# SOLAR SAIL TRAJECTORIES AND ORBIT PHASING OF MODULAR SPACECRAFT FOR SEGMENTED TELESCOPE ASSEMBLY ABOUT SUN-EARTH L2 

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#### Abstract

In-space assembly of a segmented primary mirror is needed to produce a large primary mirror bigger than LUVOIR, about 30m in diameter. We propose a novel mission concept for a segmented space telescope where each identical mirror segment is placed on modular spacecraft. Individual modules are launched as payloads of opportunity that self-assemble about the Sun-Earth L2 point. They use a solar sail as a means of continuous thrust propulsion. After docking, the solar sails are steered to overlap and create a planar sun shield for the telescope. We provide the framework for minimizing the total mission assembly time.


## INTRODUCTION

Future space telescopes will require larger primary mirrors to replicate and augment the sensitivity and resolution capabilities of current space telescopes like Hubble and the future James Webb Telescope. Manufacturing and launch costs prevent scaling up the size of not just monolithic mirrors fabricated on Earth but also segmented mirrors assembled before launch. ${ }^{1}$ In-space assembly of a segmented primary mirror is therefore needed to go beyond the $\sim 15 \mathrm{~m}$ diameter designs currently being evaluated for the next generation of space telescopes such as the Large Ultraviolet/Optical/Infrared Surveyor (LUVOIR) mission. ${ }^{2,3}$ The LUVOIR interim report predicts that the 15 m design option is capable of observing the main sequence turnoff in the local group in the V and I bands at 100 hours of integration time; a 31 m telescope could effectively perform similar observations throughout the observable universe. ${ }^{4}$ We propose a novel mission concept for a segmented space telescope where each identical mirror segment is placed on modular spacecraft. Individual modules are launched as payloads of opportunity that self-assemble about the Sun-Earth L2 point. Each module uses a solar sail as a means of continuous-thrust propulsion. After docking, the solar sails are steered to overlap and create a planar sun shield for the telescope.

We present a framework for solving for the full trajectories and minimizing total flight time starting from Earth orbit to rendezvous on a Lissajous orbit in the circular three-body frame of the Sun and Earth. ${ }^{5}$ The trajectories are broken up into segments:

1. Earth escape trajectory from an Earth orbit
2. Injection into and out of an invariant manifold directed towards L2
[^0]
## 3. Rendezvous between modules to within 1000 km for docking

To get initial conditions for the first trajectory segment, we generate a distribution of potential Earth orbits for payloads of opportunity based on historical launch data from 2016-2018, and draw random samples equal to the number of required launches. ${ }^{6}$ We solve the equations of motion for this initial value problem with an energy maximization control law in the Sun-Earth rotating frame until the Jacobi constant of an L2 manifold is reached (what we define as "escape time"). ${ }^{7}$ We use an ideal solar sail model in the dynamics. ${ }^{8}$ Earth escape trajectories are simulated for a range of solar sail performance values $\beta$ and produce a nearly one-to-one relationship with escape time. Sail performance is also quantified by the size and mass of the sail and spacecraft; we create contours of escape times over the sail parameters and use them as a design tool to determine final mirror sizes and required sail masses to minimize total mission assembly time. The second trajectory segment is achieved by branching off the Earth escape trajectories and targeting the closest point of the invariant manifold. Optimal control theory is used to find minimum time trajectories to target the manifold under the constraints of the equations of motion. ${ }^{8}$ A multiple shooting algorithm is used after integration of the state and costate equations of motion to converge onto the manifold. ${ }^{9}$ TThese shooting methods are conducted starting with large solar sail performance values that are decreased (after each convergence) until the desired levels are achieved. The same process is conducted to inject the spacecraft from the manifold to the desired L2 orbit. We will also present full mission simulations from Earth orbit to L2 for a subset of the total number of modules.

Finally, the rendezvous segment is solved within an L2 quasi-periodic orbit. A Lissajous orbit is chosen since its path nearly intersects itself multiple times as it revolves around L2; modules inserted at different locations on the Lissajous encounter one another depending on orientation on the Lissajous and clump together to form the final primary mirror. Rendezvous between individual modules are simulated by placing two modules on different parts of a Lissajous orbit about L2 and maneuvering one module towards another (assumed to be on the nominal orbit) using solar sail propulsion. The modules will have a relative distance of 1000 km with minimal relative velocities to facilitate docking maneuvers. These trajectories are solved using a multiple shooting strategy. ${ }^{9}$

## TRAJECTORY OF A SINGLE SPACECRAFT

## Dynamic Model of a Solar Sail

The trajectory design is framed within the Circular Three Body equations of motion between the Sun and the Earth. ${ }^{5}$ The two primaries are assumed to be on circular orbits about their mutual center of mass. A frame $\mathcal{R}$ —with orthogonal unit vectors $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$, Cartesian coordinates $(x, y, z)$, and origin $O$ at the barycenter of the primary masses-is defined to rotate with the two primaries relative to an inertial frame $\mathcal{I}$ with unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, as shown in Figure 1. Non-dimensional units are used for distance and time as stated in Ref. 5. The reduced mass fraction $\mu$ is defined as the smaller primary mass scaled by the mass sum. The differential equations for each of the state variables are therefore

$$
\dot{\mathbf{x}}=\mathbf{f}(t, \mathbf{x})=\left[\begin{array}{c}
\dot{\mathbf{r}}  \tag{1}\\
\dot{\mathbf{v}}
\end{array}\right]=\left[\begin{array}{c}
\dot{x} \\
\dot{y} \\
\dot{z} \\
2 \dot{y}+\frac{\partial \Omega}{\partial x} \\
-2 \dot{x}+\frac{\partial \Omega}{\partial y} \\
\frac{\partial \Omega}{\partial z}
\end{array}\right]
$$

where $\Omega$ is an effective potential term in the rotating frame dependent only on spatial coordinates:

$$
\begin{equation*}
\Omega(x, y, z)=\frac{1}{2}\left(x^{2}+y^{2}\right)+\frac{1-\mu}{r_{1}}+\frac{\mu}{r_{2}} . \tag{2}
\end{equation*}
$$

The terms $r_{1}$ and $r_{2}$ are the distances from each respective primary to the third object defined as

$$
\begin{align*}
& r_{1}=\sqrt{(\mu+x)^{2}+y^{2}+z^{2}}  \tag{3}\\
& r_{2}=\sqrt{(1-\mu-x)^{2}+y^{2}+z^{2}} . \tag{4}
\end{align*}
$$

An energy integral of motion, the Jacobi integral, is

$$
\begin{equation*}
C=-\left(\dot{x}^{2}+\dot{y}^{2}+\dot{z}^{2}\right)-2 \Omega \tag{5}
\end{equation*}
$$

which is a function of the position and velocity coordinates in the rotating frame.


Figure 1. Isometric view of the rotating frame including definition of clock and pitch angles for a solar sail relative to the S -frame axes.

Each modular spacecraft is propelled by a solar sail in order to rendezvous and dock into the large segmented primary mirror. We assume a square, ideal solar sail model where the solar radiation pressure force is perfectly reflected from the surface of the sail. ${ }^{8,10}$ The acceleration due to the radiation pressure force on the solar sail is given by

$$
\begin{equation*}
\mathbf{a}_{S}=\beta \frac{1-\mu}{r_{1}^{2}}\left(\hat{\mathbf{r}}_{1} \cdot \hat{\mathbf{n}}\right)^{2} \hat{\mathbf{n}} \tag{6}
\end{equation*}
$$

in canonical units. As shown in Figure 1, $\hat{\mathbf{r}}_{1}$ is the position vector from the first primary to the spacecraft and $\hat{\mathbf{n}}$ is a unit vector normal to the solar sail. We express the components of $\hat{\mathbf{n}}$ in an auxiliary $\mathcal{S}$ frame:

$$
\begin{align*}
& \hat{\mathbf{s}}_{1}=\hat{\mathbf{r}}_{1}  \tag{7}\\
& \hat{\mathbf{s}}_{2}=\frac{\hat{\mathbf{z}} \times \hat{\mathbf{s}}_{1}}{\left|\hat{\mathbf{z}} \times \hat{\mathbf{s}}_{1}\right|}  \tag{8}\\
& \hat{\mathbf{s}}_{3}=\hat{\mathbf{s}}_{1} \times \hat{\mathbf{s}}_{2} \tag{9}
\end{align*}
$$

where $\hat{\mathbf{z}}$ is the unit vector perpendicular to the Earth and Sun's orbit as defined in the $\mathcal{R}$ frame. In this new $\mathcal{S}$ frame, the components of $\hat{\mathbf{n}}$ are represented as

$$
\hat{\mathbf{n}}=\left[\begin{array}{c}
\cos (\alpha)  \tag{10}\\
\sin (\alpha) \cos (\delta) \\
\sin (\alpha) \sin (\delta)
\end{array}\right]
$$

where $\alpha$ and $\delta$ are spherical angles representing the pitch and clock of the solar sail attitude. With this definition, the solar sail acceleration is rewritten as

$$
\begin{equation*}
\mathbf{a}_{S}=\beta \frac{1-\mu}{r_{1}^{2}} \cos ^{2}(\alpha) \hat{\mathbf{n}} \tag{11}
\end{equation*}
$$

The parameter $\beta$ is a non-dimensional number that represents the sail performance. It is defined as

$$
\begin{equation*}
\beta=\frac{L_{\odot}}{2 \pi G M_{\odot} \sigma}=\frac{\sigma^{*}}{\sigma} \tag{12}
\end{equation*}
$$

where $\sigma$ is the total areal loading factor of the spacecraft and $\sigma^{*}$ a critical loading factor defined by $L_{\odot}$, the solar luminosity at 1 AU , and $M_{\odot} \cdot{ }^{8}$ For our design study, the total spacecraft mass $m_{T}$ is split into a payload mass and a solar sail mass: the latter constitutes the sail and all the structures needed to pack and unfurl the sail, the former represents all other subsystems including the structure and mirror payload. The total sail loading factor then becomes

$$
\begin{equation*}
\sigma=\frac{m_{T}}{A_{s}}=\frac{m_{p}+m_{s}}{A_{s}}=\frac{m_{p}}{A_{s}}+\sigma_{s} \tag{13}
\end{equation*}
$$

where $m_{p}$ is the payload mass, $m_{s}$ is the sail and sail structure combined mass, $A_{s}$ is the area of the square sail, and $\sigma_{s}$ is defined as the sail density.

The solar radiation pressure acceleration can be expressed as an additional acceleration on the original Circular Three Body equations of motion as follows:

$$
\dot{\mathbf{x}}=\mathbf{f}(t, \mathbf{x}, \mathbf{u})=\left[\begin{array}{c}
\dot{x}  \tag{14}\\
\dot{y} \\
\dot{z} \\
2 \dot{y}+\frac{\partial \Omega}{\partial x}+\mathbf{a}_{S} \cdot \hat{\mathbf{x}} \\
-2 \dot{x}+\frac{\partial \Omega}{\partial y}+\mathbf{a}_{S} \cdot \hat{\mathbf{y}} \\
\frac{\partial \Omega}{\partial z}+\mathbf{a}_{S} \cdot \hat{\mathbf{z}}
\end{array}\right]
$$

where $\mathbf{u}$ is the vector of input variables $\left[\begin{array}{ll}\alpha & \delta\end{array}\right]^{T}$. The equations can be integrated forward or backwards in time for a given set of input variables; we now select initial and final conditions, then try to identify the control history of the pitch and clock angles that lead to our desired trajectories.

## Parking Orbit for Mirror Assembly

A parking orbit is required to assemble the multitude of spacecraft into a $\sim 31 \mathrm{~m}$ primary mirror. The geometries of quasi-periodic orbits about the Lagrange points, specifically L2, benefit this assembly: individual spacecraft, injected into different portions of these orbits, are likely to come into close proximity over time. We chose the Lissajous orbit, a quasi-periodic bifurcation of a fully periodic Vertical Lyapunov orbit, as the parking orbit for the mirror assembly.

We calculate periodic and quasi-periodic orbits in the C3BP equations of motion in Eq. (1) using an iterative differential control algorithm. Differential control refines initial conditions to a desired reference trajectory using the state transition matrix of the system dynamics. For the initial conditions, we use a first-order approximation of the C3BP equations of motion that geometrically traces a Lissajous-type trajectory. The state transition matrix maps the difference in state vectors between the current trajectory and a desired trajectory as follows

$$
\begin{equation*}
\delta \mathbf{x}_{1}=\mathbf{\Phi}\left(t_{1}, t_{0}\right) \delta \mathbf{x}_{0} \tag{15}
\end{equation*}
$$

where $\delta \mathbf{x}_{0}$ and $\delta \mathbf{x}_{1}$ are the differences at times $t_{0}$ and $t_{1}$ respectively. The state transition matrix is the solution to the variational differential equations

$$
\begin{equation*}
\dot{\boldsymbol{\Phi}}\left(t, t_{0}\right)=\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \boldsymbol{\Phi}\left(t, t_{0}\right) \tag{16}
\end{equation*}
$$

The Jacobian in Equation (16) is

$$
\frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}}=\left[\begin{array}{cc}
\mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3}  \tag{17}\\
-\mathcal{U} & 2 \Delta
\end{array}\right]
$$

where $\mathbf{I}_{3 \times 3}$ is a 3 by 3 identity matrix,

$$
\Delta=\left[\begin{array}{ccc}
0 & 1 & 0  \tag{18}\\
-1 & 0 & 0 \\
0 & 0 & 0
\end{array}\right],
$$

and

$$
\mathcal{U}=\left[\begin{array}{lll}
\Omega_{x x} & \Omega_{x y} & \Omega_{x z}  \tag{19}\\
\Omega_{y x} & \Omega_{y y} & \Omega_{y z} \\
\Omega_{z x} & \Omega_{z y} & \Omega_{z z}
\end{array}\right]
$$

the Hessian of the effective potential energy term in Equation (2). To obtain the state transition matrix, we must therefore simultaneously solve 36 additional differential equations.

The quasi-periodic Lissajous orbits were found using a two-step differential correction process given in Reference 11. The first order linear approximation of the Lissajous is used as an initial guess and iterated until convergence. The two-step process divides the full trajectory into segments and, using differential correction, iteratively connects them in position space and then in velocity space until convergence. The final Lissajous orbit is shown in Figure 2.

## Invariant Manifold from Earth to L2

Transfers to the Sun-Earth L2 are facilitated by targeting an invariant manifold of a periodic orbit. They can extend to near Earth orbit and, once a spacecraft is injected into a manifold, the natural dynamical flow guides them to L2 without the need of propulsion. We first create a vertical Lyapunov orbit using a differential control algorithm. We then calculate an invariant manifold through the monodromy matrix-the state transition matrix $\boldsymbol{\Phi}\left(T, t_{0}\right)$, where $T$ is the period of the orbit-of the vertical Lyapunov. The monodromy matrix has six eigenvalues with the following


Figure 2. Lissajous trajectory about the Sun-Earth L2 point (in the rotating frame) produced through two-step differential correction process shown for 2 and 20 revolutions (left and right, respectively).
properties: ${ }^{5}$

$$
\begin{align*}
& \lambda_{1}>1  \tag{20}\\
& \lambda_{2}=\frac{1}{\lambda_{1}}  \tag{21}\\
& \lambda_{3}=\lambda_{4}=1  \tag{22}\\
& \lambda_{5}=\bar{\lambda}_{6},\left|\lambda_{5}\right|=1 . \tag{23}
\end{align*}
$$

The first two eigenvalues correspond to unstable and stable behavior, respectively. Their corresponding eigenvectors are used to find the invariant manifolds extending towards Earth. A state on the periodic orbit $\mathbf{x}_{0}^{P}$ can be offset from the periodic orbit along the direction of one of these eigenvectors and integrated forwards or backwards through time to create the desired trajectory. The new initial state of the unstable manifold, for instance, is

$$
\begin{equation*}
\mathbf{x}_{0}^{\mathcal{U}}\left(\mathbf{x}_{0}^{P}\right)=\mathbf{x}_{0}^{P}+\epsilon \mathbf{Y}^{\mathcal{U}}\left(\mathbf{x}_{0}^{P}\right) \tag{24}
\end{equation*}
$$

where $\epsilon$ corresponds to a small displacement and $\mathbf{Y}^{\mathcal{U}}$ is the unstable eigenvector. The displacements are conducted at varying points along the periodic orbit to obtain an approximation of the invariant manifold; a subset of these trajectories is shown in Figure 3. From the integration, it is noted that a trajectory starting from the closest point on the manifold to the Earth and ending near the vertical Lyapunov has a flight time of approximately 90 days.

## Earth Escape Trajectories

After launch, each individual spacecraft will be injected onto an initial Earth-centered orbit. The solar sails must accelerate the spacecraft sufficiently to match the position and velocity of the manifold before entering the manifold towards the Sun-Earth L2. The Earth-escape trajectories are computed starting from an initial Earth-centered orbit using a control law for energy maximization as in References 7,12,13 but applied to the Circular Three Body Problem. The control law is found through their same optimization of

$$
\begin{equation*}
\max _{\hat{\mathbf{n}}} \mathbf{a}_{S}(\hat{\mathbf{n}}) \cdot \mathcal{S}_{\mathbf{v}} \tag{25}
\end{equation*}
$$



Figure 3. A subset of the vertical Lyapunov invariant manifold found by integrating the unstable manifold backwards in time (left). The zoomed-in plot (right) shows the vertical Lyapunov orbit and the initial conditions of the invariant manifold.
finding the direction of the normal sail vector needed to maximize the projection of the solar radiation pressure acceleration on the velocity of the spacecraft, though the velocity is projected onto the $\mathcal{S}$ frame defined in Equations (7), (8), and (9). The optimization is solved as a constrained parameter optimization problem with the Hamiltonian

$$
\begin{equation*}
H=\beta \frac{1-\mu}{r_{1}^{2}}\left(\hat{\mathbf{r}}_{1} \cdot \hat{\mathbf{n}}\right)^{2} \hat{\mathbf{n}} \cdot \overrightarrow{\mathbf{v}}+\boldsymbol{\eta}(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}-1) \tag{26}
\end{equation*}
$$

where a Lagrange multiplier $\boldsymbol{\eta}$ is added for the constraint that the normal sail vector must be of unit length. The optimal pointing of the normal vector is found by requiring that

$$
\begin{equation*}
\frac{\partial H}{\partial \hat{\mathbf{n}}}=\mathbf{0} \tag{27}
\end{equation*}
$$

which leads to

$$
\begin{align*}
& n_{x}=\frac{\left|v_{y}\right|}{\sqrt{v_{y}^{2}+\xi^{2}\left(v_{y}^{2}+v_{z}^{2}\right)}},  \tag{28}\\
& n_{y}=\xi n_{x}, \text { and }  \tag{29}\\
& n_{z}=\frac{v_{z}}{v_{y}} n_{y} \tag{30}
\end{align*}
$$

where

$$
\begin{equation*}
\xi=\frac{-3 v_{x} v_{y} \pm v_{y} \sqrt{9 v_{x}^{2}+8\left(v_{y}^{2}+v_{z}^{2}\right)}}{4\left(v_{y}^{2}+v_{z}^{2}\right)} . \tag{31}
\end{equation*}
$$

Taking the positive sign in the $\xi$ definition leads to an energy gain trajectory while the negative sign leads to energy loss; a sample energy gain trajectory is shown in Figure 4 as a function of time. We integrate initial conditions on an Earth orbit forwards in time using this energy gain control law; an event function terminates the trajectory once the spacecraft matches the Jacobi constant of the invariant manifold.


Figure 4. Sample escape trajectory from Earth (left). Jacobi constant of a sample Earth escape trajectory using an solar sail energy gain control law (right). Jacobi constant for invariant manifold is also plotted for reference.

## Entering the Manifold

The algorithms used to transfer a spacecraft onto and out from the invariant manifold use the same techniques; we will therefore only summarize the former algorithm in depth as an example. The transfer from the Earth-escape trajectory onto the invariant manifold is computationally challenging to produce with the low-thrust propulsion of a solar sail. We therefore solve an easier problem and iterate over it until we converge onto a trajectory with our desired parameters.

Optimal Control with an Unconstrained Thruster First, we model the propulsion of the spacecraft as an unconstrained thruster and solve the transfer trajectories indirectly using optimal control theory. The control inputs are therefore

$$
\begin{equation*}
\mathbf{u}=u_{1} \hat{\mathbf{x}}+u_{2} \hat{\mathbf{y}}+u_{3} \hat{\mathbf{z}} . \tag{32}
\end{equation*}
$$

which are the accelerations produced by the unconstrained thruster in the $\mathcal{R}$ frame. ${ }^{14}$ The equations of motion for the unconstrained thruster are

$$
\dot{\mathbf{x}}=\mathbf{f}(t, \mathbf{x}, \mathbf{u})=\left[\begin{array}{c}
\dot{x}  \tag{33}\\
\dot{y} \\
\dot{z} \\
2 \dot{y}+\frac{\partial \Omega}{\partial x}+u_{1} \\
-2 \dot{x}+\frac{\partial \Omega}{\partial y}+u_{2} \\
\frac{\partial \Omega}{\partial z}+u_{3}
\end{array}\right] \text {. }
$$

The control effort of the thruster is minimized with the cost function

$$
\begin{equation*}
J=\int_{t_{0}}^{t_{F}} \frac{1}{2} \mathbf{u} \cdot \mathbf{u} d t \tag{34}
\end{equation*}
$$

where $\mathbf{u}$ is the continuous control inputs throughout the trajectory and $t_{0}$ and $t_{F}$ are the initial and final time of the trajectory. We solve a constrained optimization problem through the Hamiltonian

$$
\begin{equation*}
H_{T}(t, \mathbf{x}, \mathbf{u}, \lambda)=\frac{1}{2} \mathbf{u} \cdot \mathbf{u}+\boldsymbol{\lambda}^{T} \mathbf{f}(t, \mathbf{x}, \mathbf{u}) \tag{35}
\end{equation*}
$$

where we minimize the cost function in Equation (34) under the constraint that the trajectories follow the equations of motion in Equation (33). The six Lagrange multipliers in $\boldsymbol{\lambda}$ are referred to as costates; they are the solution to the differential equation

$$
\begin{equation*}
\dot{\boldsymbol{\lambda}}=-\left(\frac{\partial H_{T}(t, \mathbf{x}, \mathbf{u}, \boldsymbol{\lambda})}{\partial \mathbf{x}}\right)^{T} \tag{36}
\end{equation*}
$$

Setting the partial derivative of the Hamiltonian with respect to the control inputs $\mathbf{u}$ to $\mathbf{0}$, as in Equation (27), leads to the following optimal control law ${ }^{15}$

$$
\begin{align*}
u_{1} & =-\lambda_{4},  \tag{37}\\
u_{2} & =-\lambda_{5},  \tag{38}\\
u_{3} & =-\lambda_{6} . \tag{39}
\end{align*}
$$

With this new control law, we integrate the full equations of motion with initial conditions for the states and costates. We compute the transfer trajectories by solving a boundary value problem with 12 initial and terminal boundary conditions: 6 position and velocity conditions each on the escape trajectory and manifold respectively. These BVPs are solved using a collocation algorithm ${ }^{16}$ to find the initial and final costates needed to realize the desired trajectory.

Lowest Thrust Solution Next, we conduct a coarse search algorithm to find the optimal transfer conditions with the unconstrained thruster. After solving a particular BVP, we map the resultant velocity costates to control inputs. We determine an optimal transfer to be one where the average magnitude of the velocity costates-and therefore the accelerations caused by the unconstrained thruster-throughout the trajectory to be minimized, allowing us to more easily transition to a lowthrust optimization problem. For a particular escape trajectory, there are four parameters that can be toggled to produce the lowest-thrust maneuver possible: the time $t_{E}$ that the spacecraft detaches from the escape trajectory, the specific manifold branch $n_{M}$ that is being targeted, the time (or location) of injection onto that specific branch of the manifold $t_{M}$ and the flight time $\Delta t$. We model the optimization as

$$
\begin{equation*}
\min _{t_{E}, n_{M}, t_{M}, \Delta t} \frac{1}{\Delta t} \int_{t_{E}}^{t_{E}+\Delta t}\left|\boldsymbol{\lambda}_{v}\right| d t . \tag{40}
\end{equation*}
$$

We find the minimum thrust trajectory by solving BVPs over the four parameters. Quick yet sufficient solutions can be found by randomly selecting $n_{M}$, selecting the latest $t_{E}$ before reaching the Jacobi constant of the manifold, and then performing a loop over the parameters $t_{M}$ and $\Delta t$.

Iterating with a Solar Sail The ideal solar sail trajectories are solved using a multiple shooting strategy: ${ }^{9}$ the trajectory is divided into $N$ segments and the pitch and clock angles are held constant throughout each trajectory. The $N$ segments are individually and simultaneously integrated forwards until the time at which the next segment begins (or, in the case of the last segment, until the termination point). The optimization is conducted under the constraint that the final state of the integrated trajectory and the initial state of the next trajectory must match. The optimization variables are defined as follows:

$$
\begin{equation*}
\mathbf{w}=\left[\alpha_{0}, \delta_{0}, \mathbf{x}_{1}, \alpha_{1}, \delta_{1}, \ldots, \mathbf{x}_{N-2}, \alpha_{N-2}, \delta_{N-2}, \alpha_{N-1}, \delta_{N-1}, \Delta t\right] \tag{41}
\end{equation*}
$$

where $\alpha_{n}$ and $\delta_{n}$ are the pitch and clock angles of the $n^{\text {th }}$ segment, $\mathbf{x}_{n}$ is the state vector of the intermediate points between segments (not defined for the first or last segment), and $\Delta t$ is the total
flight duration. Each segment is defined in equal portions of time from $t_{E}$ to $t_{E}+\Delta t$. We use the optimal unconstrained thruster trajectory from the previous section as an initial guess for the optimization variables. The velocity costates $\boldsymbol{\lambda}_{v}$ are converted into angles by projecting it onto the $\mathcal{S}$ frame and rewriting the corresponding unit vector as pitch and clock angles as follows:

$$
\boldsymbol{\mathcal { S }}_{\boldsymbol{\lambda}_{v}}=\left[\begin{array}{l}
\lambda_{4, \mathcal{S}}  \tag{42}\\
\lambda_{5, \mathcal{S}} \\
\lambda_{6, \mathcal{S}}
\end{array}\right]=\left[\begin{array}{c}
\cos \bar{\alpha} \\
\sin \bar{\alpha} \cos \bar{\delta} \\
\sin \bar{\alpha} \sin \bar{\delta}
\end{array}\right] .
$$

These relative angles are used as an initial guess for the pitch and clock angles of the solar sail. The multiple shooting optimization is conducted using the optimal unconstrained thruster BVP solutions as an initial guess but with an infeasibly powerful sail $(\beta=1)$. Once that optimization converges (meaning a continuous trajectory is formed out of the $N$ segments), the multiple shooting process is conducted again but with a lower $\beta$ value using the previous optimization variables as an initial guess. This is done iteratively until the desired final $\beta$ converges. Future optimization will seek to minimize the flight time of these transfers.

## SIMULATING THE ENTIRE MISSION

A full mission simulation was conducted using the previous techniques to develop end-to-end trajectories for multiple spacecraft with randomized initial conditions. A 31 m primary requires 840 hexagonal 1 m mirror segments. ${ }^{17}$ In the following sections we detail the methods for determining launch schedules, designing the solar sail and results of sending multiple solar sails from Earth orbit to the Lissajous.

## Launch Scheduling

Predicting future launch capability is central to determining how many segments can be launched as payloads of opportunity, and how many would have to be launched on rockets dedicated to this mission. To predict future launch capability, a historical launch analysis was performed, which was targeted at recent launches (2016-2018) from the United States. Launch data was collected from the Orbital Launch Log, ${ }^{6}$ last updated in May of 2018. It includes data on launch dates, sites, vehicles, and the Committee on Space Research (COSPAR) designation for each payload on the launch. Payload data was collected from the Union of Concerned Scientists' satellite catalogue, ${ }^{18}$ which contains orbital elements and payload mass for most satellites in order. Satellites were matched to launches using COSPAR designations.

For this analysis, launches from Kennedy Space Center/Cape Canaveral (KSC/CC), Wallops (W), and Vandenburg Airforce Base (V/VS) were selected, if they took place between January 1st, 2016, and December 31st, 2017. Once the payloads for each launch were identified based on their COSPAR designation, the mass of each payload, and the inclination and semi-major axis of their orbit were collected. The semi major axis was used to calculate the $C_{3}$ value for each payload. Launches where payload data was missing were removed from the data pool. For launches with a full data set, an array of total payload mass, inclination, and $C_{3}$ value was calculated. The $C_{3}$ value for the launch was taken to be the a weighted average of the $C_{3}$ for each payload, weighted by mass. The inclination launch was taken to be the average inclination for each payload item.

Next, launches were categorized based on launch vehicle. Then, for each launch, a spare payload was calculated. This calculation was performed conservatively, by defining spare payload as the
difference between a given launch's payload, and the maximum payload carried by the same launch vehicle, on a launch with an equal or higher $C_{3}$. This definition means that we considered many flights to have zero spare payload, but ensures that the spare payload estimate is based off of the payload capabilities that have been demonstrated by each launch vehicle. For recent years (2016, 2017), many launches took place on launch vehicle variants that only had one flight. Many launches also took place on launch vehicles that had few enough flights that no spare payload could be calculated. For 2016-2017, only the Falcon 9 and Atlas V 401 had enough launches to define spare payload. Therefore, only these launch vehicles were used to predict future spare payload capacity.


Figure 5. Histograms of randomly generated launch attributes. The figure on the left is a histogram of launch C3, the center figure is a histogram of launch inclination, and the figure on the right is a histogram of spare payload capacity.

To predict future launch capacity, launches were randomly generated based on the historical launch data. The distribution of launch vehicles in 2016 and 2017 was used to randomly select a launch vehicle from the pool of launch vehicles for which spare payload was calculated. Once a launch was chosen, a launch profile was randomly selected from the launches for that vehicle from 2016 to 2017. The payload, spare payload, launch site, inclination, and c3 for that launch were taken to be the values for the randomly generated launch. Figure 5 shows 1000 randomly generated launches describing the projected distribution of future launches. The results from this analysis were that the majority of launches have little to no spare payload, and are launched on geosynchronous transfer orbits (GTO), which have little to no inclination. Fewer launches are to low Earth orbit, or are launched to a high inclination or polar orbit, and $43 \%$ of launches have more than 500 kg of spare payload.

To determine the number of modules that could be launched, random launches were generated for 7 years, with each year containing 34 launches (the total number of relevant US launches in 2018, and therefore a conservative estimate that doesn't project any growth in the orbital launch market). The spare payloads were then run through the methods described previously; the current rate of launches would be sufficient to launch all 840 modules in less than 7 years, with no dedicated
launches required.

## Escape Time as a Sail Design Metric

The time to escape the Earth, or at least reach an appropriate Jacobi constant, can be used as a good metric for measuring the performance of the solar sail. Simulations of Earth escape trajectories were conducted by seeding random initial Earth-centered orbits from the launch catalog and deploying the solar sail by solving an IVP with the terminating event that the Jacobi constant of the invariant manifold is reached. The beta values were varied for 200 simulations. The results are presented in Figure 6, showing an inverse relation between beta and escape time.


Figure 6. Escape times are plotted on the left for 200 simulations of Earth escape trajectories with varying $\beta$ values. On the right is a contour plot of $\beta$ values as a function of sail length and payload masses, with $\beta$ replaced by escape times. Heatmap shows corresponding sail mass with sail density of $25 \mathrm{~g} / \mathrm{m}^{2}$.

As mentioned previously, the beta values can be parameterized using the payload mass, sail area and sail density. A visualization of the parameter space for the sail loading is shown in Figure 6. Sail density is assumed a constant (value of $25 \mathrm{~g} / \mathrm{m}^{2}$ ) while payload mass and sail length are varied in the 2D plot. The corresponding sail mass is shown in the color scale. Each point on the grid corresponds to a beta value and, from Figure 6, the beta and escape time relationship is nearly one-to-one. Contour lines are drawn on the figure for escape times corresponding to the shown beta value. Parameters can be chosen to target feasible escape times under 6 years, for example.

## Full Mission Simulation

We conducted a mission simulation for 840 spacecraft launching within a 7 year period. We chose a 1 m mirror design and estimate the payload mass $m_{P}$ to be approximately 150 kg . With a feasible escape time of approximately 2 years, the contour map in Figure 7 indicates a sail length of approximately 35 m and sail mass of 45 kg (corresponding to a $\beta$ of 0.01 ). The launch windows are sampled from the gathered launch data and shown in Figure 7. The number of spacecraft launched as payloads of opportunity are also shown for each launch. Almost every launch was performed by the Falcon 9 vehicle because of the likelihood for spare payload corresponding to our spacecraft mass.

Each trajectory, from Earth orbit to Lissajous, was computed using 16 parallel 3.7 GHz cores in approximately 27 hours. Unfortunately, not every solar sail converged at a $\beta$ of 0.01 ; most of


Figure 7. Histogram of launch schedule throughout simulated mission with number of launched spacecraft shown per launch.
the trajectories shown converged at approximately 0.06 . More refined optimization techniques are needed to conclude whether this limitation is a physical one or one due to the algorithm used. We also assumed that, whenever multiple satellite are launched at a single date, they all follow the same trajectory. Future simulations may decide to stagger the solar sail deployment while on the initial Earth orbit. A subset of trajectories are shown in Figure 8, approximately 44. The trajectories take each module from Earth orbit, to an Earth escape orbit, transfer into the manifold, follow the natural flow of the manifold, then branch off and transfer into the Lissajous orbit. A histogram is also shown in Figure 9 showing the full mission flight times of all the modules. Every successful mission was completed within less than 4.5 years total, meaning that in the worst case scenario, launching all modules within a 7 year span, all modules would be on the Lissajous in under 12 years without needing any dedicated launches.

## TELESCOPE ASSEMBLY

## Rendezvous

Analysis of module rendezvous is motivated by the need to determine which maneuvers can be accomplished with solar sails. The first step in rendezvous analysis was identifying the phasing of modules such that a close approach would be induced. To do this, a Lissajous orbit that would last 50 years was generated. This was necessary due to the quasi-periodic behaviour of the selected Lissajous orbit. With this orbit created, the distance between each point on the orbit, and every other point, was calculated. This was done with the goal of identifying locations on the orbit that had a close approach, where two modules would come within 1000 km of each other. Points within $\pm 30$ days of a given point on the orbit were ignored for this part of the analysis, to prevent close approaches from being identified in parts of the Lissajous orbit immediately adjacent to a given point. This analysis showed that close approaches would be induced if modules were phased by 177.5 days on the orbit. This means that if a module is randomly placed on the Lissajous orbit, it will have a close approach with a module placed 177.5 days ahead of it on the orbit, or 177.5 days behind it. For the following rendezvous simulations, modules were placed near where close approaches were identified, to reduce the amount of the trajectory that needed to be simulated.


Figure 8. Plot of subset of mission trajectories from Earth orbit to Lissajous injection, centered about the Earth. Total of 44 trajectories are shown.

Once two modules were appropriately placed on the Lissajous orbit, they were assigned roles. One was arbitrarily designated as the leader, which would move passively on the Lissajous orbit, and the other was designated as the follower, which uses control to match the leader's position and velocity. The first attempt at developing rendezvous trajectories involved modeling the propulsion system as an unconstrained thruster as was done during the manifold injection trajectories. Rendezvous trajectories and control inputs were generated using the same collocation algorithm, with the time to rendezvous set to 1.45 days.

These trajectories and control inputs were used as the initial guess for the multiple shooting method, which was used to solve for the trajectories and control input for a module with a solar sail. Additionally, the multiple shooting method was set to minimize time to rendezvous, and used Equation (43) as the cost function to minimize. Here, $t_{0}$ is the time at close approach, and $t_{f}$ is the time at rendezvous. In contrast to the unconstrained thruster optimal control solution, this method set the time at rendezvous, $t_{f}$, as a free parameter, to be updated every iteration along with the positions and velocities at each segment. Since the position and velocity of the lead satellite at rendezvous is a function of $t_{f}$, these values were updated as final time changed, requiring the terminal constraint equation to be updated every iteration. The terminal constraint can be seen in Equation 44. Here, $\mathbf{x}_{\mathbf{1}}\left(t_{f}\right)$ and $\dot{\mathbf{x}}_{\mathbf{1}}\left(t_{f}\right)$ are the position and velocity of the lead satellite at $t_{f} . \mathbf{x}_{\mathbf{2}}\left(t_{f}\right)$ and $\dot{\mathbf{x}}_{\mathbf{2}}\left(t_{f}\right)$ are the position and velocity of the follower at the final time.


Figure 9. Histogram of module flight times from Earth to Lissajous.

$$
\begin{gather*}
L=\int_{t_{0}}^{t_{f}} d t  \tag{43}\\
{\left[\begin{array}{l}
\mathbf{x}_{\mathbf{1}}\left(t_{f}\right)-\mathbf{x}_{\mathbf{2}}\left(t_{f}\right) \\
\dot{\mathbf{x}}_{\mathbf{1}}\left(t_{f}\right)-\dot{\mathbf{x}}_{\mathbf{2}}\left(t_{f}\right)
\end{array}\right]=\mathbf{0}} \tag{44}
\end{gather*}
$$

Rendezvous were generated throughout the Lissajous trajectory, to ensure the solar sail was capable of producing a rendezvous trajectory regardless of the direction of the initial and final velocity, as this varies greatly depending on the location on the Lissajous orbit.


Figure 10. Histogram of the time to rendezvous for modules using a solar sail. The $x$ axis is the time to rendezvous in days, the blue bar represents the percentage of simulated rendezvous that took that amount of time to rendezvous.

Simulations of 100 rendezvous were conducted; a histogram of the time to rendezvous for these simulations can be seen in Figure 10. This indicates that modules can rendezvous regardless of their
position on the orbit. A peak is identified for final times between 1.5 days and 2 days. The multiple shooting algorithm is not deterministic; increasing the number iterations for each rendezvous increases the chances of identifying the true minimum time trajectory. However, the majority of the rendezvous events simulated have a time to rendezvous greater than the initial guess of 1.45 , so it is reasonable to assume that this is close to the true minimum time to rendezvous for most of the simulated events, as no rendezvous events were found to have a time to rendezvous of less that one day.

## Docking

Analysis of the docking procedure is required to understand how larger groups of modules should rendezvous. The concept of operations for this mission calls for the assembly of clusters of modules, which will then rendezvous with one another for the final assembly of the primary mirror. The first step in assessing docking is to identify how many modules should be in a cluster before it can rendezvous for final assembly. It is desirable to have a low number of "precision docking events", where a module needs to navigate to a specific location on a cluster. However, it is necessary for some number of precision docking events to take place, to prevent vacant positions from being surrounded and made inaccessible, and to ensure the correct final mirror shape is created. Additionally, it is desirable for docking events to occur on docking sites where the vacant position is adjacent to three modules. When three modules are adjacent to a docking site, the incoming trajectory can be anywhere within a 60 degree arc, allowing for some error in the incoming module's trajectory. If four modules are adjacent to a docking site, there is only one allowable approach direction, allowing for no error in the approach trajectory. This can be seen in Figure 11.


Figure 11. Top view of modules docking in two configurations. In the configuration on the left, an incoming module's trajectory may be anywhere within a 60 degree arc. However, in the configuration on the right, an incoming module only has one available trajectory.

These conditions produces a simple docking scheme, where modules are allowed to randomly add themselves to any available docking site on a cluster, unless there is a docking site that is adjacent to three modules. In that case, a precision docking maneuver must take place, where the incoming module must dock at the site with three adjacent modules. Figure 12 shows a cluster growing using this scheme. Each frame shows the state after 5 modules have been added. This docking scheme, while simple, prevents difficult docking situations, while still allowing some modules to dock to at random sites.


Figure 12. Top views of growing clusters of modules. Frame (a) shows the cluster after the first five modules have docked, and frames (b) through (f) show the state sequentially, with five modules docking in between each frame. The black hexagons represent docked modules, and the red dashed hexagons represent the available docking sites.

## CONCLUSION

We demonstrate a basic concept of operations for the in-space autonomous assembly of modular spacecraft about Sun-Earth L2. Each spacecraft carries a 1m sized hexagonal mirror and, when all 840 are assembled, form a 31 m primary mirror for a space telescope. With our conservative projection of future launches, all spacecraft can be launched within a period of 7 years as payloads of opportunity. Transfers from Earth to the Sun-Earth L2 Lissajous orbit have a flight time of approximately 4 years if using a 35 m solar sail and a 200 kg spacecraft (including sail). Rendezvous trajectories can also be achieved within days. Full assembly can be accomplished within an 11-12 year period. Further work is needed to refine the transfers to the manifold, including a higher level optimization which incorporates the initial Earth orbit.

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