3 - Orbits in Time and Space

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Orbits in Time and Space

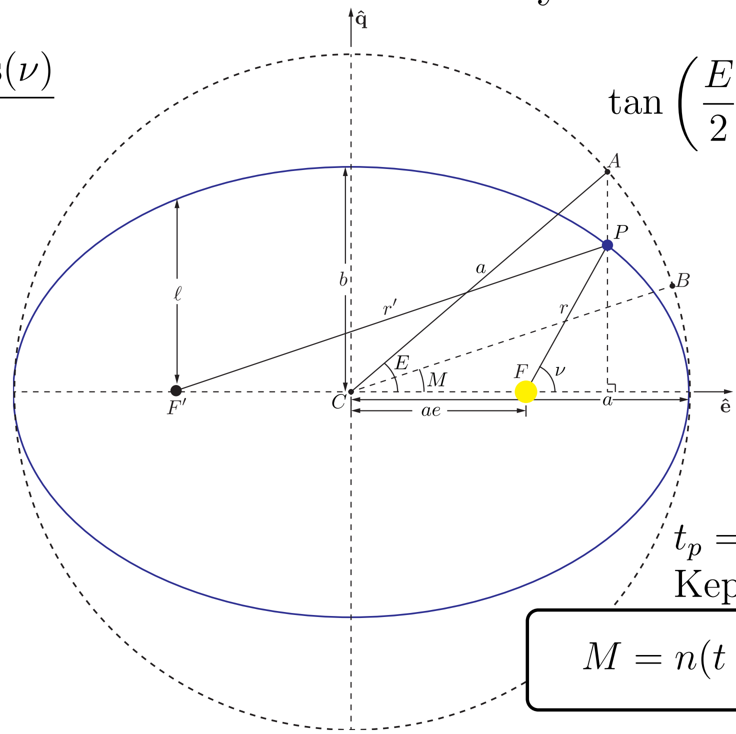
The two-body system is fully integrable, and has an exact solution for all time, but that doesn't mean that we can write down an analytical, algebraic solution for the position and velocity of our orbiting body at any arbitrary time (although we can express these solutions as infinite series). Much of the history of astrodynamics has been devoted to coming up with more accurate and computationally efficient solutions to this problem, which we will explore here.

The Auxiliary Circle and Mean Anomaly

Eccentric Anomaly:

$$\cos(E) = \frac{ae + r\cos(\nu)}{a}$$

$$\tan\left(\frac{E}{2}\right) = \sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\nu}{2}\right)$$



t_p = time of periapsis

Kepler's Time Equation:

$$M = n(t - t_p) = E - e\sin(E)$$

Mean Motion:

$$n \triangleq \frac{2\pi}{T_P} = \sqrt{\frac{\mu}{a^3}}$$

Solving Kepler's Time Equation

Kepler's time equation is still a transcendental one, and so cannot be analytically inverted to solve for eccentric anomaly (and therefore true anomaly) as a function of mean anomaly (time). The benefit of this equation is that it is more easily numerically solvable than attempting to find true anomaly directly from time. Much of the history of astrodynamics has been devoted to coming up with new and better approaches for inverting Kepler's time equation. While literally dozens of distinct methods exist, here we will focus on just one: Newton-Raphson iteration. This approach has the benefit of being easy to implement in almost any computer language, is relatively computationally efficient, and, with the proper choice of initial conditions, is typically guaranteed to converge to any desired precision within a finite number of iterations.

Newton-Raphson Iteration

- Given: $x : f(x) = 0, x \in \mathbb{R}; \quad f'(x) = \frac{\mathrm{d}f}{\mathrm{d}x}$
- Iterate: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
- Until converged (answer stops changing to your desired precision)

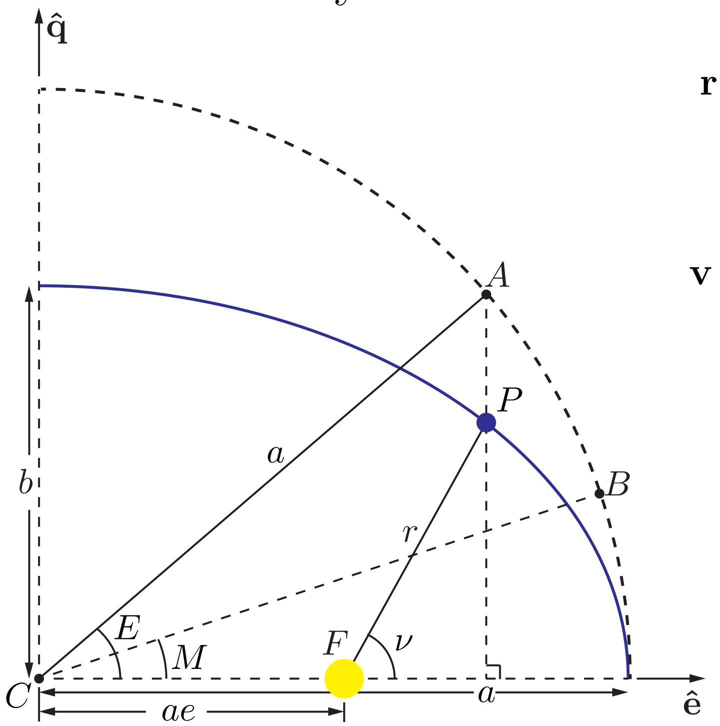
Newton-Raphson Iteration for Kepler's Time Equation

$$M - (E - e\sin(E)) = 0$$

$$E_{n+1} = E_n - \frac{M - E_n + e\sin(E_n)}{e\cos(E_n) - 1}$$

$$E_0 = \begin{cases} \dfrac{M}{1-e} & \dfrac{M}{1-e} < \sqrt{\dfrac{6(1-e)}{e}} \\ \left(\dfrac{6M}{e}\right)^{\frac{1}{3}} & \text{else} \end{cases}$$

Eccentric Anomaly



$$\mathbf{r} = r\cos(\nu)\hat{\mathbf{e}} + r\sin(\nu)\hat{\mathbf{q}}$$
$$= a\left(\cos(E) - e\right)\hat{\mathbf{e}} + b\sin(E)\hat{\mathbf{q}}$$

$$\mathbf{v} = -a\dot{E}\sin(E)\hat{\mathbf{e}} + b\dot{E}\cos(E)\hat{\mathbf{q}}$$
$$= \frac{an}{r}\left(-a\sin(E)\hat{\mathbf{e}} + b\cos(E)\hat{\mathbf{q}}\right)$$

$$n \triangleq \frac{2\pi}{T_P} = \sqrt{\frac{\mu}{a^3}}$$

$$\boxed{\dot{E} = \frac{n}{1 - e\cos(E)}}$$

f and g Functions

$$\left.\begin{aligned}\mathbf{r}(t+\Delta t) &= f\mathbf{r}(t) + g\mathbf{v}(t)\\ \mathbf{v}(t+\Delta t) &= \dot{f}\mathbf{r}(t) + \dot{g}\mathbf{v}(t)\end{aligned}\right\} f\dot{g} - \dot{f}g = 1$$

$$\mathbf{r}_0 \triangleq \mathbf{r}(t) \qquad \mathbf{r} \triangleq \mathbf{r}(t+\Delta t) \qquad r_0 \triangleq \|\mathbf{r}_0\| \qquad r \triangleq \|\mathbf{r}\|$$

$$f = 1 - \frac{r}{\ell}\left(1 - \cos(\Delta\nu)\right) = \frac{a}{r_0}\left(\cos(\Delta E) - 1\right) + 1$$

$$g = \frac{r r_0}{\sqrt{\mu\ell}}\sin(\Delta\nu) = \frac{1}{n}\left(\sin(\Delta E) - \Delta E\right) + \Delta t$$

$$\dot{f} = \sqrt{\frac{\mu}{\ell}}\tan\left(\frac{\Delta\nu}{2}\right)\left(\frac{1-\cos(\Delta\nu)}{\ell} - \frac{1}{r_0} - \frac{1}{r}\right) = -\frac{a^2 n}{r r_0}\sin(\Delta E)$$

$$\dot{g} = 1 - \frac{r_0}{\ell}\left(1 - \cos(\Delta\nu)\right) = \frac{a}{r}\left(\cos(\Delta E) - 1\right) + 1$$

Series Solutions to f and g Functions

$$\mathbf{r} = \mathbf{r}_0 + \frac{{}^{\mathcal{I}}\mathrm{d}}{\mathrm{d}t}\mathbf{r}_0 \Delta t + \frac{{}^{\mathcal{I}}\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{r}_0 \frac{(\Delta t)^2}{2!} + \frac{{}^{\mathcal{I}}\mathrm{d}^3}{\mathrm{d}t^3}\mathbf{r}_0 \frac{(\Delta t)^3}{3!} + \ldots$$

$$\sigma \triangleq \frac{\mu}{r^3} \qquad p \triangleq \frac{\mathbf{r}\cdot\mathbf{v}}{r^2} = \frac{\dot{r}}{r} \qquad q \triangleq \frac{\mathbf{v}\cdot\mathbf{v}}{\mathbf{r}\cdot\mathbf{r}} - \sigma$$

$$\dot{\sigma} = -3\sigma p \qquad \dot{p} = q - 2p^2 \qquad \dot{q} = -p(2q+\sigma)$$

$$\frac{{}^{\mathcal{I}}\mathrm{d}^{(n)}}{\mathrm{d}t^{(n)}}\mathbf{r} = f_n \mathbf{r} + g_n \mathbf{v} \quad \Longrightarrow \quad \frac{{}^{\mathcal{I}}\mathrm{d}^{(n+1)}}{\mathrm{d}t^{(n+1)}}\mathbf{r} = \dot{f}_n \mathbf{r} + f_n \mathbf{v} + \dot{g}_n \mathbf{v} - g_n \sigma \mathbf{r}$$

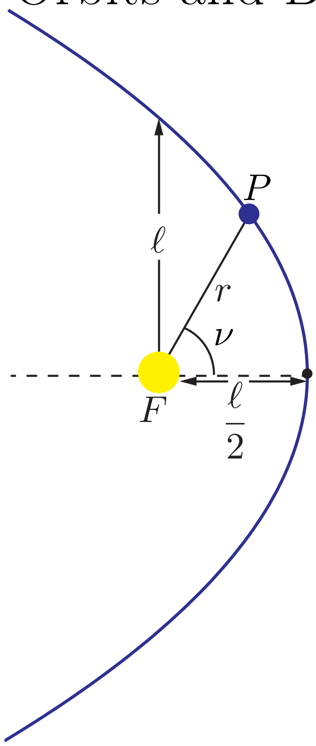
$$\boxed{\begin{aligned} &f_{n+1} = \dot{f}_n - \sigma g_n & &f = \sum_{n=0}^{\infty} \frac{1}{n!} f_n (\Delta t)^n & &\dot{f} = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} f_n (\Delta t)^{n-1} \\ &g_{n+1} = f_n + \dot{g}_n & & & & \\ &f_0 = 1,\ g_0 = 0 & &g = \sum_{n=0}^{\infty} \frac{1}{n!} g_n (\Delta t)^n & &\dot{g} = \sum_{n=1}^{\infty} \frac{1}{(n-1)!} g_n (\Delta t)^{n-1} \end{aligned}}$$

f and g to 8$^{\text{th}}$ order

$$\begin{aligned} f \approx\, & 1 - \frac{\Delta t^2 \sigma}{2} + \frac{\Delta t^3 p\sigma}{2} + \frac{\Delta t^4 \sigma\left(-15p^2 + 3q + \sigma\right)}{24} + \frac{\Delta t^5 p\sigma\left(7p^2 - 3q - \sigma\right)}{8} \\ & + \frac{\Delta t^6 \sigma\left(-945p^4 + 630p^2 q + 210p^2\sigma - 45q^2 - 24q\sigma - \sigma^2\right)}{720} \\ & + \frac{\Delta t^7 p\sigma\left(165p^4 - 150p^2 q - 50p^2\sigma + 25q^2 + 14q\sigma + \sigma^2\right)}{80} + \frac{\Delta t^8 \sigma}{40320}\left(-135135p^6 + \right. \\ & 155925p^4 q + 51975p^4\sigma - 42525p^2q^2 - 24570p^2q\sigma - 2205p^2\sigma^2 + 1575q^3 + \\ & \left. 1107q^2\sigma + 117q\sigma^2 + \sigma^3\right) \end{aligned}$$

$$\begin{aligned} g \approx\, & \Delta t - \frac{\Delta t^3 \sigma}{6} + \frac{\Delta t^4 p\sigma}{4} + \frac{\Delta t^5 \sigma\left(-45p^2 + 9q + \sigma\right)}{120} + \frac{\Delta t^6 p\sigma\left(14p^2 - 6q - \sigma\right)}{24} \\ & + \frac{\Delta t^7 \sigma\left(-4725p^4 + 3150p^2 q + 630p^2\sigma - 225q^2 - 54q\sigma - \sigma^2\right)}{5040} \\ & + \frac{\Delta t^8 p\sigma\left(495p^4 - 450p^2 q - 100p^2\sigma + 75q^2 + 24q\sigma + \sigma^2\right)}{320} \end{aligned}$$

Parabolic Orbits and Barker's Equation



$$B \triangleq \tan\left(\frac{\nu}{2}\right)$$

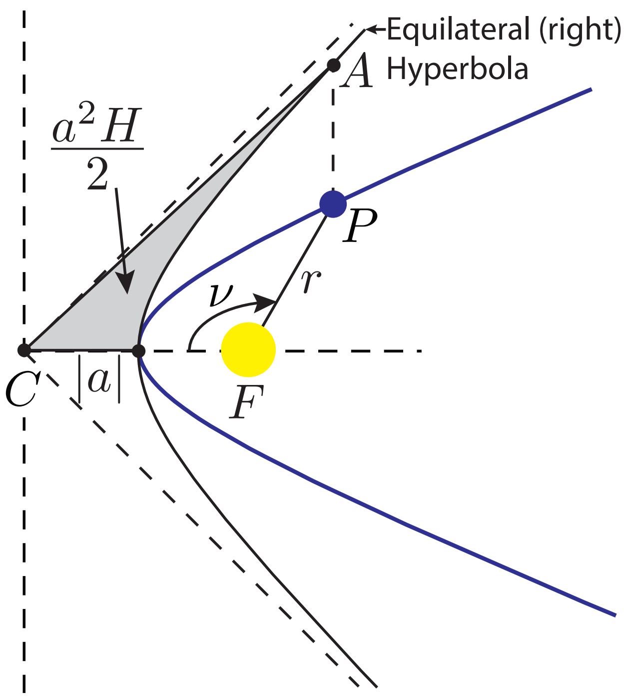
$$r = \frac{\ell}{2}\left(1 + B^2\right)$$

$$\nu = \sin^{-1}\left(\frac{\ell B}{r}\right)$$

$$n_p \triangleq 2\sqrt{\frac{\mu}{\ell^3}}$$

$$\boxed{n_p(t - t_p) = B + \frac{B^3}{3}}$$

Hyperbolic Orbits



$$\sinh(H) = -\frac{r\sin(\nu)}{a\sqrt{e^2 - 1}}$$

$$\cosh(H) = \frac{ae + r\cos(\nu)}{a}$$

$$r = a\left(1 - e\cosh(H)\right)$$

$$\tan\left(\frac{\nu}{2}\right) = \sqrt{\frac{e+1}{e-1}}\tanh\left(\frac{H}{2}\right)$$

$$n_h \triangleq \sqrt{-\frac{\mu}{a^3}}$$

$$\boxed{n_h\left(t - t_p\right) = e\sinh(H) - H}$$

Universal Variable

$$\dot{\chi} \triangleq \frac{\sqrt{\mu}}{r} \implies \mathrm{d}\chi = \mathrm{d}r\left(2r - \ell - \frac{r^2}{a}\right)^{-1/2} \implies \chi + c_o = -\sqrt{a}\sin^{-1}\left(\frac{-2r/a + 2}{\sqrt{-4\ell/a + 4}}\right)$$

Assuming $a > 0$ — Constant of Integration

$$r = a\left(1 + e\sin\left(\frac{\chi + c_o}{\sqrt{a}}\right)\right)$$

$$\sqrt{\mu}\Delta t = a\left(\chi - \sqrt{a}e\cos\left(\frac{\chi + c_o}{\sqrt{a}}\right)\right)\Bigg|_{\chi(t_0)\equiv 0}^{\chi(t)}$$

$$\left.\begin{aligned} \frac{r_0}{a} - 1 &= \sin\left(\frac{c_o}{\sqrt{a}}\right) \\ \frac{\mathbf{r}_0 \cdot \mathbf{v}_0}{\sqrt{\mu a}} &= e\cos\left(\frac{c_o}{\sqrt{a}}\right) \end{aligned}\right\} \begin{aligned} r &= a\left(1 + \sin\left(\frac{\chi}{\sqrt{a}}\right)\frac{\mathbf{r}_0 \cdot \mathbf{v}_0}{\sqrt{\mu a}} + \cos\left(\frac{\chi}{\sqrt{a}}\right)\left(\frac{r_0}{a} - 1\right)\right) \\ \sqrt{\mu}\Delta t &= a\left(\chi - \sqrt{a}\sin\left(\frac{\chi}{\sqrt{a}}\right)\right) + \frac{\mathbf{r}_0 \cdot \mathbf{v}_0}{\sqrt{\mu a}}a\left(1 - \cos\left(\frac{\chi}{\sqrt{a}}\right)\right) \\ &\quad + r_0\sqrt{a}\sin\left(\frac{\chi}{\sqrt{a}}\right) \end{aligned}$$

Universal Variable Continued

$$\psi \triangleq \frac{\chi^2}{a} \implies a = \frac{\chi^2}{\psi}$$

$$c_2 \triangleq \begin{cases} \dfrac{1 - \cos\left(\sqrt{\psi}\right)}{\psi} & \psi \geq 0 \\ \dfrac{1 - \cosh\left(\sqrt{-\psi}\right)}{\psi} & \psi < 0 \end{cases} \qquad c_3 \triangleq \begin{cases} \dfrac{\sqrt{\psi} - \sin\left(\sqrt{\psi}\right)}{\sqrt{\psi}^3} & \psi \geq 0 \\ \dfrac{\sinh\left(\sqrt{-\psi}\right) - \sqrt{-\psi}}{\sqrt{-\psi}^3} & \psi < 0 \end{cases}$$

$$\boxed{\begin{aligned} r &= \chi^2 c_2 + \frac{\mathbf{r}_0 \cdot \mathbf{v}_0}{\sqrt{\mu}}\chi(1 - \psi c_3) + r_0\left(1 - \psi c_2\right) \\ n\Delta t &= c_3\left(\frac{\chi}{\sqrt{a}}\right)^3 + c_2\frac{\mathbf{r}_0 \cdot \mathbf{v}_0}{\sqrt{\mu a}}\left(\frac{\chi}{\sqrt{a}}\right)^2 + \frac{r_o}{a}\left(\frac{\chi}{\sqrt{a}}\right)(1 - \psi c_3) \end{aligned}}$$

Universal Variable Propagation

$$\chi = \begin{cases} \sqrt{a}\Delta E \\ \sqrt{\ell}\Delta B \\ \sqrt{-a}\Delta H \end{cases} \qquad \chi_0 = \begin{cases} \dfrac{\sqrt{\mu}\Delta t}{a} & \text{Closed} \\ \sqrt{\ell}\tan\left(\dfrac{B_0}{2}\right) & \text{Parabola} \\ \dfrac{\sqrt{-a}}{\operatorname{sgn}(\Delta t)}\ln\left(\dfrac{-2\mu/a\,\Delta t}{\mathbf{r}_0\cdot\mathbf{v}_0 + \operatorname{sgn}(\Delta t)\sqrt{-\mu a}\,(1 - r_0/a)}\right) & \text{Hyperbola} \end{cases}$$

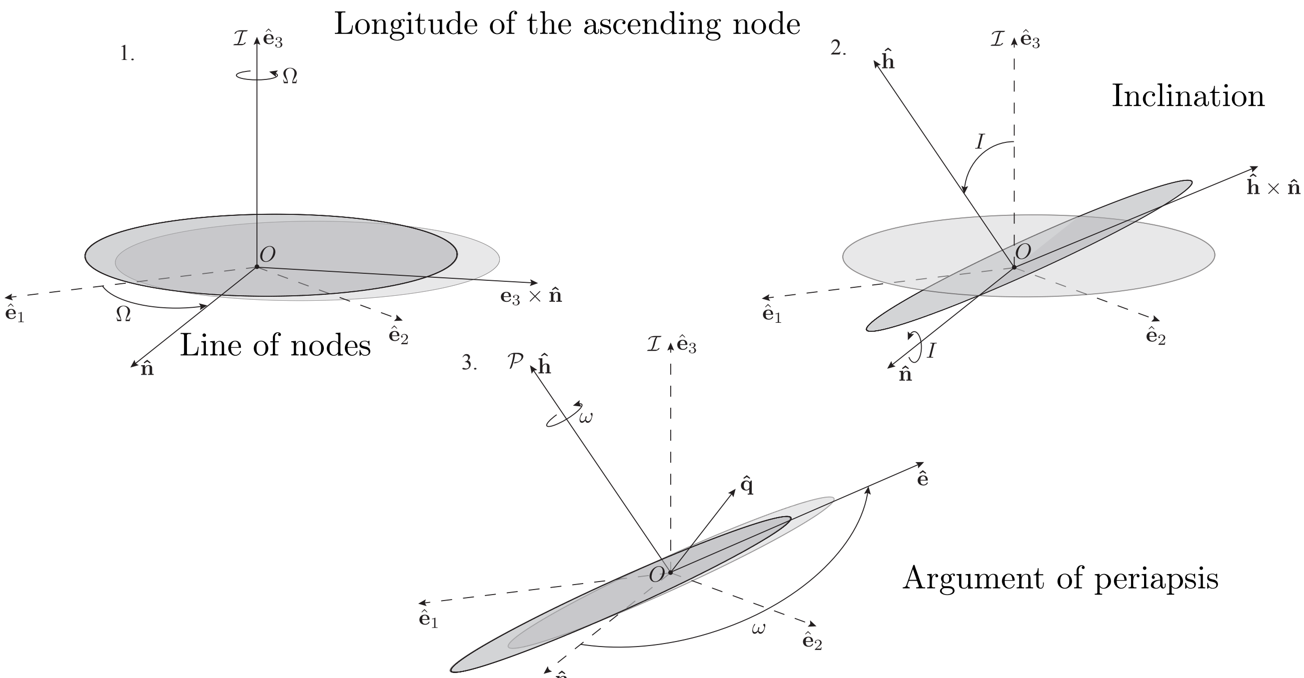
$$\chi_{n+1} = \chi_n + \frac{1}{r}\left(\sqrt{\mu}\Delta t - \chi_n^3 c_3 - \frac{\mathbf{r}_0\cdot\mathbf{v}_0}{\sqrt{\mu}}\chi_n^2 c_2 - r_0\chi_n(1-\psi c_3)\right)$$

$$f = 1 - \frac{\chi^2}{r_0}c_2 \qquad g = \Delta t - \frac{\chi^3}{\sqrt{\mu}}c_3 \qquad \mathbf{r} = f\mathbf{r}_0 + g\mathbf{v}_0$$

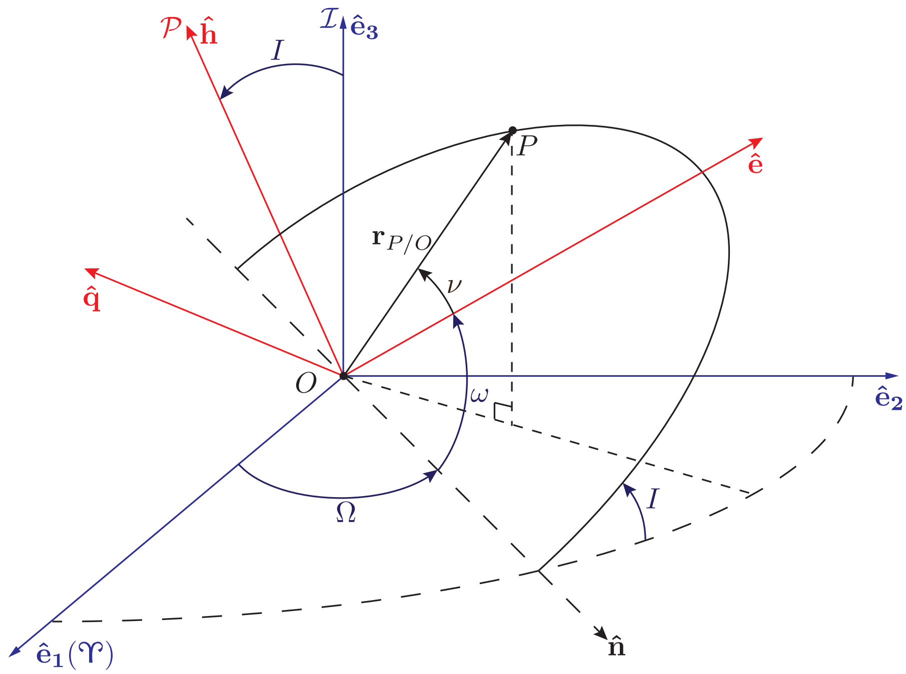
$$\dot{f} = \frac{\sqrt{\mu}}{r r_0}\chi(\psi c_3 - 1) \qquad \dot{g} = 1 - \frac{\chi^2}{r}c_2 \qquad \mathbf{v} = \dot{f}\mathbf{r}_0 + \dot{g}\mathbf{v}_0$$

$$1 = f\dot{g} - \dot{f}g$$

Inertial $\rightarrow$ Perifocal: 3-1-3 (Ω, I, ω) Body Rotation



Orbits in 3D



Orbits in 3D (Math Version)

$$^{\mathcal{P}}C^{\mathcal{I}} = \begin{bmatrix} \cos(\omega) & \sin(\omega) & 0 \\ -\sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(I) & \sin(I) \\ 0 & -\sin(I) & \cos(I) \end{bmatrix} \begin{bmatrix} \cos(\Omega) & \sin(\Omega) & 0 \\ -\sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} -\sin(\Omega)\sin(\omega)\cos(I) + \cos(\Omega)\cos(\omega) & \sin(\Omega)\cos(\omega) + \sin(\omega)\cos(I)\cos(\Omega) & \sin(I)\sin(\omega) \\ -\sin(\Omega)\cos(I)\cos(\omega) - \sin(\omega)\cos(\Omega) & -\sin(\Omega)\sin(\omega) + \cos(I)\cos(\Omega)\cos(\omega) & \sin(I)\cos(\omega) \\ \sin(I)\sin(\Omega) & -\sin(I)\cos(\Omega) & \cos(I) \end{bmatrix}$$

$$\left[\mathbf{r}_{P/O}\right]_{\mathcal{I}} = {}^{\mathcal{I}}C^{\mathcal{P}} \begin{bmatrix} r\cos\nu \\ r\sin\nu \\ 0 \end{bmatrix}_{\mathcal{P}} = r \begin{bmatrix} \cos(\Omega)\cos(\nu+\omega) - \sin(\Omega)\sin(\nu+\omega)\cos(I) \\ \sin(\Omega)\cos(\nu+\omega) + \sin(\nu+\omega)\cos(I)\cos(\Omega) \\ \sin(I)\sin(\nu+\omega) \end{bmatrix}_{\mathcal{I}}$$

Special Cases

- $I = 0$, Define Longitude of Periapsis:

$$\pi \equiv \varpi \triangleq \omega + \Omega$$

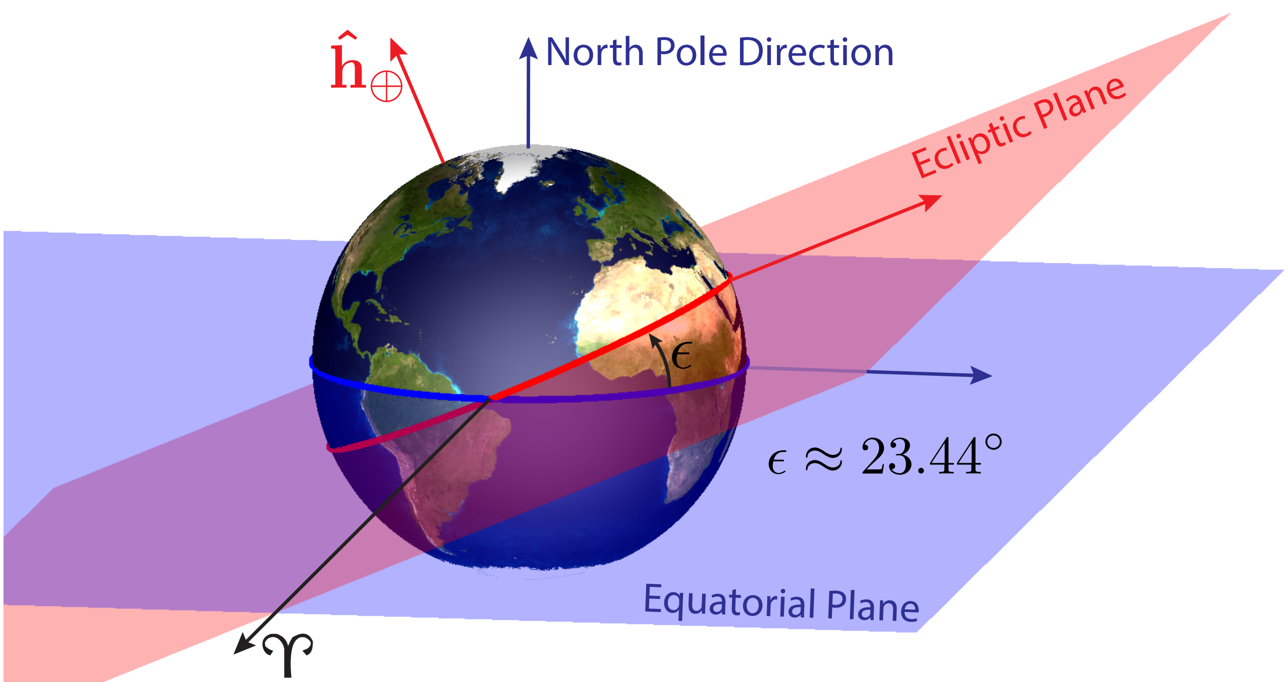
- $e = 0$, Define Argument of Lattitude:

$$u \equiv \theta \triangleq \nu + \omega$$

- $e = I = 0$, Define True Longitude:

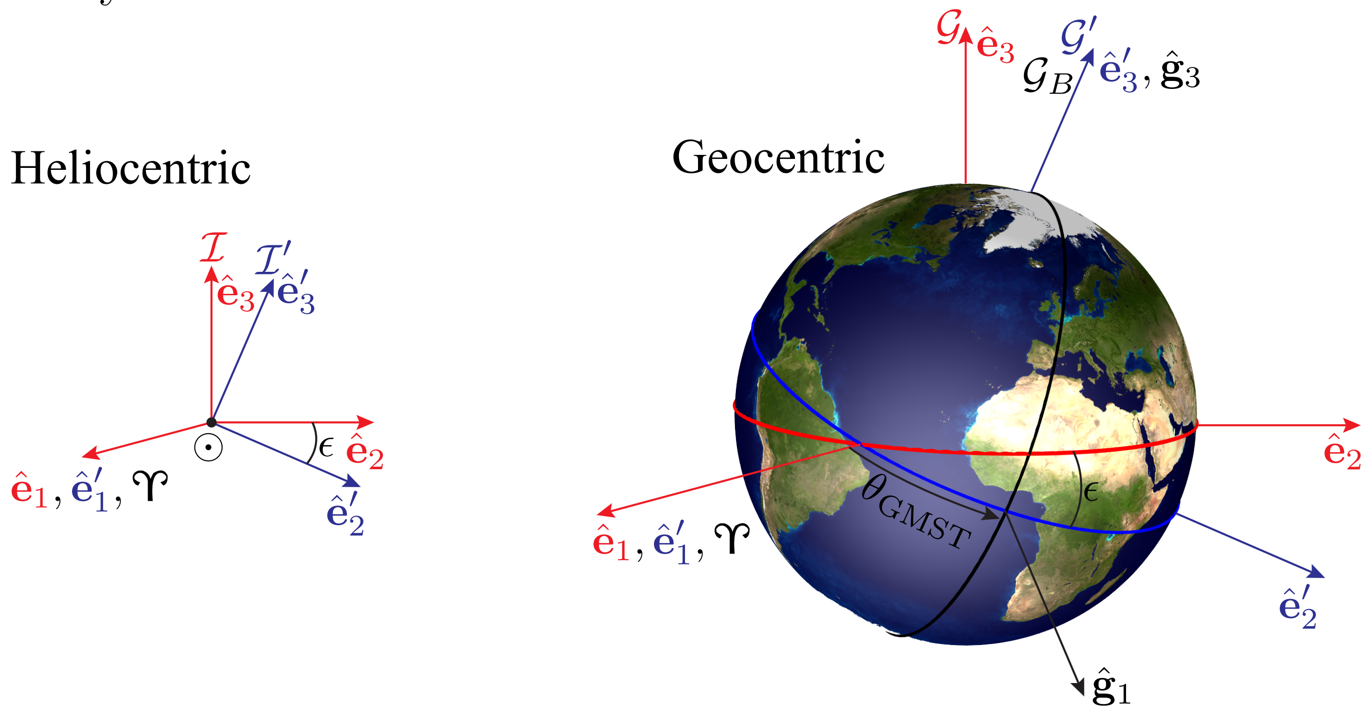
$$l \triangleq \varpi + \nu = \Omega + \omega + \nu$$

Solar System Reference Planes



ϵ is known as the *obliquity of the ecliptic*.

Solar System Reference Frames

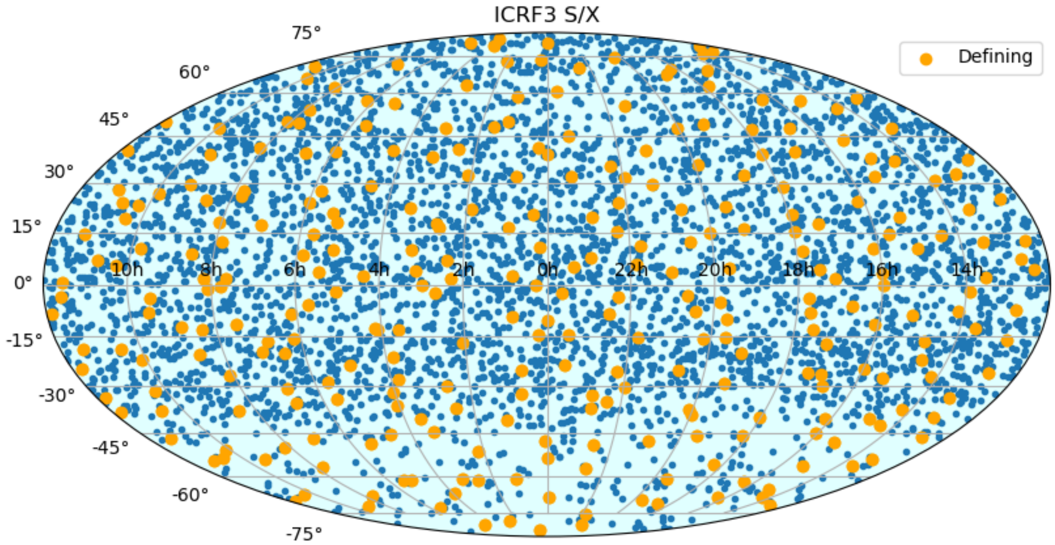


$\mathcal{I}, \mathcal{G}$ Ecliptic, Inertial; $\mathcal{I}', \mathcal{G}'$ Equatorial, Inertial; $\mathcal{G}_B$ Equatorial, Non-Inertial

Spherical Coordinate Systems

Name	Origin	Reference Plane	Prime Direction	Azimuth Angle	Elevation Angle
Geographic	Geocentric	Equator	Prime Meridian	Longitude (λ)	Latitude (φ or L)
Horizontal (Topocentric)	Observer Location	Horizon	North	Azimuth (Az)	Altitude/Elevation (Alt/El)
Equatorial	Geocentric or Heliocentric	Celestial Equator	Vernal Equinox	Right Ascension (α)	Declination (δ)
Ecliptic	Geocentric or Heliocentric	Ecliptic	Vernal Equinox	Ecliptic Longitude (λ)	Ecliptic Latitude (β)
Galactic	Heliocentric	Galactic Plane	Galactic Center	Galactic Longitude (l)	Galactic Latitude (b)

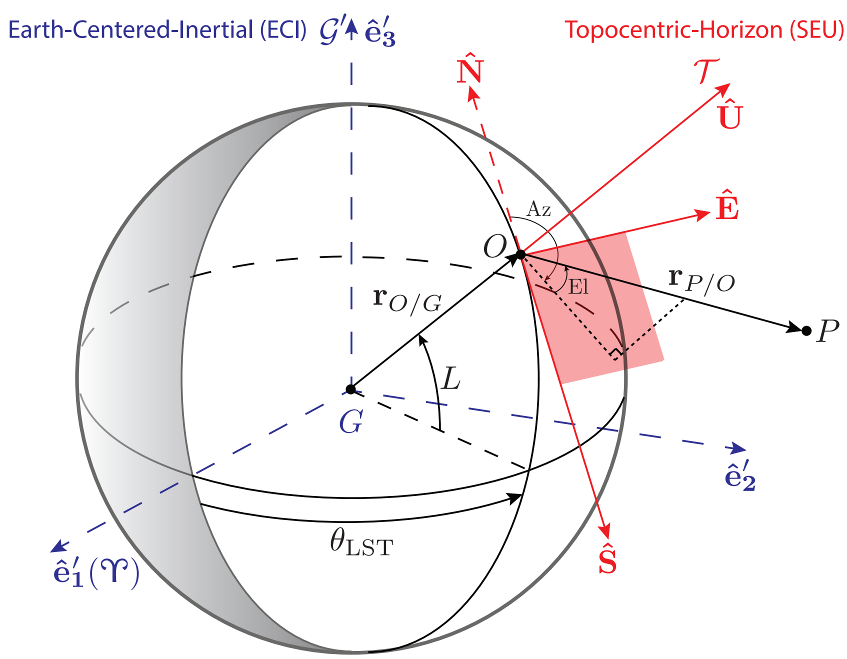
International Celestial Reference System (ICRS)



ICRF3: 4,356 Extragalactic Sources, 303 defining.
http://hpiers.obspm.fr/icrs-pc/newwww/icrf/index.php

ICRS is the standard by which the reference frame is defined. ICRF1-3 are realizations fo the standard based on updated measurements. ICRS attempts to approximate equatorial coordiantes, with a coordinate origin at the solar system barycenter, a pole direction approximating the north pole direction, and an equinox direction approximating ♈.

Topocentric-Horizon Coordinate System



The Reference Geoid

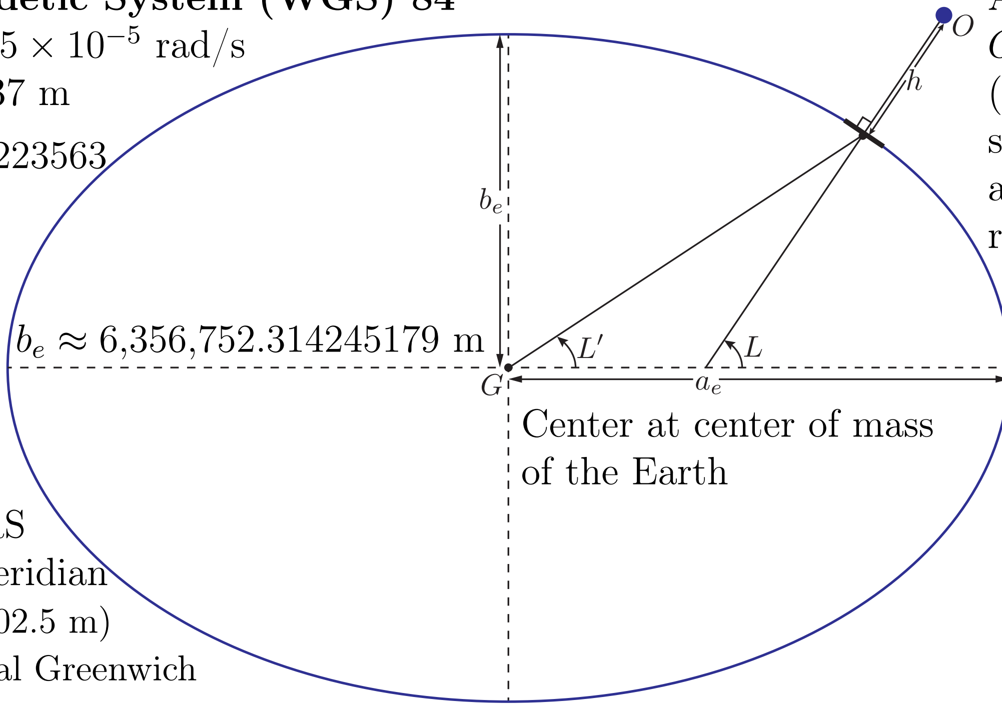
World Geodetic System (WGS) 84

$\omega_e = 7.292115 \times 10^{-5}$ rad/s

$a_e = 6{,}378{,}137$ m

$\frac{1}{f} = 298.257223563$

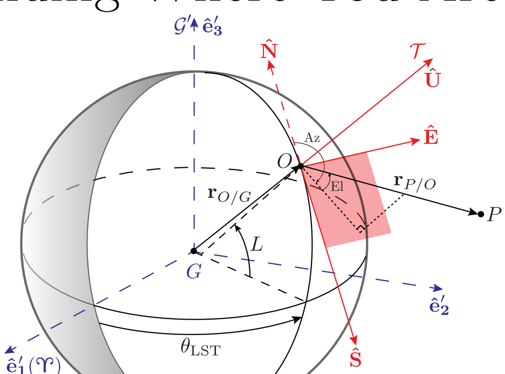
$f \triangleq \frac{a-b}{a}$

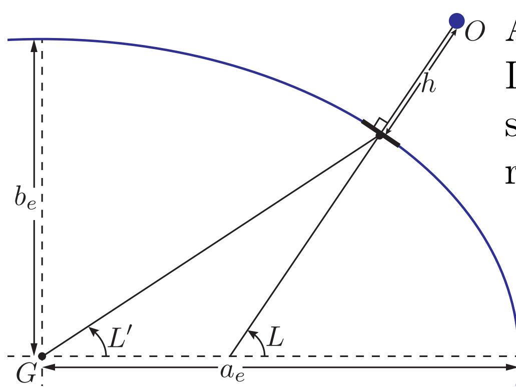


A surface point O at Lon/Lat (λ, L) is at some height h above the reference geoid

$\lambda = 0$ at IERS Reference Meridian $\sim$5.3 arcsec(102.5 m) East of original Greenwich

Finding Where You Are





A surface point O at Lon/Lat (λ, L) is at some height h above the reference geoid

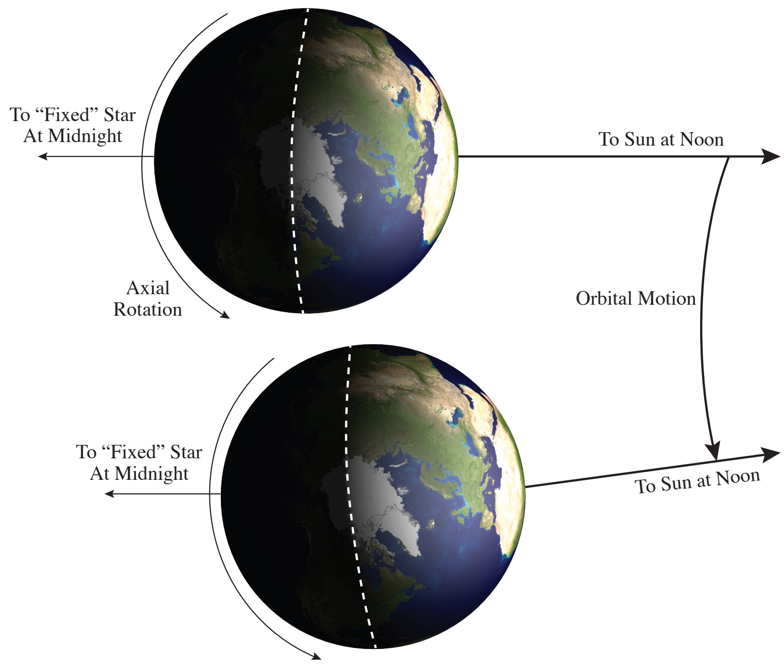
Geoid described by a_e and e_e where:
$e_e^2 = 2f - f^2$

$$[\mathbf{r}_{O/G}]_{\mathcal{G}'} = \begin{bmatrix} x\cos\theta_{\text{LST}} \\ x\sin\theta_{\text{LST}} \\ z \end{bmatrix}_{\mathcal{G}'}$$

$$x = \left(\frac{a_e}{\sqrt{1 - e_e^2 \sin^2 L}} + h\right)\cos L$$

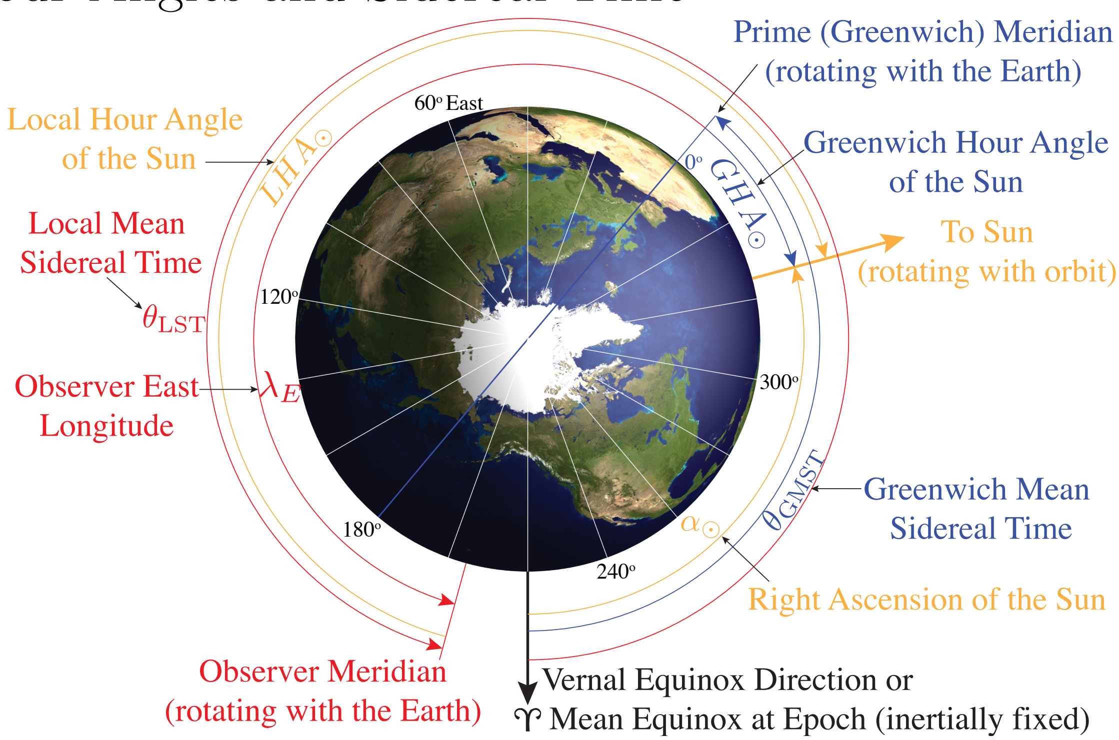
$$z = \left(\frac{a_e(1 - e_e^2)}{\sqrt{1 - e_e^2 \sin^2 L}} + h\right)\sin L$$

Solar vs Sidereal Time



- Mean Solar Day (d): 24 SI hours = 86400 SI seconds
- Solar (Tropical) Year: 365.242190402 d
- Mean Sidereal Day: 23h56m4.09054s
- Sidereal Year: 365.256363004 d

Hour Angles and Sidereal Time



Hour Angle

is defined as the time from when an object was directly overhead. Negative hour angles imply that the object is approaching.

Time Measurements

- Local Hour Angle and Greenwich Hour Angle: $LHA = GHA + \lambda_E$
- Local Solar Time (local midnight is 0 hours): $LHA_\odot + 12^\mathrm{h}$
- Greenwich Solar Time: $\theta_\mathrm{GMST} - \alpha_\odot + 12^\mathrm{h}$ where θ_GMST is the location of the Prime (Greenwich) Meridian with respect to the vernal equinox
- There are two different 'suns': The **apparent** sun (where the sun actually is) and the **fictitious mean sun** (a sun moving uniformly along the celestial equator). We can define apparent and mean solar times.
- The **mean** solar time at Greenwich is defined as **Universal Time**:

$$\mathrm{UT0} = GHA_\odot + 12^\mathrm{h} = LHA_\odot + 12^\mathrm{h} - \lambda_E$$

- The motion of Earth's pole affects all these measurements. Correcting for this gives you UT1 ($|\mathrm{UT1} - \mathrm{UT0}| \approx 30$ ms)

Finding When You Are

$$\theta_\mathrm{LST} = \theta_{g0} + \omega_e(t - t_0) + \lambda_E$$

Reference Value at Epoch

OR

MEAN SIDEREAL TIME, 2019

Greenwich mean sidereal time at 0^h UT

		h			h			h			h
Jan.	0	6·6250	Apr.	0	12·5389	July	0	18·5185	Oct.	0	0·5638
Feb.	0	8·6620	May	0	14·5102	Aug.	0	20·5555	Nov.	0	2·6008
Mar.	0	10·5019	June	0	16·5472	Sept.	0	22·5925	Dec.	0	4·5721

Greenwich mean sidereal time (GMST) on day d of month at hour t UT

$= \text{GMST at } 0^\mathrm{h} \text{ UT on day } 0 + 0^\mathrm{h}.06571\, d + 1^\mathrm{h}.00274\, t$

Local mean sidereal time = GMST $\pm$ ($+$ east, $-$ west) longitude

https://aa.usno.navy.mil/publications/reports/ap19_for_web.pdf

$\omega_e = 7.292115 \times 10^{-5}$ rad/s (WGS84)

$$\theta_{g0} = 100.4606184^\circ + 36,000.77005361 T_\mathrm{UT1} + 0.00038793 T_\mathrm{UT1}^2 - 2.6 \times 10^{-8} T_\mathrm{UT1}^3$$

T_UT1 = number of Julian centuries from J2000.0

Julian Date

- Days since January 1, 4713 BCE, 12^{h} UT

$$\mathrm{JD} = 367\mathrm{Y} - \mathrm{int}\left(\frac{7\left(\mathrm{Y} + \mathrm{int}\left(\frac{\mathrm{M}+9}{12}\right)\right)}{4}\right) + \mathrm{int}\left(\frac{275\mathrm{M}}{9}\right) + \mathrm{D} + 1721013.5 + \frac{\mathrm{UT}}{24}$$
$$- \frac{1}{2}\,\mathrm{sgn}\left(100\mathrm{Y} + \mathrm{M} - 190002.5\right) + \frac{1}{2}$$

$$\mathrm{int}(x) = \begin{cases} \lfloor x \rfloor & x \geq 0 \\ \lceil x \rceil & x < 0 \end{cases} \qquad \mathrm{sgn}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

- 1 Julian year is *exactly* 365.25 days, 1 Julian century is 100 Julian years
- Define: MJD = JD -2,400,000.5

More Time Systems

- **Coordinated Universal Time** (UTC) is an approximation to UT1 defined such that $|\mathrm{UT1} - \mathrm{UTC}| < 0.9$ seconds

- UTC is based on **International Atomic Time** (TAI), a weighted average of >400 atomic clocks in over 50 national laboratories worldwide

- Leap seconds are added to TAI to get UTC. In 2021, TAI is 37 seconds ahead of UTC, with the last leap second added on 12/31/2016 23:59:60 UTC

- GPS time is UTC as of 1/16/1980. As of 2021, GPS - UTC = 18 seconds. TAI - GPS will equal 19 seconds forever.