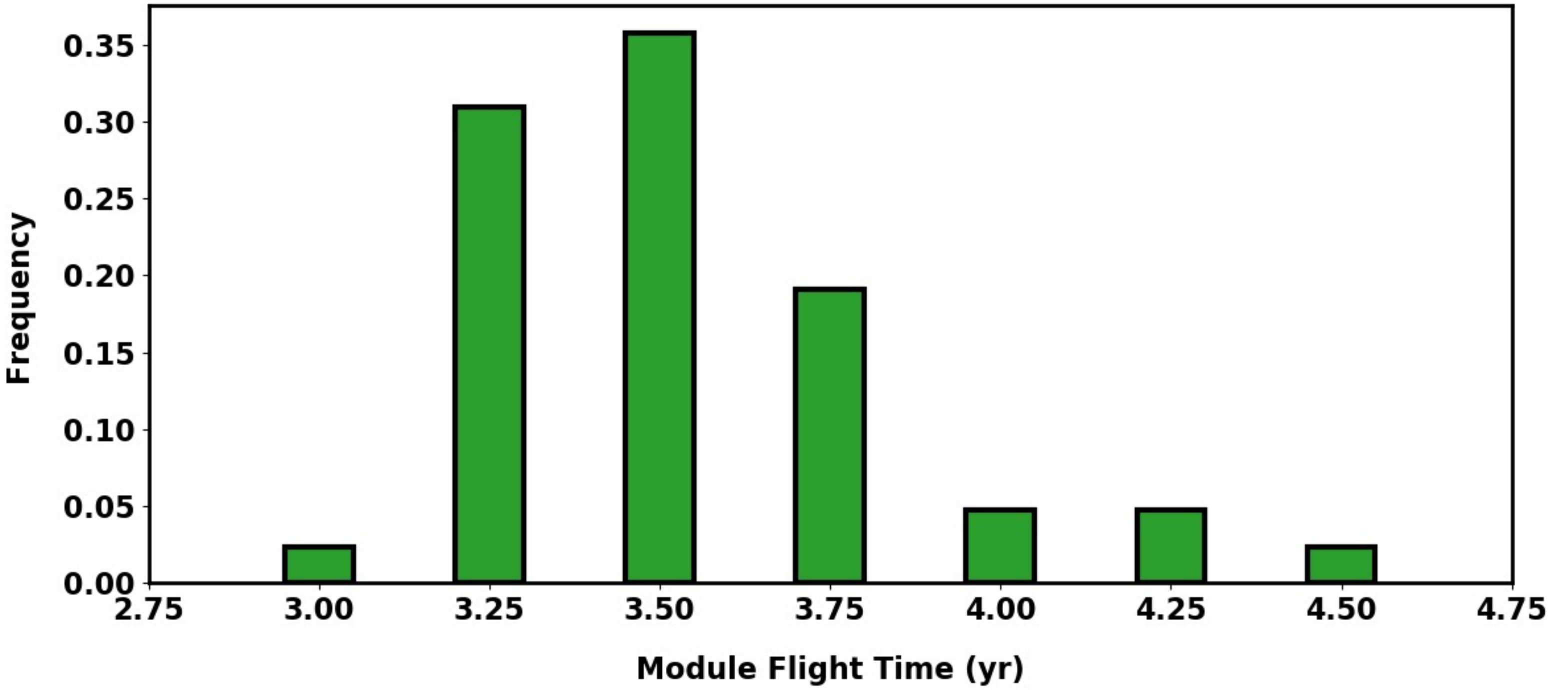




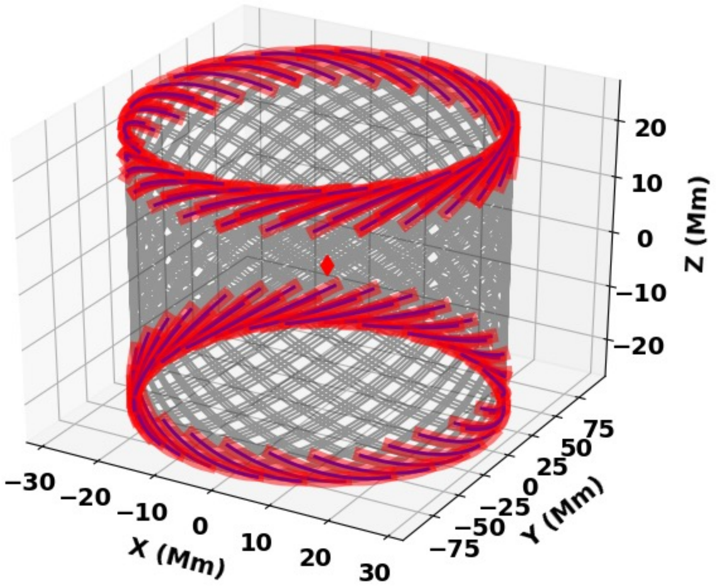
Full Trajectories from Earth to L2



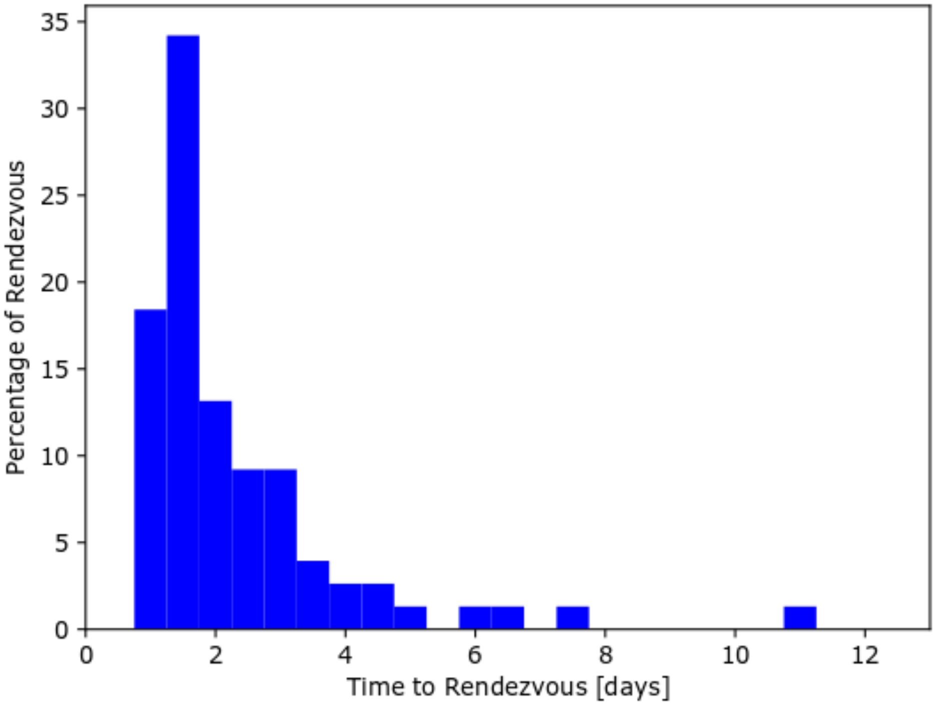
Full Trajectories from Earth to L2



Rendezvous

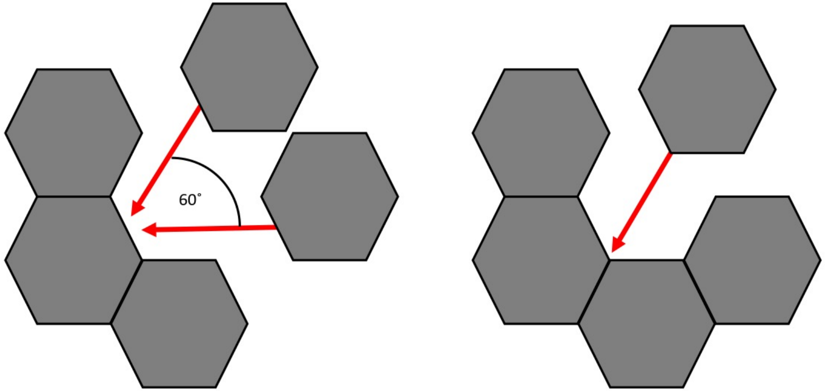


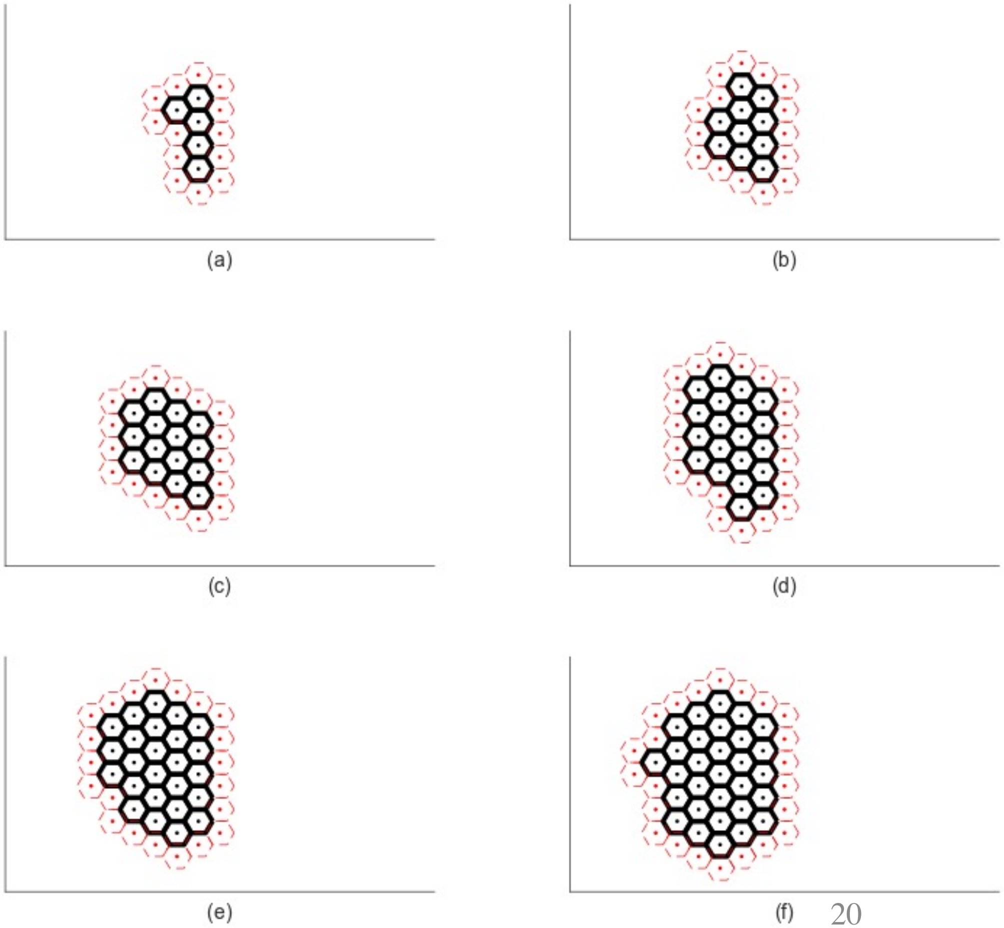
20
10
0
−10
−20
Z (Mm)
−30
−20
−10
0
10
20
30
X (Mm)
75
50
25
0
−25
−50
−75
Y (Mm)



Percentage of Rendezvous
35
30
25
20
15
10
5
0
0
2
4
6
8
10
12
Time to Rendezvous [days]

Docking





Conclusions

- Simulate future launch schedules using 2016-2018 launch data
  - 840 modules launched within 6-7 years
  - All injected into Lissajous within 11 years

- Developed tools to simulate full mission from Earth to L2 Lissajous orbits
  - Uses standard Python packages including `numpy` and `scipy`
  - Design tools for selecting sail parameters coupled with Earth escape times

- Close encounters (within 1000 km) occur when spacecraft are n periods apart
  - Solar sail rendezvous takes, on average, about 2 days

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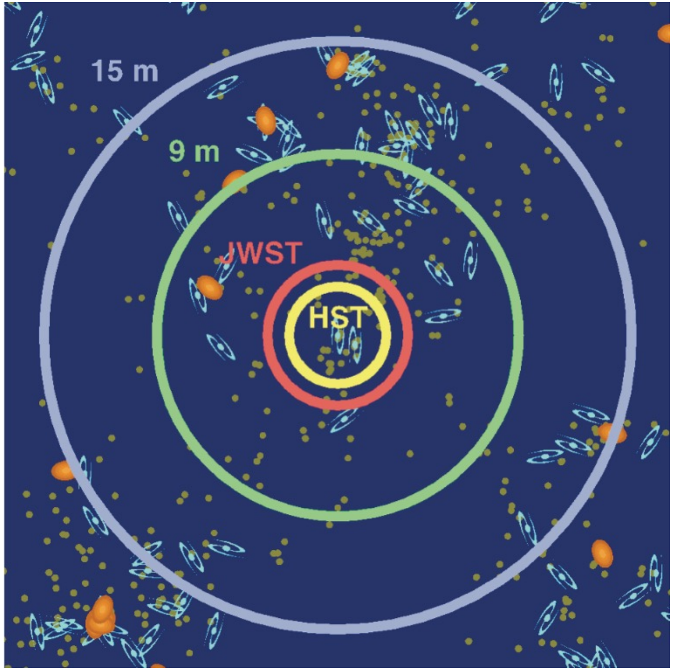
This work was supported by NIAC Grant 80NSSC18K0869.

My website: soto.sioslab.com

Backup Slides

Observe Main Sequence Turnoff Point



From: LUVOIR Interim Report[1].
Figure by T. Brown.

- Observability of main sequence turnoff at different telescope scales
  - SNR = 5
  - 100 hours of integration time
  - V and I bands
  - Captures the local group
- 31 m telescope would effectively be sensitive throughout the entire observable universe.

Invariant Manifold Analysis

- Differential Correction

$$\dot{\mathbf{\Phi}}(t, t_0) = \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \mathbf{\Phi}(t, t_0)$$

$$\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \mathbf{0}_{3\times 3} & \mathbf{I}_{3\times 3} \\ -\mathcal{U} & 2\Delta \end{bmatrix}$$

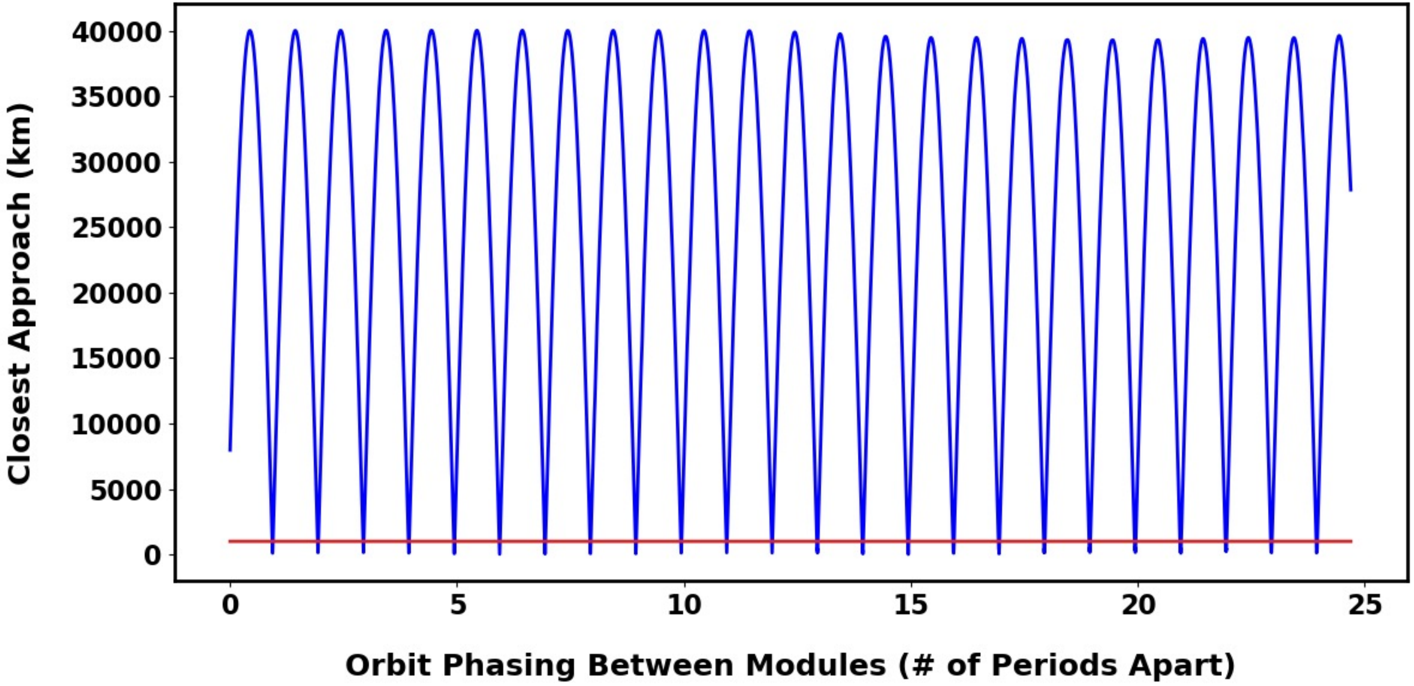
$$\Delta = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathcal{U} = \begin{bmatrix} \Omega_{xx} & \Omega_{xy} & \Omega_{xz} \\ \Omega_{yx} & \Omega_{yy} & \Omega_{yz} \\ \Omega_{zx} & \Omega_{zy} & \Omega_{zz} \end{bmatrix}$$

- Monodromy Matrix

$$\mathbf{\Phi}(T, t_0) \begin{cases} \lambda_1 > 1 \\ \lambda_2 = \dfrac{1}{\lambda_1} \\ \lambda_3 = \lambda_4 = 1 \\ \lambda_5 = \bar{\lambda}_6 \ , \ |\lambda_5| = 1. \end{cases}$$

$$\mathbf{x}_0^{\mathcal{U}}(\mathbf{x}_0^P) = \underline{\mathbf{x}_0^P} + \epsilon \underline{\mathbf{Y}^{\mathcal{U}}(\mathbf{x}_0^P)}$$

Orbit Phasing



Full Trajectories in the Inertial Frame