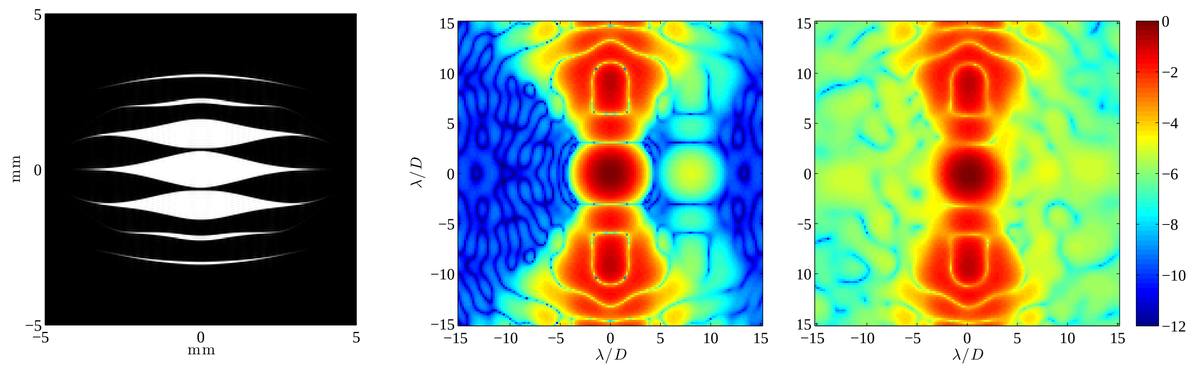
Experimental Verification of Bayesian Planet Detection Algorithms with a Shaped Pupil Coronagraph

Dmitry Savransky, Tyler D. Groff, and N. Jeremy Kasdin[1]

[1]Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ USA

Problem



(a) Pupil Mask (b) Star + Planet (c) Star + Planet + Speckles

Simulation of star and planet images with a shaped pupil coronagraph [Kasdin et al., 2003]. Planet is 10^5 times dimmer than star, and ~ 4 times brighter than speckle average, but is still difficult to pick out from speckles. Image plane figures are Log(Intensity).

Linear Filters and Cross-Correlation

- Model observation as:

$$\mathbf{z}(x,y) = C_p\bar{P}(x-\xi, y-\eta) + \boldsymbol{\nu}$$

where C_p is the mean photon count at planet location - pixel (ξ,η), $\bar{P}$ is the normalized PSF, and $\boldsymbol{\nu}$ is the noise. [Kasdin and Braems, 2006]

- Seek filter h to maximize signal-to-noise (SNR):

$$\mathrm{SNR} = \frac{\langle \mathbf{s}, \mathbf{s}\rangle}{\mathcal{E}\left\{\langle \mathbf{n}, \mathbf{n}\rangle\right\}}$$

where $\langle\cdot,\cdot\rangle$ is the inner product, $\mathcal{E}\{\}$ is the expectation and

$$\mathbf{s} = \langle h, C_p\bar{P}\rangle \quad \text{and} \quad \mathbf{n} = \langle h, \boldsymbol{\nu}\rangle$$

- The optimal (matched) filter is then

$$h = \alpha R^{-1} C_p \bar{P}$$

for constant α and noise covariance (or autocorrelation) R, with filter output given by the convolution:

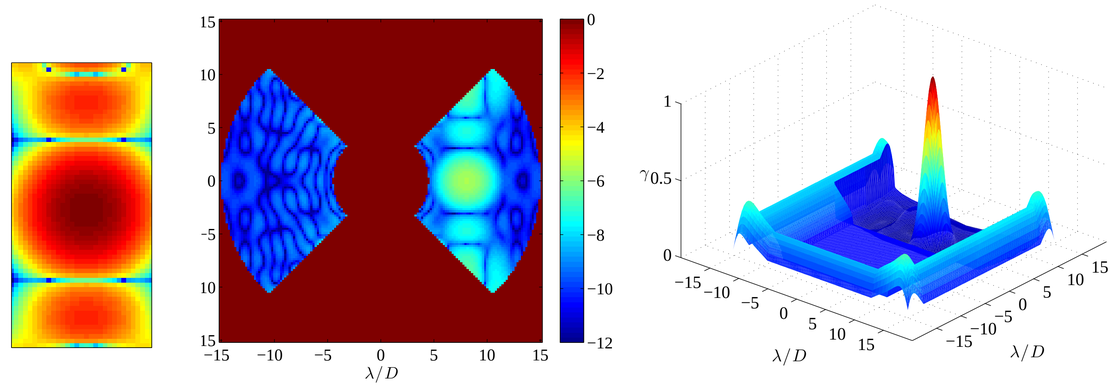
$$\mathbf{y} = h * \mathbf{z}$$

- A matched filter can be replaced by correlation operations for template matching. [Ziemer and Tranter, 2002]
- Define a normalized cross-correlation as:

$$\gamma = \frac{(\mathbf{z}-\bar{\mathbf{z}}')*(T-\bar{T})}{\sqrt{\langle \mathbf{z}-\bar{\mathbf{z}}', \mathbf{z}-\bar{\mathbf{z}}'\rangle \langle T-\bar{T}, T-\bar{T}\rangle}}$$

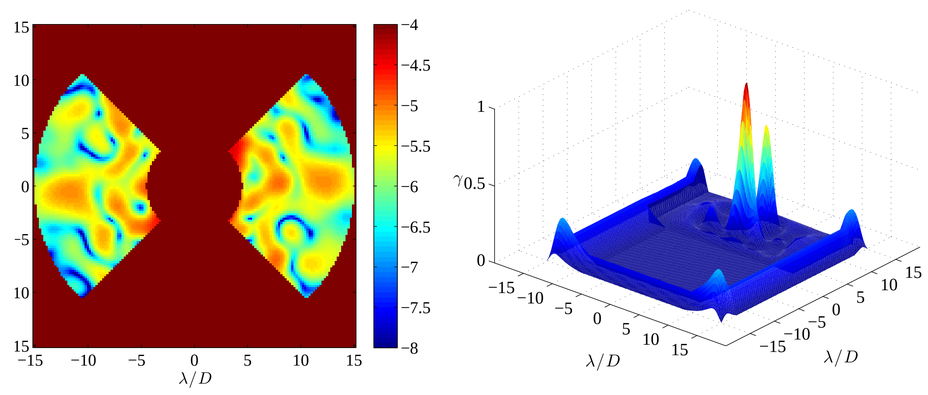
where T is the template, $\mathbf{z}'$ is the section of the image beneath the template, and $\bar{x}$ denotes normalization. [Lewis, 1995]

Simulation



(a) Template (b) Image (c) Cross-correlation

Simulation of cross-correlation applied to planet image with no speckle (and star removed with focal plane mask) using normalized PSF as the template. Template and image are Log(Intensity). Corner peaks are due to edge effects.



(a) Image (b) Cross-correlation

Simulation of cross-correlation applied to planet image with speckle. Planet is at 99% mean speckle intensity and second cross-correlation peak is due to strong speckle.

- How do we determine significance of a peak?

References

- Groff, T., Carlotti, A., and Kasdin, N. (2010). Progress on broadband control and deformable mirror tolerances in a 2-dm system. In *Proceedings of SPIE*, volume 7731, page 77314S.
- Kasdin, N., Vanderbei, R., Spergel, D., and Littman, M. (2003). Extrasolar planet finding via optimal apodized-pupil and shaped-pupil coronagraphs. *ApJ*, 582(2):1147–1161.
- Kasdin, N. J. and Braems, I. (2006). Linear and bayesian planet detection algorithms for the terrestrial planet finder. *ApJ*, 646:1260–1274.
- Lewis, J. (1995). Fast normalized cross-correlation. In *Vision Interface*, volume 10, pages 120–123. Citeseer.
- Navarro, R., Nestares, O., and Valles, J. (2004). Bayesian pattern recognition in optically degraded noisy images. *Journal of Optics A: Pure and Applied Optics*, 6:36.
- Ziemer, R. and Tranter, W. (2002). *Principles of communications*. Wiley New York.

Probability

- Express the filtered observation as

$$\mathbf{y} = (\mathbf{k}*\mathbf{f})*h + \boldsymbol{\nu}*h \approx C_pP_h(x-\xi, y-\eta) + \boldsymbol{\nu}*h$$

where $\mathbf{k}$ is the optical system impulse response, $\mathbf{f}$ is the original pattern and $P_h = \mathbf{k}*h$. [Navarro et al., 2004]

- Assuming a constant prior for impulse response, the posterior for the input pattern is:

$$p(\mathbf{f}, \{C_p,\xi,\eta\}\,|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{f}, \{C_p,\xi,\eta\})p(\mathbf{f})$$

- Seek the probability that the input pattern matches the template:

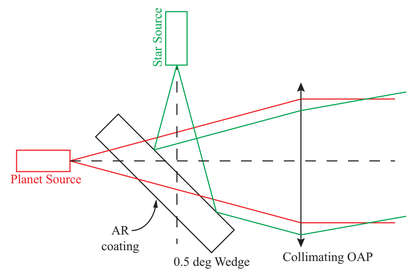
$$p(\mathbf{f} = T, \{C_p,\xi,\eta\}\,|\mathbf{y})$$

- Maximizing this probability is equivalent to minimizing Euclidian distance error function so:

$$\max p(\mathbf{f} = T, \{C_p,\xi,\eta\}\,|\mathbf{y}) \propto \exp\left(R^{-1}\left(\langle \mathbf{z}', \mathbf{z}'\rangle\right)^{-1}\langle\langle \mathbf{y}, C_pP_h(x-\xi,y-\eta)\rangle, \langle \mathbf{y}, C_pP_h(x-\xi,y-\eta)\rangle\rangle\right)$$

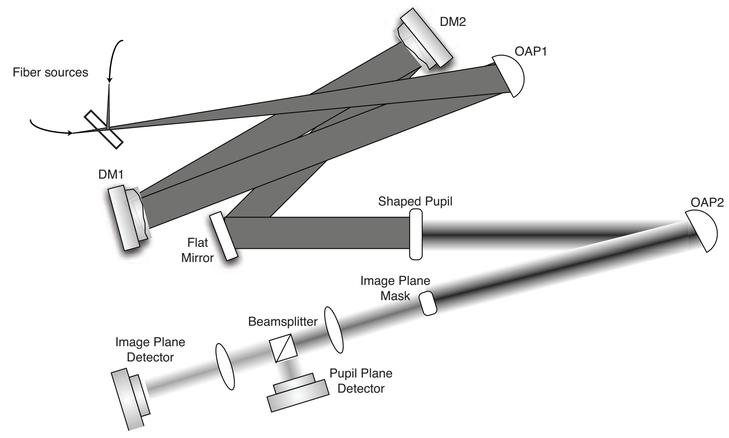
- Choose threshold for cross-correlation based on desired minimum probability of match.

Lab Setup

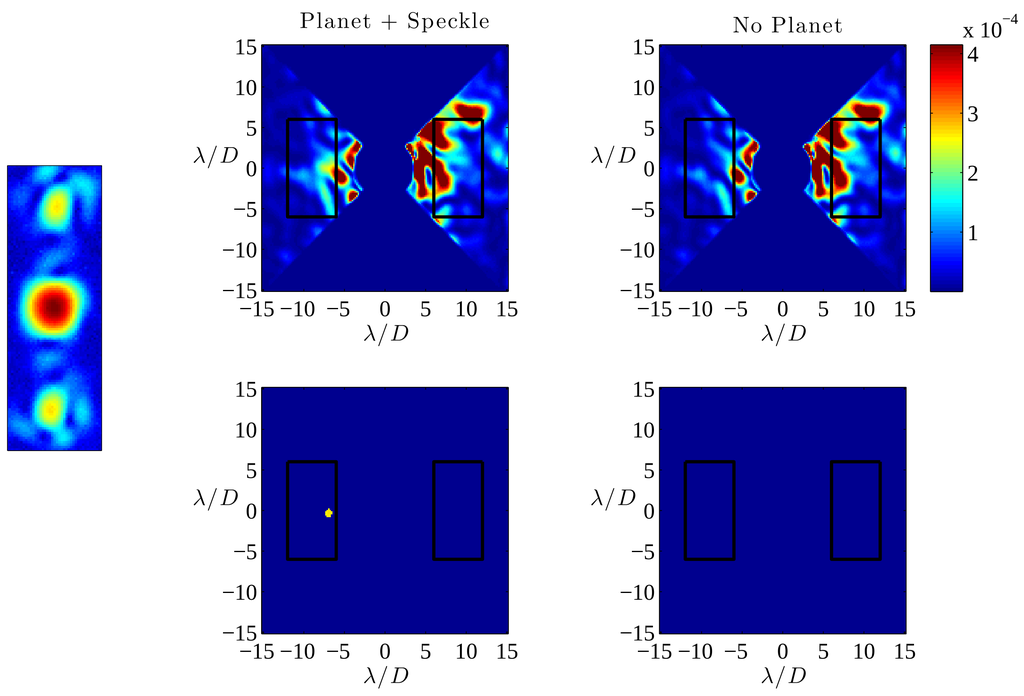


Separate laser sources are used to generate the star and planet, one transmitted and one reflected through a 0.5° glass wedge to a collimating optic (OAP 1 in the figure below).

High Contrast Imaging Lab (HCIL). [Groff et al., 2010]

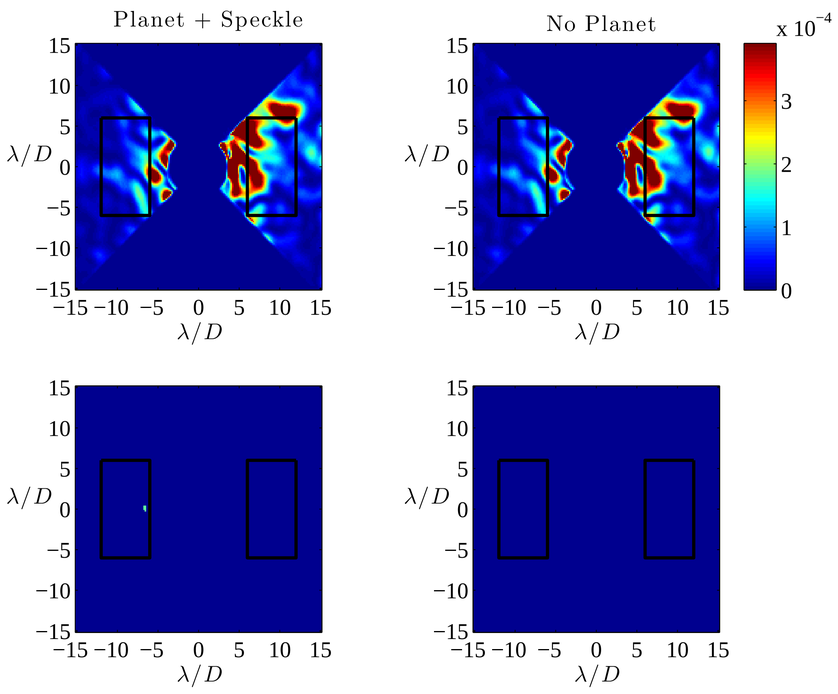


Experimental Results



(a) Template (b) Planet at Mean Speckle Intensity, p = 0.95

Top row: lab images with and without planet signal. Bottom row: filter applied to areas in black boxes with stated probability threshold. Images are linearly scaled.



(c) Planet at One Tenth Mean Speckle Intensity, p = 0.55