

# Verification of a Satellite Observing Event-Based Sensor Model

## A Examination

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Rachel Oliver

31 August 2022

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# Space Domain Awareness (SDA)

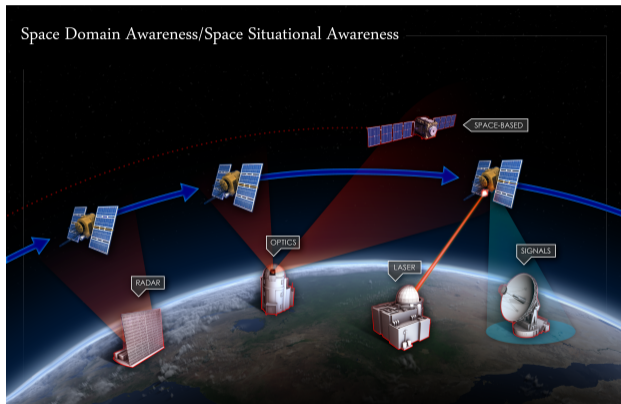


Figure: Weisbarth, 2020,  
<https://media.defense.gov/2020/Mar/15/2002264814/-1/-1/0/190111-F-HF064-002.PNG>



Figure: MDA, 2008,  
<http://www.mda.mil/mdalink/pdf/too164.pdf>

# Growing Complexity

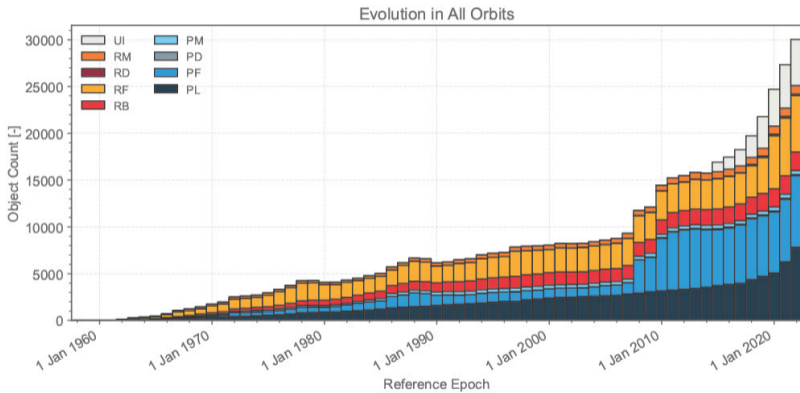
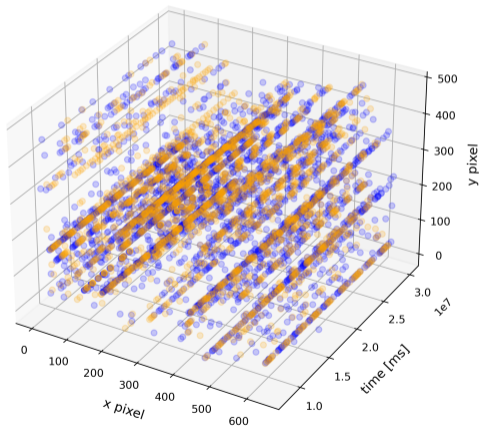


Figure: ESA, ESA's Space Environment Report, 2022

# Event-Based Sensors



- Asynchronous data collection
- Records time series events,  $e$ 
  - $e = \{t, x, y, \rho\}$
  - recording timestamp,  $t$
  - pixel location,  $x$  and  $y$
  - change polarity,  $\rho$

# Event-Based Sensors Advantages in SDA

- Event-Based Sensor Advantages for SDA
  - Temporal Sensitivity
  - High Dynamic Range
  - Space-Based
    - ▶ Low SWaP for Space-Based
    - ▶ Maximize Information for Downlink
    - ▶ Enable Onboard Computation
  - Ground-Based
    - ▶ Low Cost for Augmenting SDA Operations



Figure: Cohen, 2019, <http://greg-cohen.com/project/astrosite/>

# Event Simulations

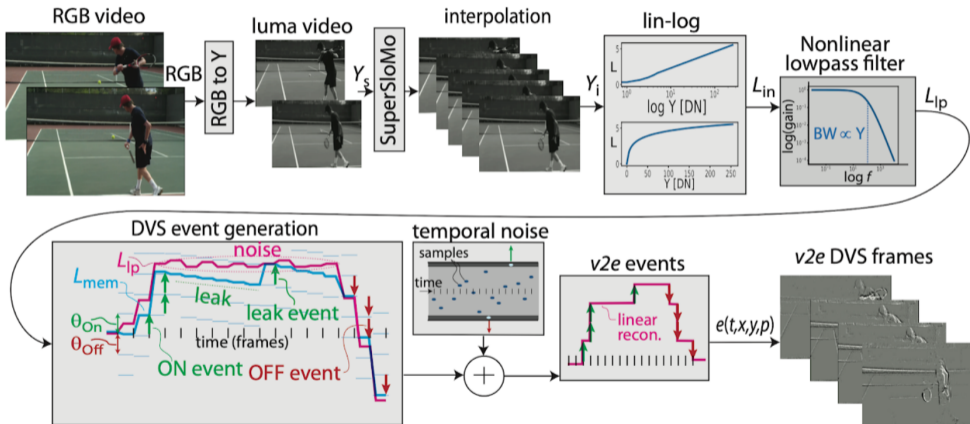
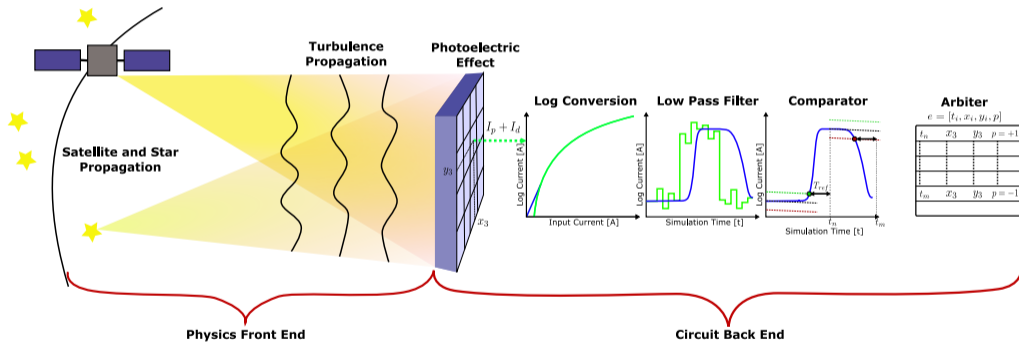
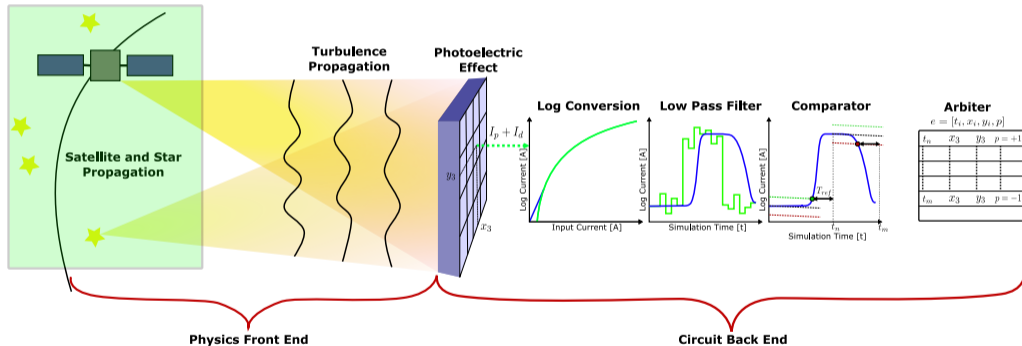


Figure: Delbruk, 2021, <https://sites.google.com/view/video2events/home>

# New Event-based Sensor Model

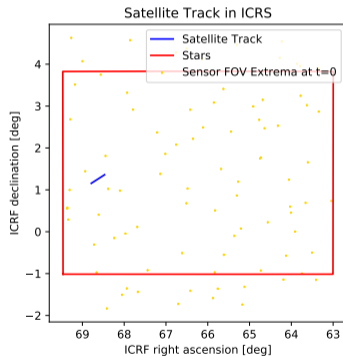
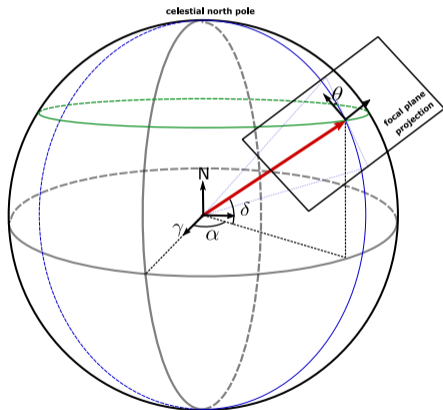


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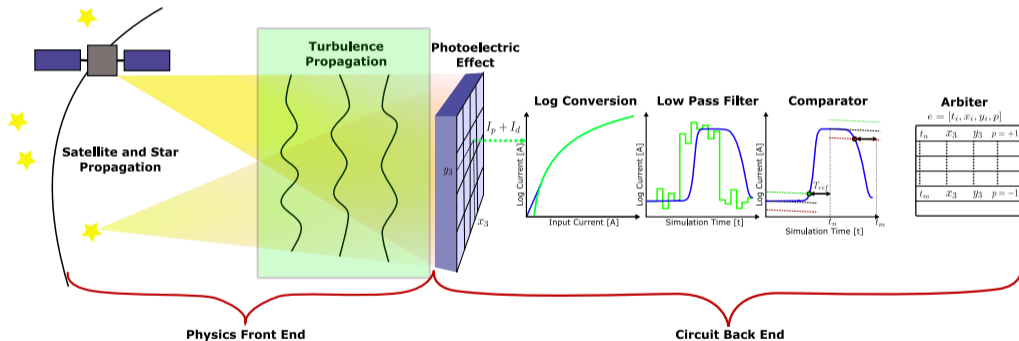




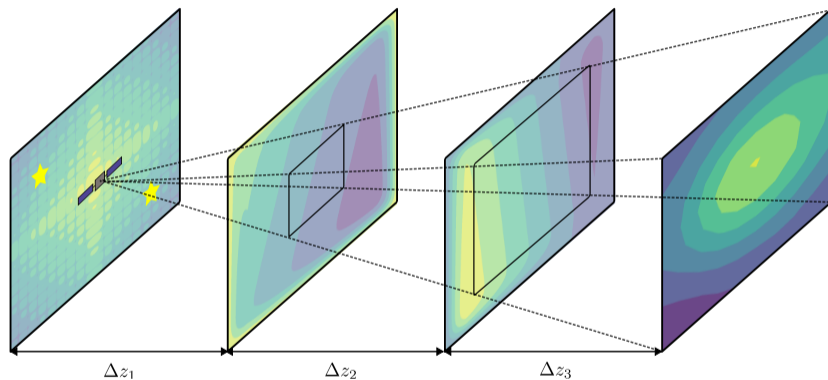
# Satellite and Star Generation



# New Event-based Sensor Model

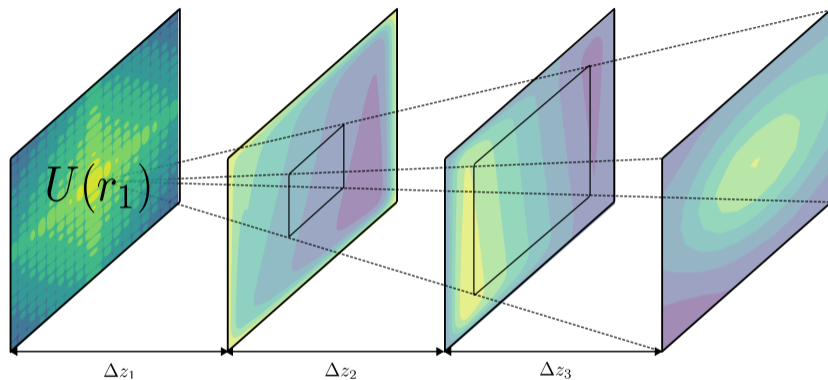


# Simulating Atmospheric Propagation



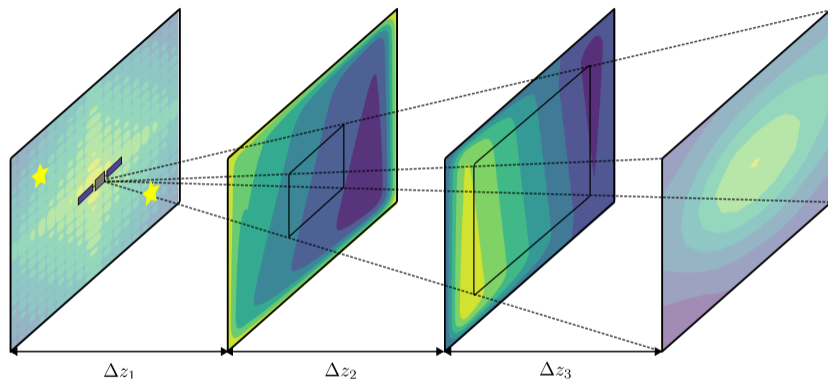
$$U(r_n) = \mathcal{Q} \left[ \frac{m_{n-1} - 1}{m_{n-1} \Delta z_{n-1}}, r_n \right] \times \prod_{i=1}^{n-1} \left\{ \mathcal{T}[z_i, z_{i+1}] \mathcal{F}^{-1} \left[ f_i, \frac{r_{i+1}}{m_i} \right] \mathcal{Q}_2 \left[ -\frac{\Delta z_i}{m_i}, f_i \right] \mathcal{F} \left[ r_i, f_i \right] \frac{1}{m_i} \right\} \times \left\{ \mathcal{Q} \left[ \frac{1 - m_1}{\Delta z_1} \right] \mathcal{T}[z_1, z_2] U(r_1) \right\}$$

# Simulating Atmospheric Propagation



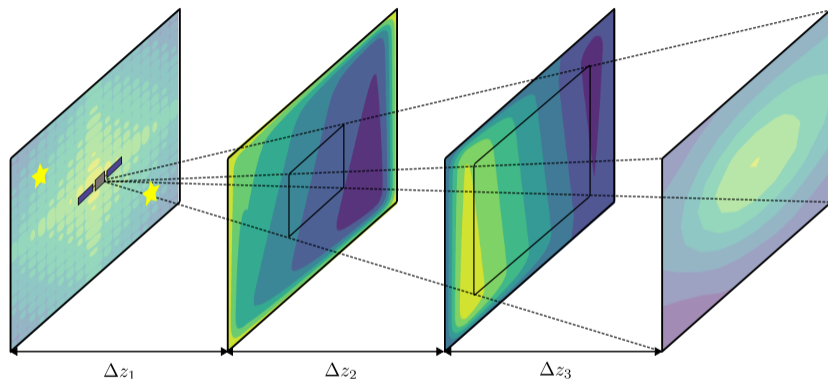
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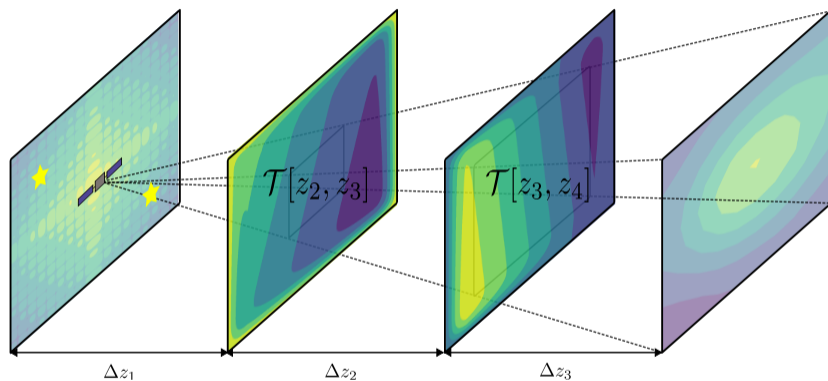
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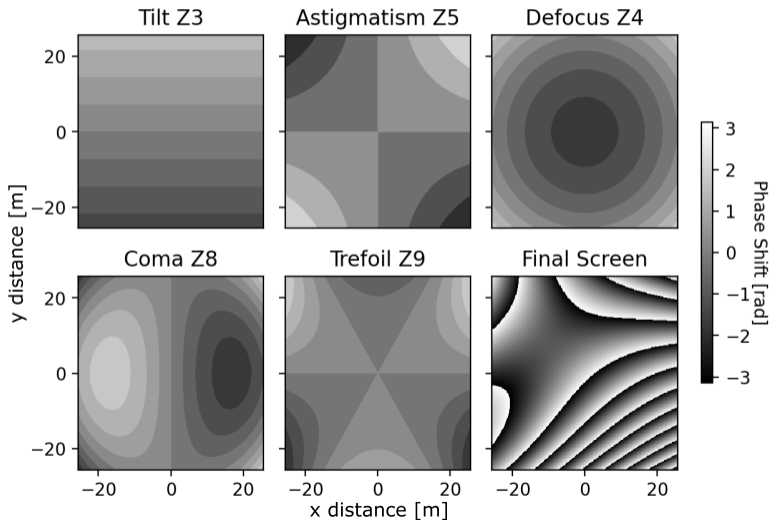
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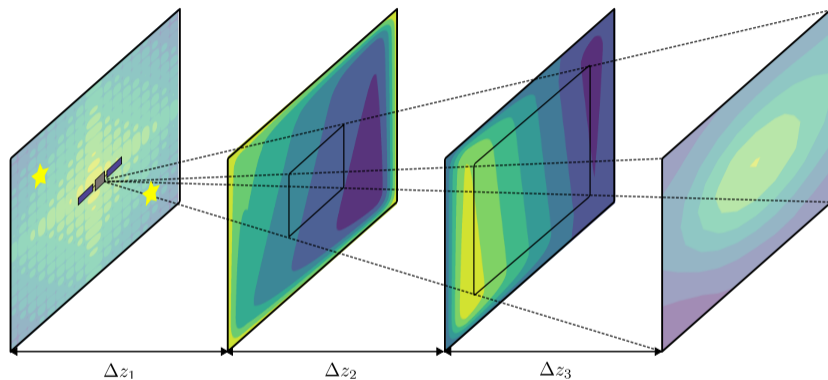
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# Zernike Mode Phase Screen



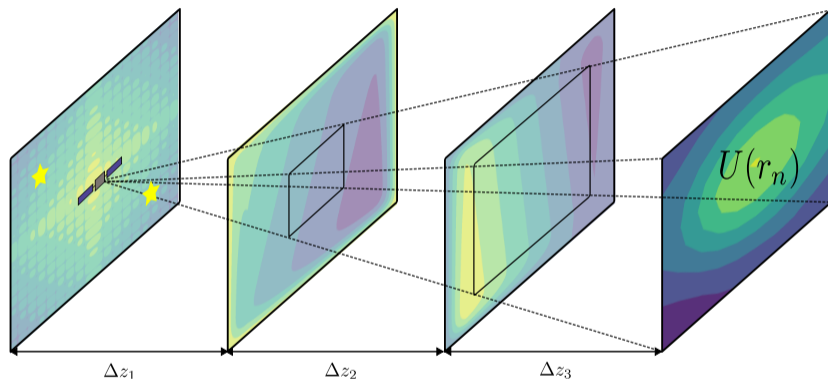


# Simulating Atmospheric Propagation



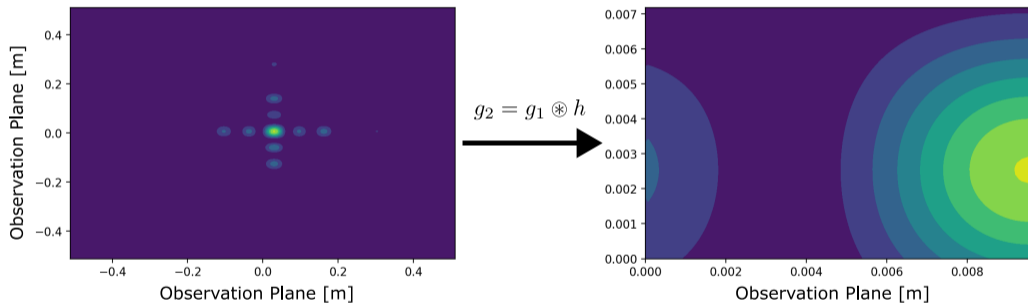
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# Simulating Atmospheric Propagation

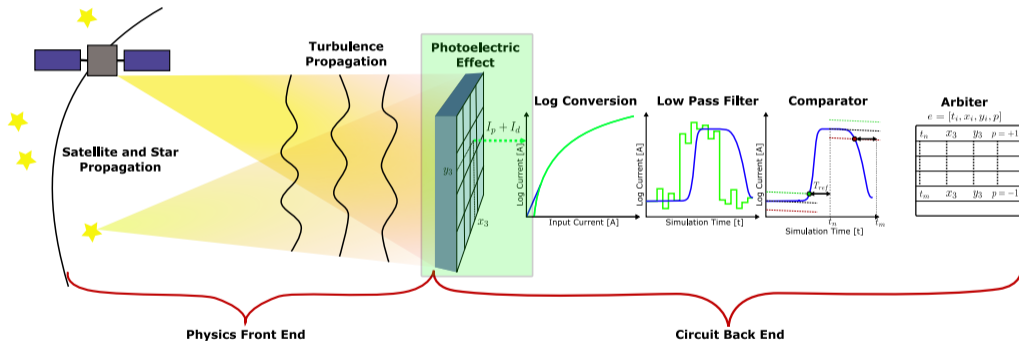


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# Simulating Atmospheric Propagation



# New Event-based Sensor Model



# Induced Photocurrent

- Responsivity linearly scales Power,  $\Phi[W]$

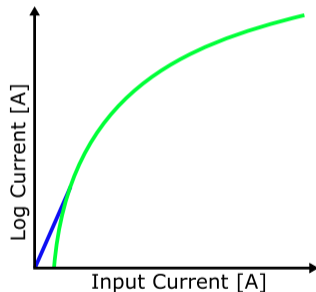
$$I_p = R_\lambda \Phi [A]$$

- Responsivity,  $R_\lambda$ , Scaled by Quantum Efficiency,  $\eta$

$$R_\lambda = \eta \frac{q}{hf} \approx \eta \frac{\lambda}{1.23985} \left[ \frac{A}{W} \right]$$

- Temperature,  $T$ , Dependent Dark Current,  $I_{darklog}$

$$I_{darklog} = \ln(I_{dark}) = constant - \frac{E_a}{kT}$$



# Event Simulations

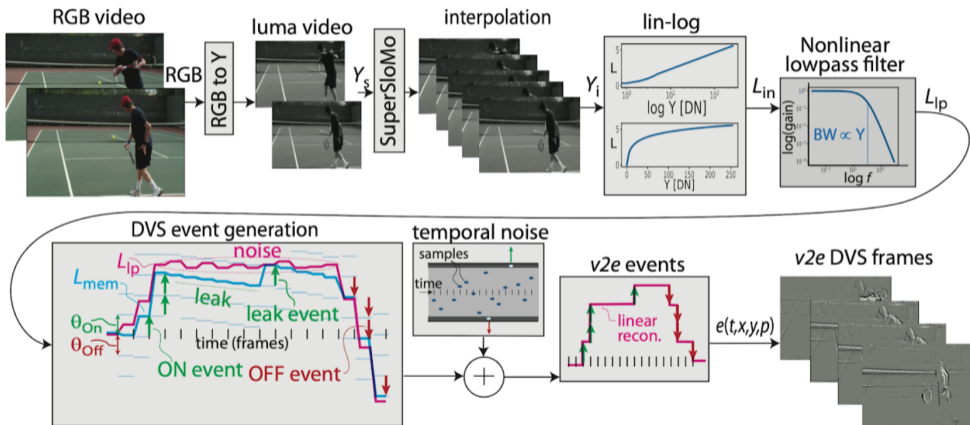


Figure: Delbruk, 2021, <https://sites.google.com/view/video2events/home>

# Gaussian White Noise

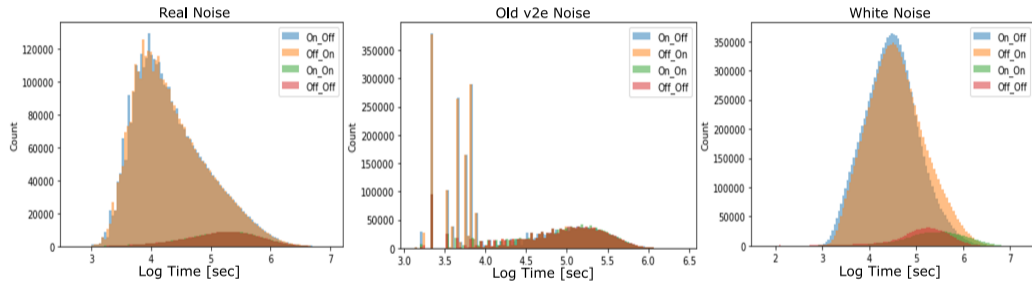
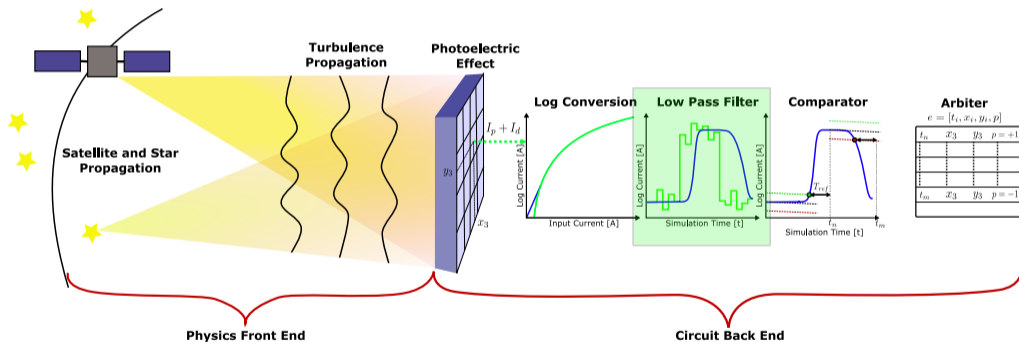


Figure: McRenolds, 2022, ETH Zurich

# New Event-based Sensor Model





# Low Pass Filter

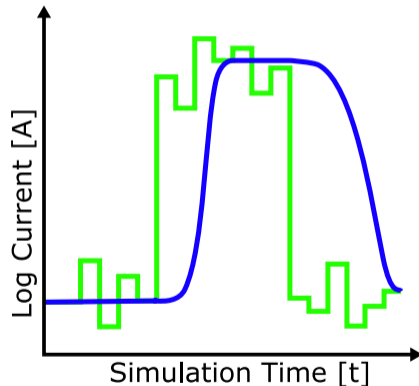
- Maximum bandwidth,  $f_{3dBmax} \approx 3000[Hz]$
- Resultant bandwidth,  $f_{3dB}$

$$f_{3dB} = \frac{I_{in} + \left(\frac{I_{max}}{10}\right)}{I_{max}} \times f_{3dBmax}$$

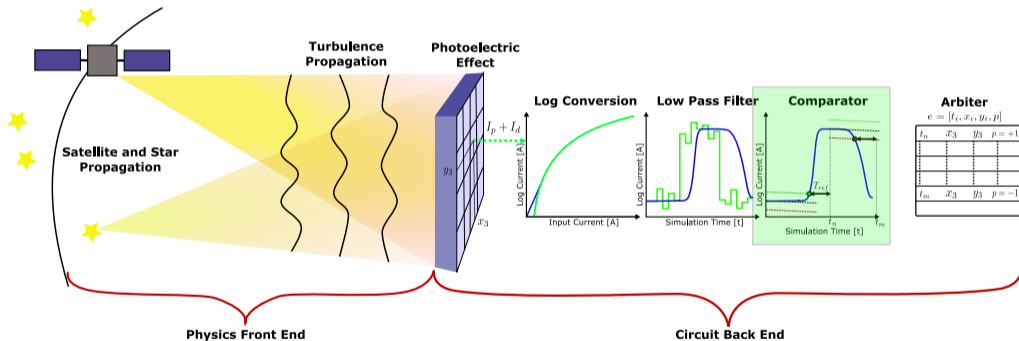
$$\epsilon = e^{2\pi * \Delta t f_{3db}}$$

$$I_{pint} \leftarrow (1 - \epsilon)I_{p-1} + \epsilon I_{in}$$

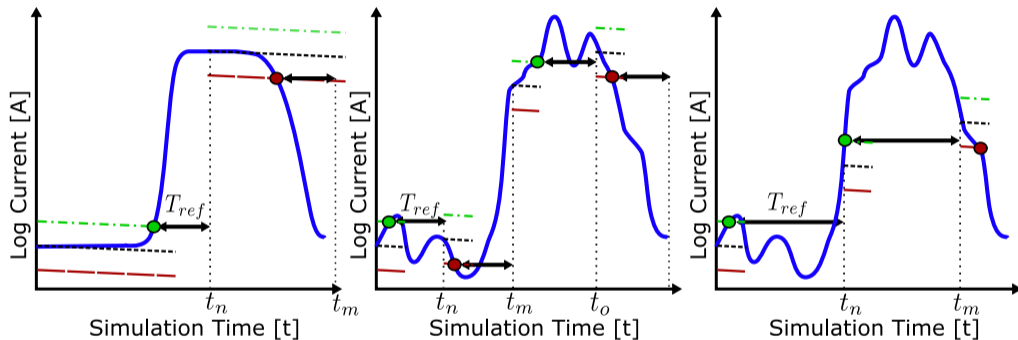
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# New Event-based Sensor Model

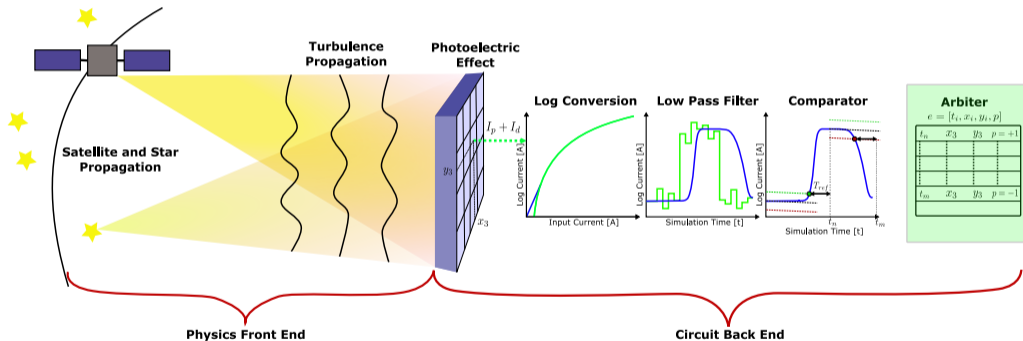


# Comparator and Refractory Circuit



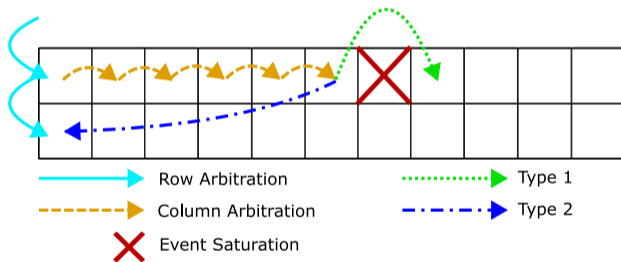
- On Threshold
- Memorized Current
- Off Threshold
- Circuit Current

# New Event-based Sensor Model



# Arbitration

- Ordered row then column query
- Restricted maximum number of events
- Allows and tracks loss of events
  - Saturation
  - Refractory Period



# Model Overview

- High Fidelity Model Innovations
  1. Inclusion of a physics based front end
  2. Induced current a function of quantum efficiency
  3. Inclusion of a temperature dependant dark current
  4. White noise scaled by photocurrent
  5. Realistic arbitration
  6. Tracks events lost during arbitration

# Verification

- Goal: Prove Model Creates Data Analogous to Real Data
  - Build trust in simulation output
  - Produce simulated data for algorithm development
  - Test camera settings and designs to maximize low-light performance
- Challenges:
  - Non-deterministic: no one-to-one matching
  - Unknown importance of data characteristics

# Data Collection

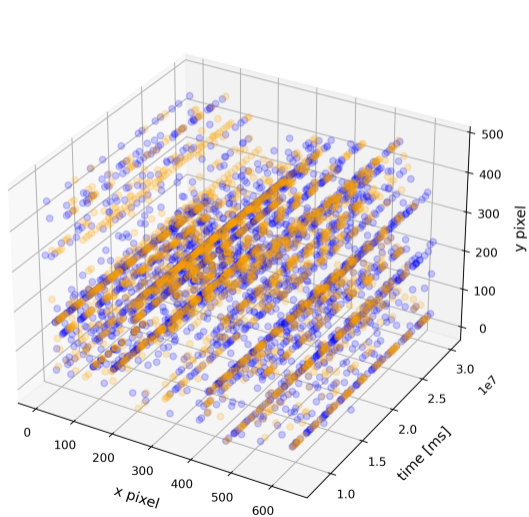
- Prophesee Gen3 (VGA) camera
- 85mm f1.4 Lens
- January - March 2021
- Collection Procedure
  1. Slew to fixed azimuth and elevation
  2. Stare in fixed position for 30 seconds
  3. Slew to next location
- Courtesy of Dr. David Monet of AFRL Space Vehicles Directorate



Figure: Sony Prophesee, 2022 <https://www.prophesee.ai/event-based-evaluation-kits/>



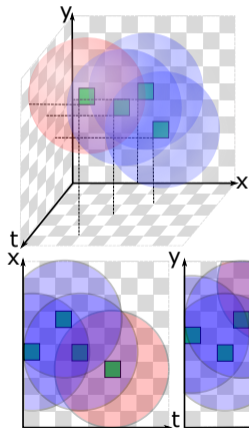
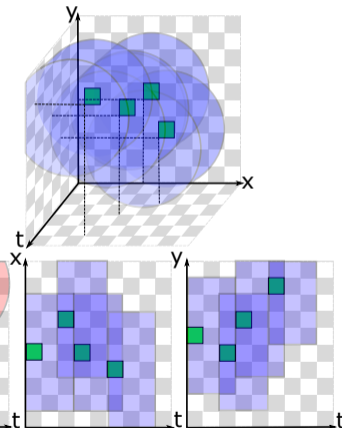
# Example Data



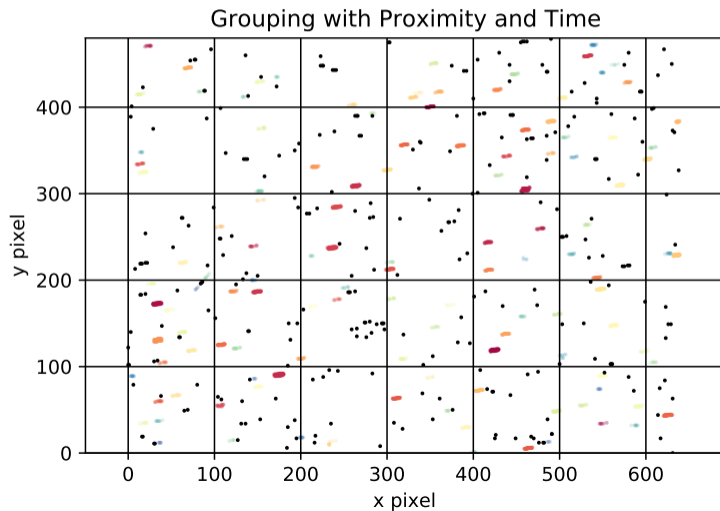
# Analysis Approaches

- Classic Post Processing
  1. Integrate data over 3 second time slice FITS image
  2. SExtractor yields:  $(column, row, flux)$
  3. Conversion of flux to visual magnitude:  $(column, row, v_{mag})$
  4. Astrometry.net maps  $(column, row)$  to  $(RA, DEC)$  and identifies reference stars and satellites
  5. Correlate satellite  $(RA, DEC)$  from each time slice with a satellite detection
- New Temporal Data Association
  1. Cluster data in 3-dimensions
  2. Filter clusters with too few pixels and/or events
  3. Combine clusters with Hough transform
  4. Assign cluster numbers to satellites
  5. Extract signature and temporal characteristics

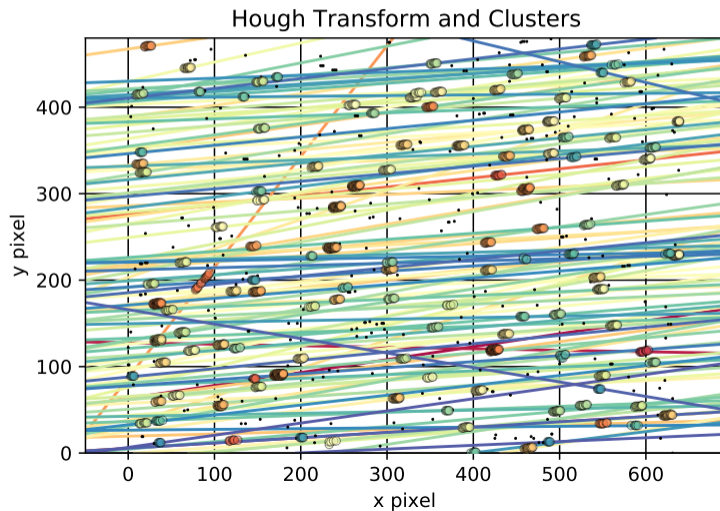
# Clustering

**a) Density Clustering****b) Time Sequential Clustering**

# Clustering

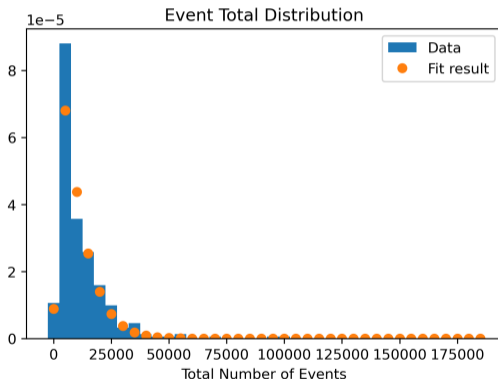


# Hough Transform



## New Approach Limitations

- Human involvement is necessary
  - Easier to access in 2-dimensions
  - Poor signal to noise datasets are hard to process
- Time clustering is more consistent, but slow

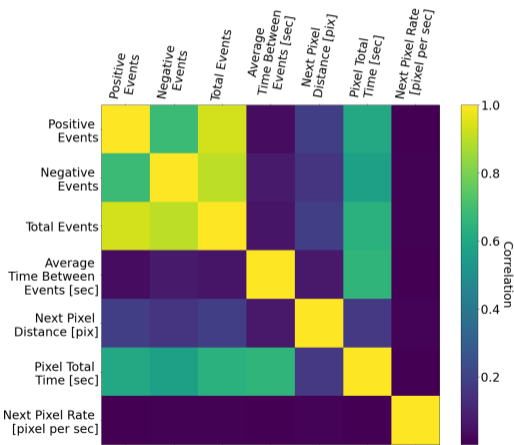


# Selected Characteristics

## Track Correlation Matrix

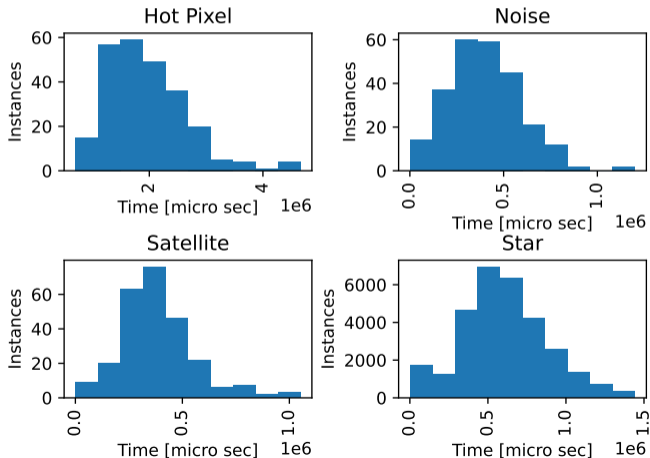


## Pixel Correlation Matrix



# Characteristic Distributions

## Average Time Between Events





# Validation Plan

1. Select at random a set of data from those post-processed
2. Simulate the physics front-end once
3. Run sensitivity analysis on camera parameters
4. Create Jacobian from sensitivity analysis
5. Find least squares fit of camera parameters
6. Use best fit parameters to simulate other collections
7. Compare the distributions with the Kolmogorov Smirnov Test

# Non-linear Least Squares

- Sensitivity analysis is a finite difference approximation

- Produces Jacobian,  $J_{ij}$

$$J_{ij} = -\frac{\delta r_i}{\delta \beta_j}$$

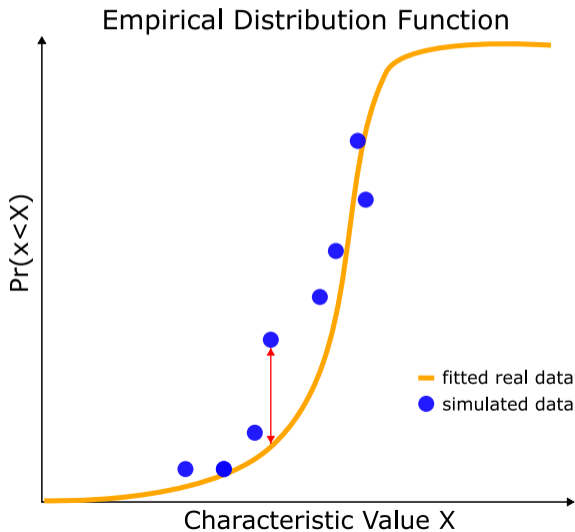
- Residuals,  $r_i$
- Input Parameters,  $\beta_j$

- Least Squares to find change in output,  $\Delta y$

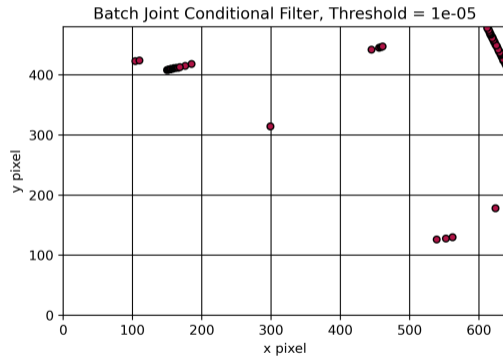
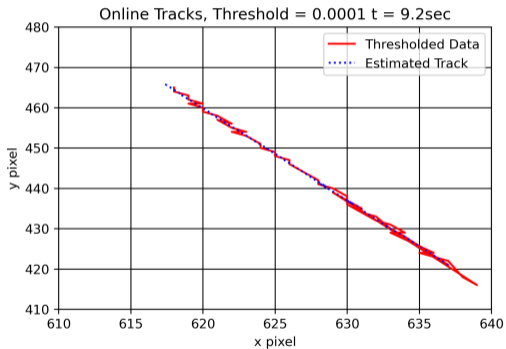
$$(J^T J)\Delta\beta = J^T \Delta y$$

$$\begin{bmatrix} \frac{\delta r_1}{\delta \beta_1} & \frac{\delta r_1}{\delta \beta_2} & \dots & \dots & \dots & \frac{\delta r_1}{\delta \beta_j} \\ \frac{\delta r_2}{\delta \beta_1} & \frac{\delta r_2}{\delta \beta_2} & & & & \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \frac{\delta r_i}{\delta \beta_1} & & & & & \frac{\delta r_i}{\delta \beta_j} \end{bmatrix}$$

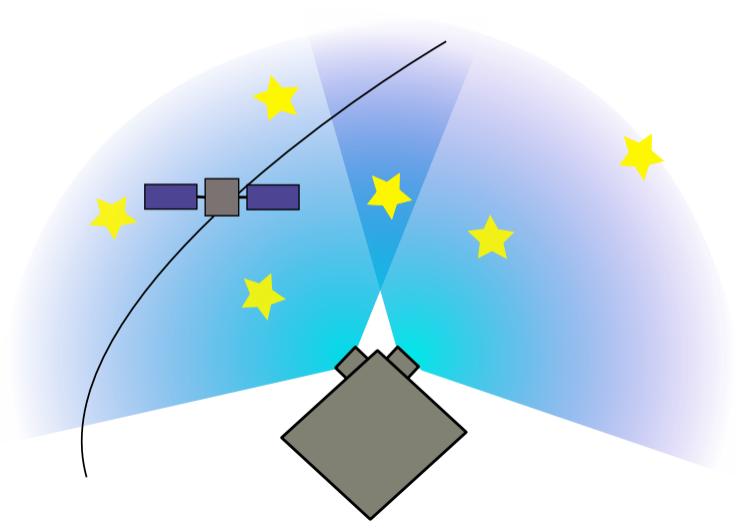
# Kolmogorov Smirnov Test



# Multiple Hypothesis Tracker



# Event-based Sensor Star Tracker



## Other Projects

- Incorporation into an optical neural network
  - Recurrent Neural Network
  - Spiked Output
- Low light noise while slewing characteristic study
- Clustering algorithm update to analyze event-based data of lightning

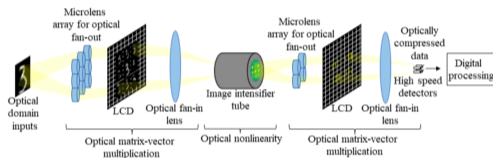


Figure: Sohani, 2021, NDSEG Proposal

## Contribution Overview

- Code: Event-based Sensor Model for Space Domain Awareness, various modules (physics front end, scaling white noise, limited arbiter)
- Conference Paper: Event-based Sensor Model for Space Domain Awareness, Advanced Maui Optical and Space Surveillance Technologies Conference 2021
- Code: Event-based post processing clustering algorithm for data association
- Code: Event-based multiple hypothesis tracker
- Conference Paper: Event-Based Sensor Multiple Hypothesis Tracker For Space Domain Awareness, Advanced Maui Optical and Space Surveillance Technologies Conference 2022
- Conference Short Course: An Introduction to Event-Based Sensors for SDA: A Hands-On Tutorial, Advanced Maui Optical and Space Surveillance Technologies Conference 2022

## Contribution Overview Continued

- Research Seminar: Finding Satellites in Event-based Data, SGRS 2022
- Journal Paper: Verification of a Satellite Observing Event-Based Sensor Model, To Be Submitted in September 2022



Thank you for listening!

Special thank you to my committee members, husband, and friends for their never-ending patience with me.

Rachel Oliver

ro234@cornell.edu

## Backup Slides - Courses Taken

- For Credit
  - AEP 5400 - Nonlinear and Quantum Optics
  - MAE 5180 - Autonomous Mobile Robots
  - MAE 5730 - Intermediate Dynamics
  - MAE 5780 - Feedback Control Systems
  - MAE 5830 - Astronautic Optimization
  - MAE 6760 - Model-Based Estimation
  - MAE 6780 - Multivariable Control Theory
- Audited
  - MAE 6280 - Adaptive and Learning Systems
  - MAE 6700 - Advanced Dynamics
  - CS 5780 - Intro to Machine Learning

## Backup Slides - Additional Graduate Level Courses

- Taken at the Air Force Institute of Technology
  - Linear Systems Analysis
  - The Space Environment
  - Methods of Applied Mathematics
  - Intermediate Dynamics
  - Space Mission Analysis and Design
  - Satellite Communications
  - Control and State Space Concepts
  - Introduction to Flight Dynamics
  - Applied Linear Algebra
  - Intermediate Space Flight Dynamics
  - Spacecraft Systems Engineering
  - Satellite Design and Test
  - Modern Methods of Orbital Determination
  - Space Surveillance
  - Chemical Rocket Propulsion

## Backup Slides - Irradiance Calculations

- Star by Johnson V:  $V_0 = q_v - 2.5 \log(F_v)$ 
  - $q_v$  zero magnitude flux scaled from Vega
  - Rearrange Equation for Flux
  - $F_v = 10^{-0.4(V_0 - q_v)} \left[ \frac{W}{cm^2 \text{Angstrom}} \right]$
  - Multiply by bandpass, filter width, and receiving area
  - $P_v = F_v \text{Area} \text{Bandpass}$
- Satellite by Point Source:
  - Power sent to hemisphere
  - $I = \frac{L \sum (A_{surf} \rho_{surf})}{2 * \pi}$
  - Calculate energy received per steradian
  - $\mathcal{E}_\lambda = \frac{I_\lambda * \cos(\theta_R)}{R^2}$
  - Multiply by receiving area
  - $P_{sat} = \mathcal{E}_\lambda \text{Area}$

## Backup Slides - Zernike Mode Weighting

$$Z_{evenj} = \sqrt{n+1}R_n^m(r)\sqrt{2}\cos(m\theta)$$

$$Z_{oddj} = \sqrt{n+1}R_n^m(r)\sqrt{2}\sin(m\theta)$$

$$Z_j = \sqrt{n+1}R_n^0(r), m = 0$$

$$\Theta_{atm}(r, \theta) = \sum_j a_j Z_j(r, \theta).$$

$$C_{jj'} = E[a_j, a_{j'}] = \frac{K_{zz'}\delta_z\Gamma[(n+n'-5/3)/2](D/r_o)^{5/3}}{\Gamma[(n-n'+17/3)/2]\Gamma[(n'-n+17/3)/2]\Gamma[(n+n'+23/3)/2]}.$$

$$a_j = U^T \cdot \vec{n}.$$

## Backup Slides - Beta Binomial Distribution Fit

- Unknown or random probability of success in each of a fixed or known number of Bernoulli trials
- Binomial distribution modification: probability of success is drawn from beta distribution
- Fit with selection of approximate  $\alpha$  and  $\beta$  values defining the continuous beta distribution
- Defined the bin size and center values of those bins
- Curve fit the center locations to a continuous beta distribution

