Unscented Filtering for Directly-Observed Exoplanet Orbits

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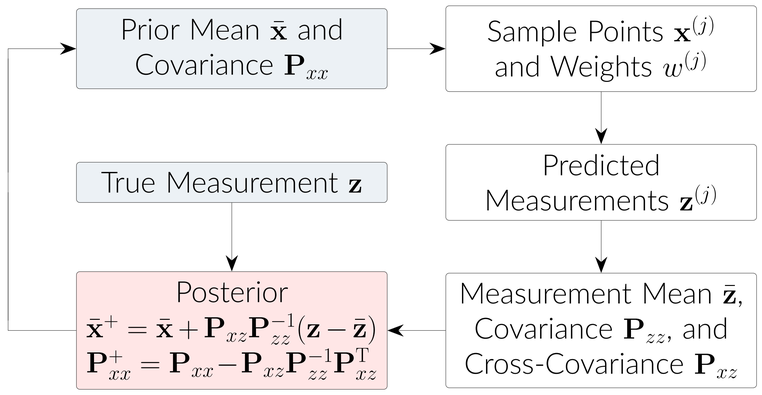
Challenges of Exoplanet Orbit Fitting

- Sparse measurements with large errors
- Highly nonlinear dynamics and measurements
- Classical orbital elements have singularities and bounds that are impractical for estimation
- Most existing techniques are computationally expensive (e.g., Monte Carlo methods)

Filtering Methods

Unscented Kalman Filter (UKF) Update [1]



Improving UKF Performance

- We use a 7th-order Gaussian cubature for $\mathbf{x}^{(j)}$ and $w^{(j)}$
- Running multiple filter passes with same measurements

Simulated Exoplanet Parameters

Semi-Major Axis	a	3.0 au
Eccentricity	e	0.1
Inclination	I	30.0 deg
Longitude of Ascending Node	Ω	120.0 deg
Argument of Periapsis	ω	45.0 deg
Mean Anomaly at Epoch	M_0	75.0 deg
Period	P	5.0 yr
Distance from Observer	d	2.5 pc

- Generated five measurements over a one-year time span
- Standard deviation of measurement noise is 5 mas

References

[1] S. Julier and J. Uhlmann, "A New Extension of the Kalman Filter to Nonlinear Systems," *Proc. SPIE 3068, Signal Processing, Sensor Fusion, and Target Recognition VI*, 1997.

[2] C. J. Cohen and E. C. Hubbard, "A Nonsingular Set of Orbital Elements," *The Astronomical Journal*, vol. 67, no. 1, 1962.

A New Set of Orbital Elements

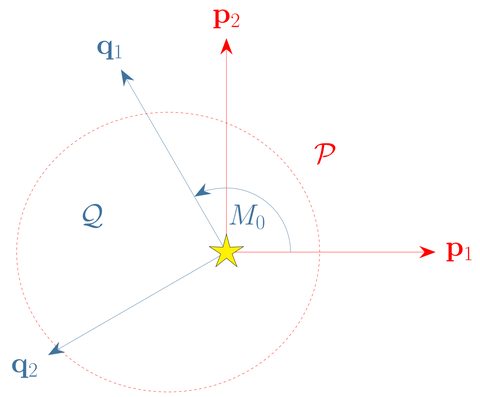
Definitions

$$\Xi_{11} = \frac{a}{d}(\cos(\omega + M_0)\cos\Omega - \sin(\omega + M_0)\sin\Omega\cos I)$$
$$\Xi_{21} = \frac{a}{d}(\cos(\omega + M_0)\sin\Omega + \sin(\omega + M_0)\cos\Omega\cos I)$$
$$\Xi_{12} = \frac{a}{d}(-\sin(\omega + M_0)\cos\Omega - \cos(\omega + M_0)\sin\Omega\cos I)$$
$$\Xi_{22} = \frac{a}{d}(-\sin(\omega + M_0)\sin\Omega + \cos(\omega + M_0)\cos\Omega\cos I)$$
$$\eta_1 = \frac{e\cos M_0}{\sqrt{1-e^2}}$$
$$\eta_2 = -\frac{e\sin M_0}{\sqrt{1-e^2}}$$
$$\lambda = \log\left(\frac{P}{P_0}\right) \quad (P_0 \text{ is an arbitrary scaling factor, e.g., 1 yr})$$

Properties

- No singularities at $e = 0$, $I = 0$, etc.
- Any values in $\mathbb{R}^7$ describe an elliptic orbit
- Based on the reference frame $\mathcal{Q}$
- Combine features of the Cohen-Hubbard nonsingular elements [2] and Thiele-Innes constants

Perifocal Frame $\mathcal{P}$ and Auxiliary Frame $\mathcal{Q}$



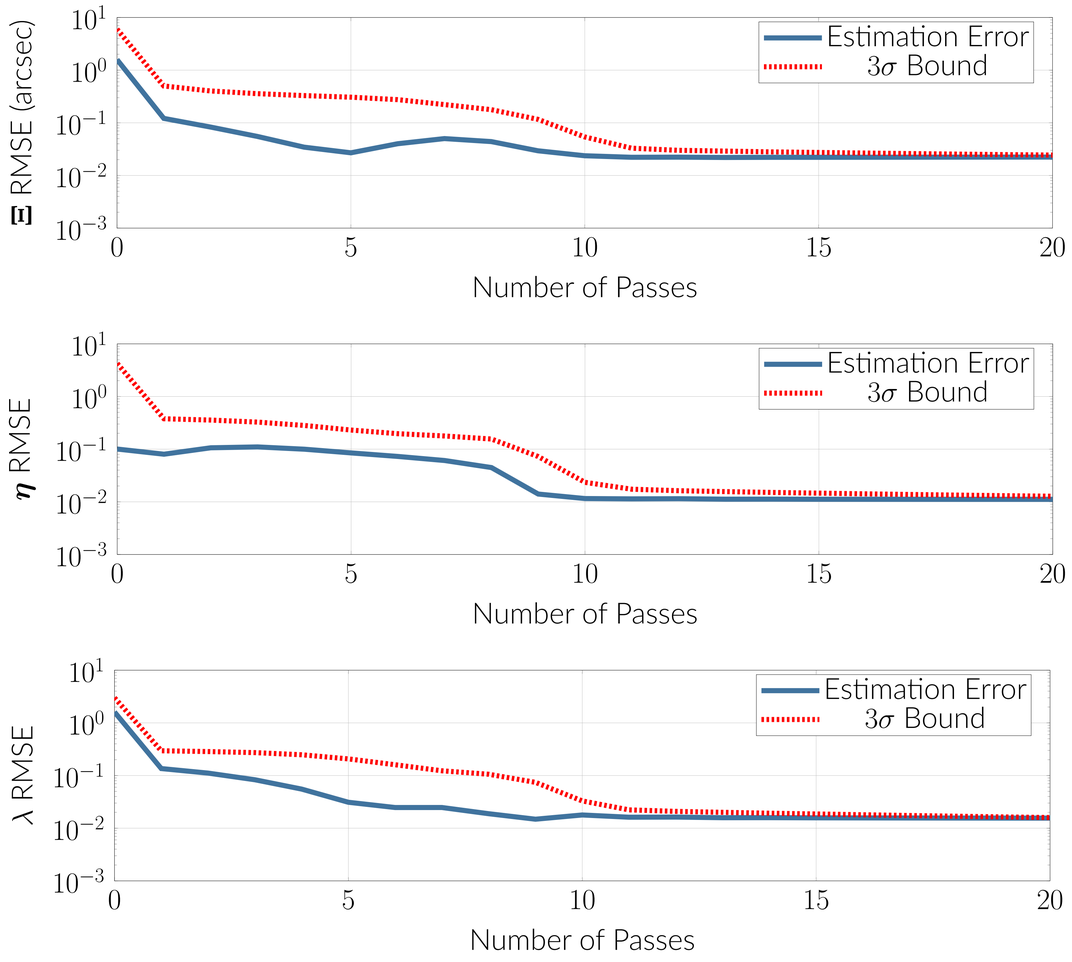
Measurement Model

$$\mathbf{z} = \boldsymbol{\Xi}\boldsymbol{\zeta}(\boldsymbol{\eta}, \lambda, t) + \mathbf{w}$$

- $\boldsymbol{\zeta}$ is the position in the orbital plane in $\mathcal{Q}$, scaled by $1/a$
- $\mathbf{w}$ is the measurement noise

Filter Results

Root Mean-Square Error (RMSE) over 100 Simulation Trials



Performance Summary

Accuracy: Relative RMSE after 20 filter passes

- 1.4% for $\boldsymbol{\Xi}$
- 11% for $\boldsymbol{\eta}$
- 1.0% for λ

Speed: ∼10,000 filter updates per second in C++ implementation

Future Work

- Testing and statistical analysis with various simulated exoplanets
- Determining convergence criteria for multiple filter passes
- Analyzing error estimates and bounds
- Testing with other UKF variants
- Comparison with other exoplanet orbit fitting methods
- Application to real exoplanet data

Code Repository

`https://github.com/SIOSlab/ExoUKF`