Automated Reflective Optical System Alignment with Focal Plane Sensing and Kalman Filters

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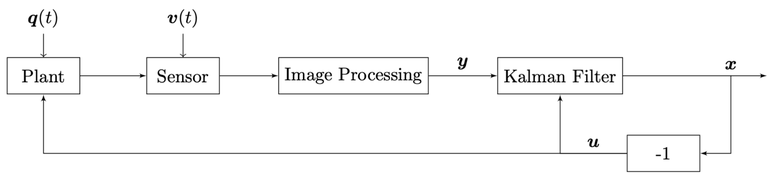
Introduction

Automated alignment can significantly reduce the time and labor costs in optical system setup and maintenance, especially for space and remote, ground-based telescopes where manual adjustment remains difficult. We demonstrate an automated alignment technique on a multi-element reflective system using pure focal plane sensing and model-based optimal estimation and control. Using either an iterated extended Kalman filter[2] (IEKF) or a square-root unscented Kalman filter[3,4] (SR-UKF), we are able to consistently bring the linear misalignments from around 1 mm to < 5 μm and the angular misalignments from around 500 arcsec to < 6 arcsec in simulation. Combined with our lab's previous work[1], we can generalize this auto-aligning technique to more complex optical systems with multiple reflective and refractive elements.



Our auto-aligning, closed-loop control system takes focal plane images from the sensor (camera), processes the images to extract key measurements **y**, employs the embedded Kalman filter to estimate the implicit misalignment states **x**, and applies a negative feedback control **u** to correct for the misalignment in the plant (optical system).

Optical Model

The two off-axis parabolic mirrors (OAP) are placed on motorized translation and tip-tilt stages, providing 10 degrees of freedom in total:

$$\boldsymbol{x} = \left[D_{x1}, D_{y1}, D_{z1}, T_{x1}, T_{z1}, D_{x2}, D_{y2}, D_{z2}, T_{x2}, T_{z2}\right]^T$$

- On-axis beam: Focal plane sensing, alignment state estimation and control
- Off-axis beam: Post-verification of repeatability, potentially additional measurements

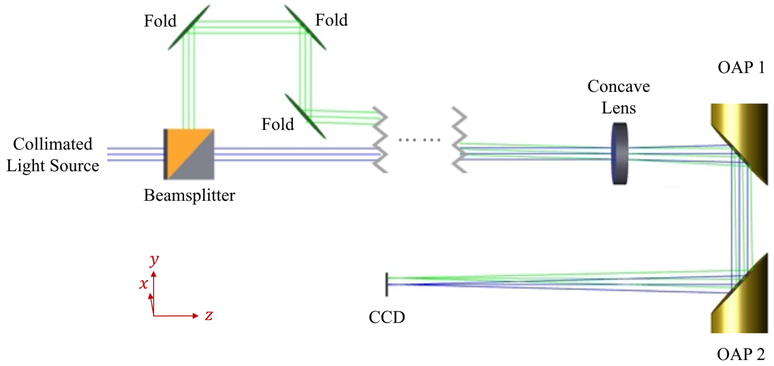
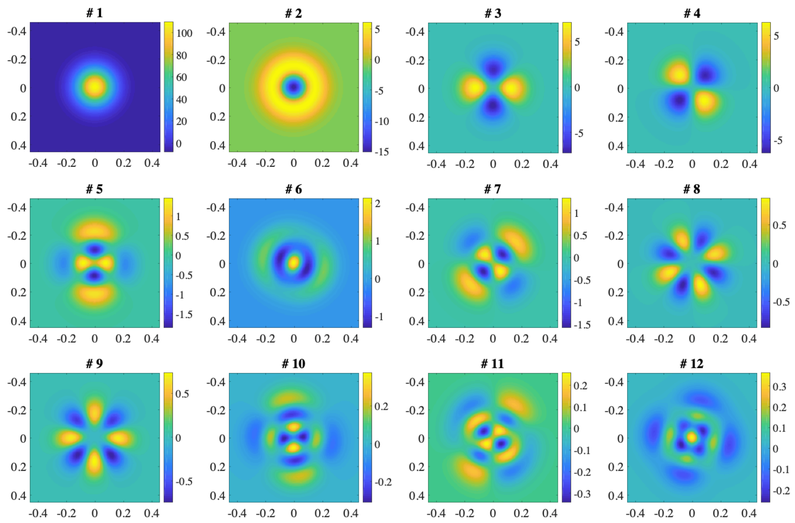


Image Processing

Misalignment of the OAPs introduces wavefront aberrations which cause deviations in the focal plane image. We include the spot center and the KL weights as our measurements:

$$\boldsymbol{y} = \left[C_x, C_y, w_1, w_2, \ldots, w_{50}\right]^T$$

- $\{C_x, C_y\}$: on-axis spot center position, determined from 2D Gaussian fit
- $\{w_i\}$: Karhunen-Loève (KL) weights in image modal reconstruction, determined from principal component analysis and KL transform
- The first 12 KL modes sorted by eigenvalues in descending order



Kalman Filter

We assumed the discrete time-invariant state-space representation of the system:

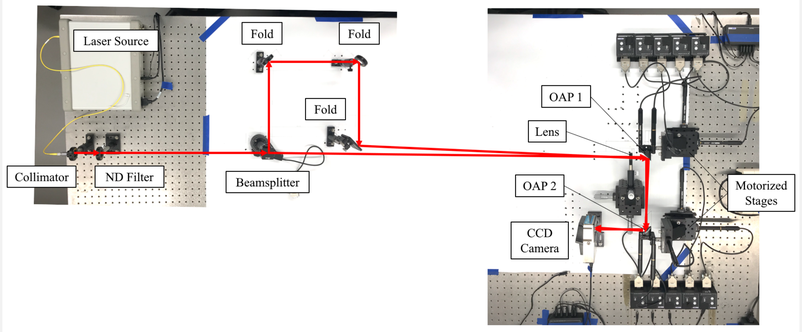
$$\boldsymbol{x}_k = \boldsymbol{F}\boldsymbol{x}_{k-1} + \boldsymbol{B}\boldsymbol{u}_k + \boldsymbol{q}_k, \qquad \boldsymbol{y}_k = \boldsymbol{h}(\boldsymbol{x}_k) + \boldsymbol{r}_k$$

- State transition matrix $\boldsymbol{F}$ and control input matrix $\boldsymbol{B}$ are identity matrices
- Measurement function $\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x})$ is a second order polynomial
- Process noise $\boldsymbol{q}_k \sim N(0, \boldsymbol{Q})$, measurement noise $\boldsymbol{r}_k \sim N(0, \boldsymbol{R})$

Due to nonlinearity in $\boldsymbol{h}(\boldsymbol{x})$, we apply IEKF and SR-UKF for state estimation:

- IEKF: local linearization, iteration inside each measurement update step
- SR-UKF: unscented transform, propagate square root of the covariance to enhance numerical stability

Experiment Setup

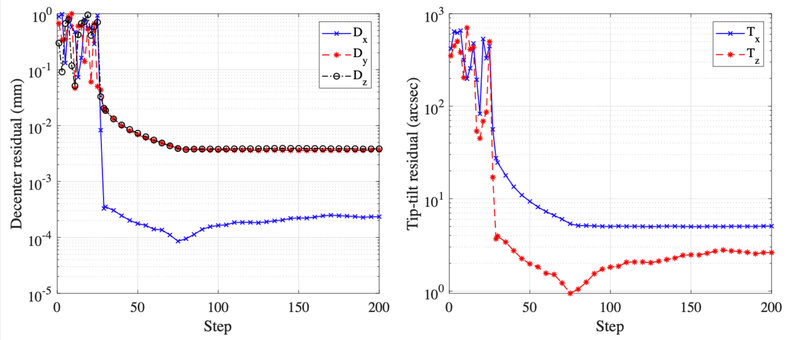


State Estimation Results

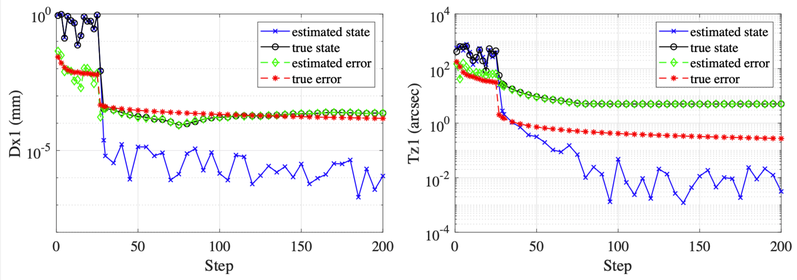
Both IEKF and SR-UKF are able to consistently bring the linear misalignments from around 1 mm down to < 5 μm and the angular misalignments from around 500 arcsec down to < 6 arcsec in closed-loop control. The similar performance of the two filters indicates that our system is close to linear.

State	IEKF RMS residual	SR-UKF RMS residual
D_{x1}	0.24 ± 0.097 μm	0.11 ± 0.075 μm
D_{y1}	3.2 ± 0.9 μm	4.2 ± 2.3 μm
D_{z1}	3.4 ± 0.7 μm	4.6 ± 2.8 μm
T_{x1}	4.4 ± 1.3 arcsec	5.7 ± 3.1 arcsec
T_{y1}	2.6 ± 1.1 arcsec	1.2 ± 0.80 arcsec
D_{x2}	0.37 ± 0.15 μm	0.17 ± 0.12 μm
D_{y2}	2.5 ± 0.7 μm	3.2 ± 1.7 μm
D_{z2}	1.8 ± 0.4 μm	2.4 ± 1.4 μm
T_{x2}	4.5 ± 1.3 arcsec	5.7 ± 3.1 arcsec
T_{y2}	2.6 ± 1.1 arcsec	1.2 ± 0.83 arcsec

- Random walk for the first 25 steps to introduce phase diversity for better wavefront estimation
- Quick convergence after step 26 when state feedback control starts
- Similar trend in D_x and T_z implicates multi-state coupling effect



- Filter consistently underestimates the misalignments and the associated estimation error, which implies multi-state coupling effect and lack of observability



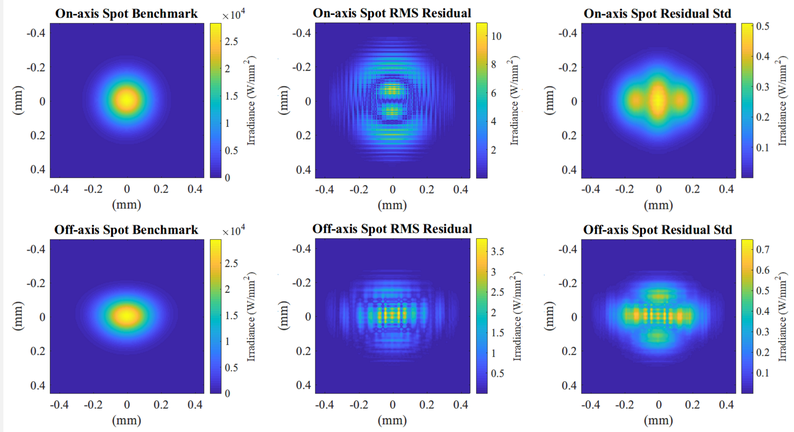
Alignment Quality

We evaluate the final alignment quality through image residuals and wavefront errors. Our automated alignment method can be applied on optical systems that can tolerate the amount of image and wavefront error shown here.

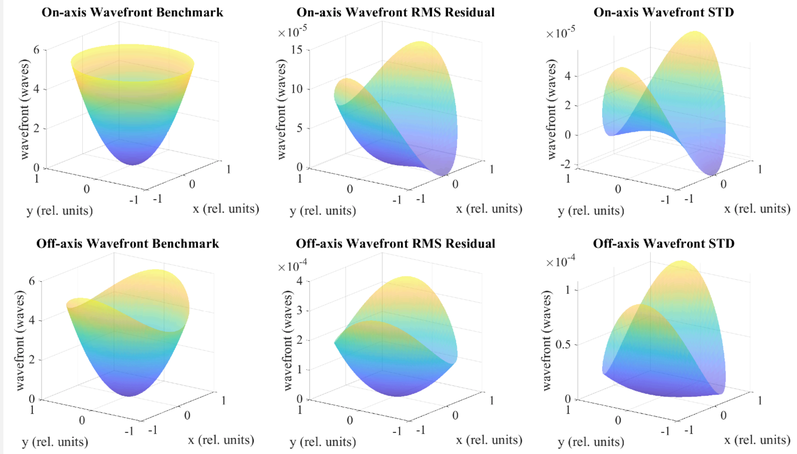
- The error of the spot position in the focal plane is on the scale of nanometers.

Measurement	RMS residual
$C_{x,\text{on-axis}}$	0.36 ± 0.31 nm
$C_{y,\text{on-axis}}$	19 ± 0.06 nm
$C_{x,\text{off-axis}}$	84 ± 35 nm
$C_{y,\text{off-axis}}$	49 ± 20 nm

- Subframe image quality. Majority of the on-axis spot residual is in coma, which is a dominating wavefront aberration associated with off-axis parabolic mirrors.



- Wavefront errors. There is an overall tip/tilt in the on-axis wavefront residual because tip/tilt has little effect on the intensity so it is hard to be captured by the focal plane image.



Conclusion

We demonstrate automated alignment of a reflective system in simulation using only focal plane sensing, with linear accuracy < 5 μm and angular accuracy < 6 arcsec, for both IEKF and SR-UKF. Automated alignment makes the assembly and adjustment of optical systems more efficient and flexible. Utilizing internal sensors reduces cost and complexity introduced by dedicated wavefront sensors or interferometric devices, increases optical throughput and eliminates non-common path error.

Future work:

- Experiments to validate simulation results
- Improve the measurement function to reduce the multi-state coupling effect
- Replace the KFs with an information filter and perform information-based observability analysis
- Train neutral networks on the mapping between **x** and **y**, which can potentially describe more convoluted relationships and capture more features from images

References

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