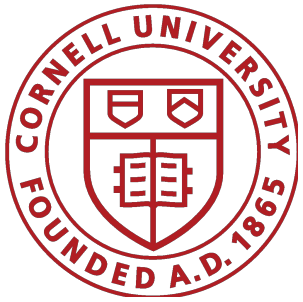


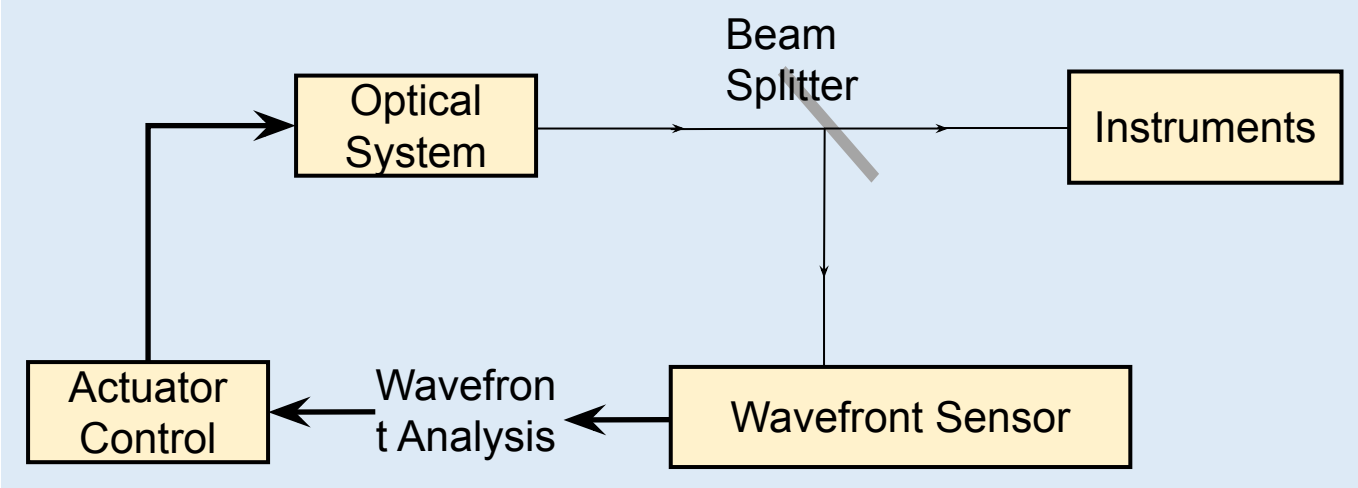
Automated Reflective Optical System Alignment: Experiments and Analysis

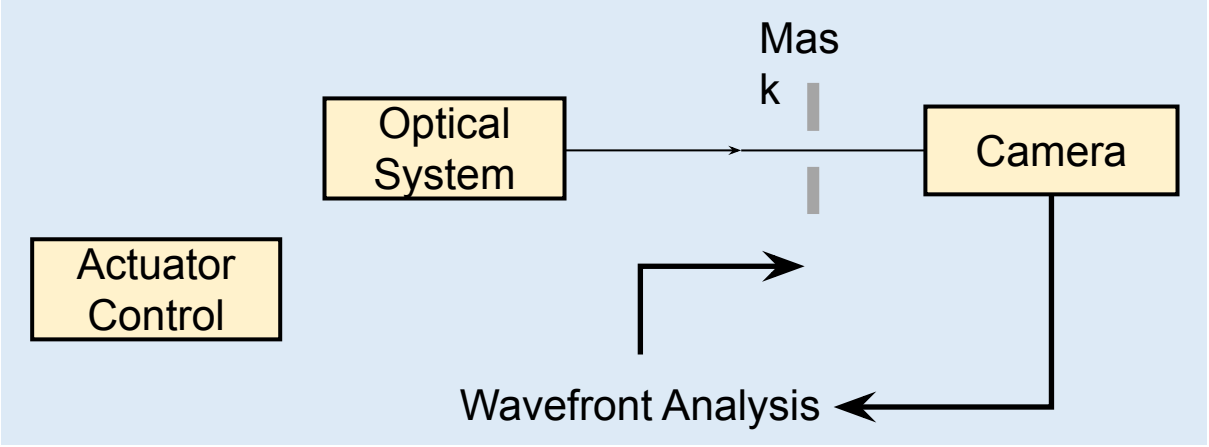
Duan Li and Dmitry Savransky

Sibley School of Mechanical and Aerospace Engineering

Cornell University

Background

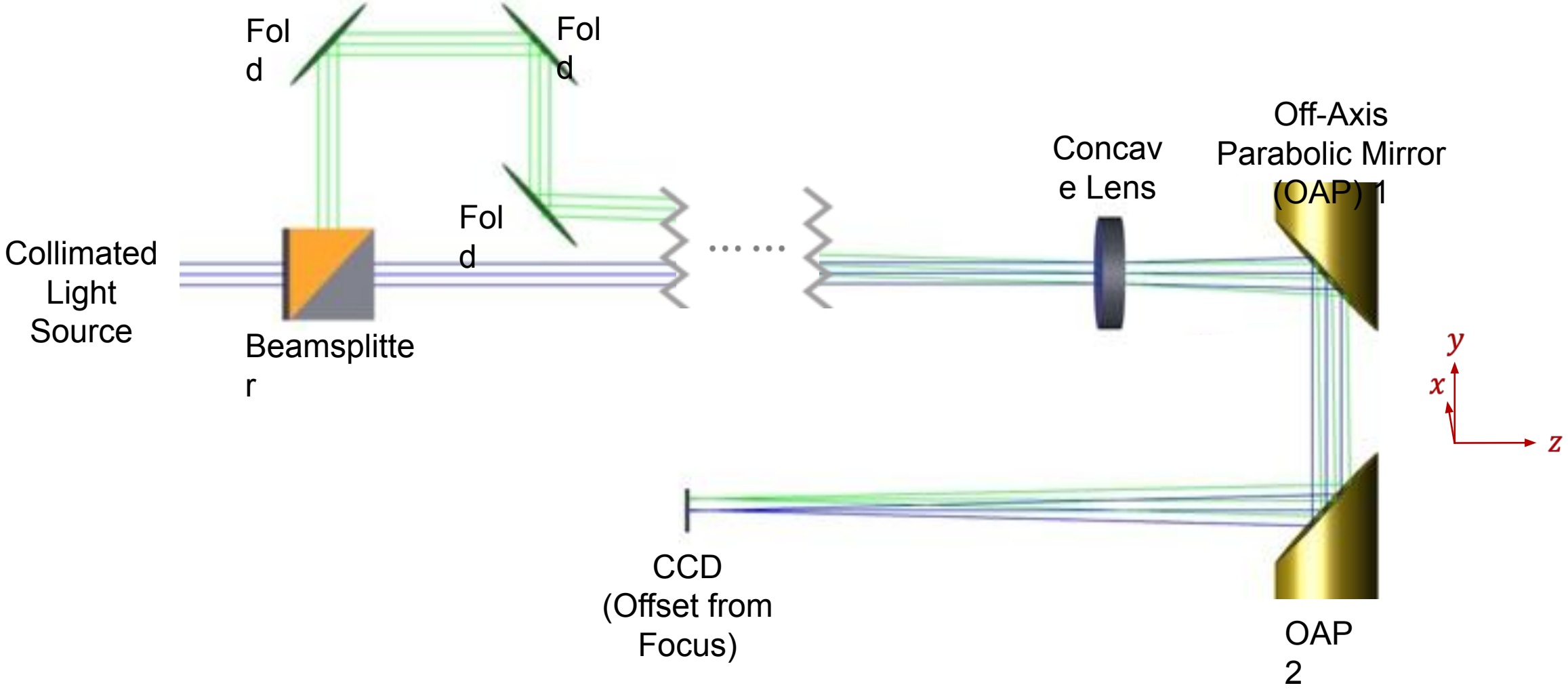




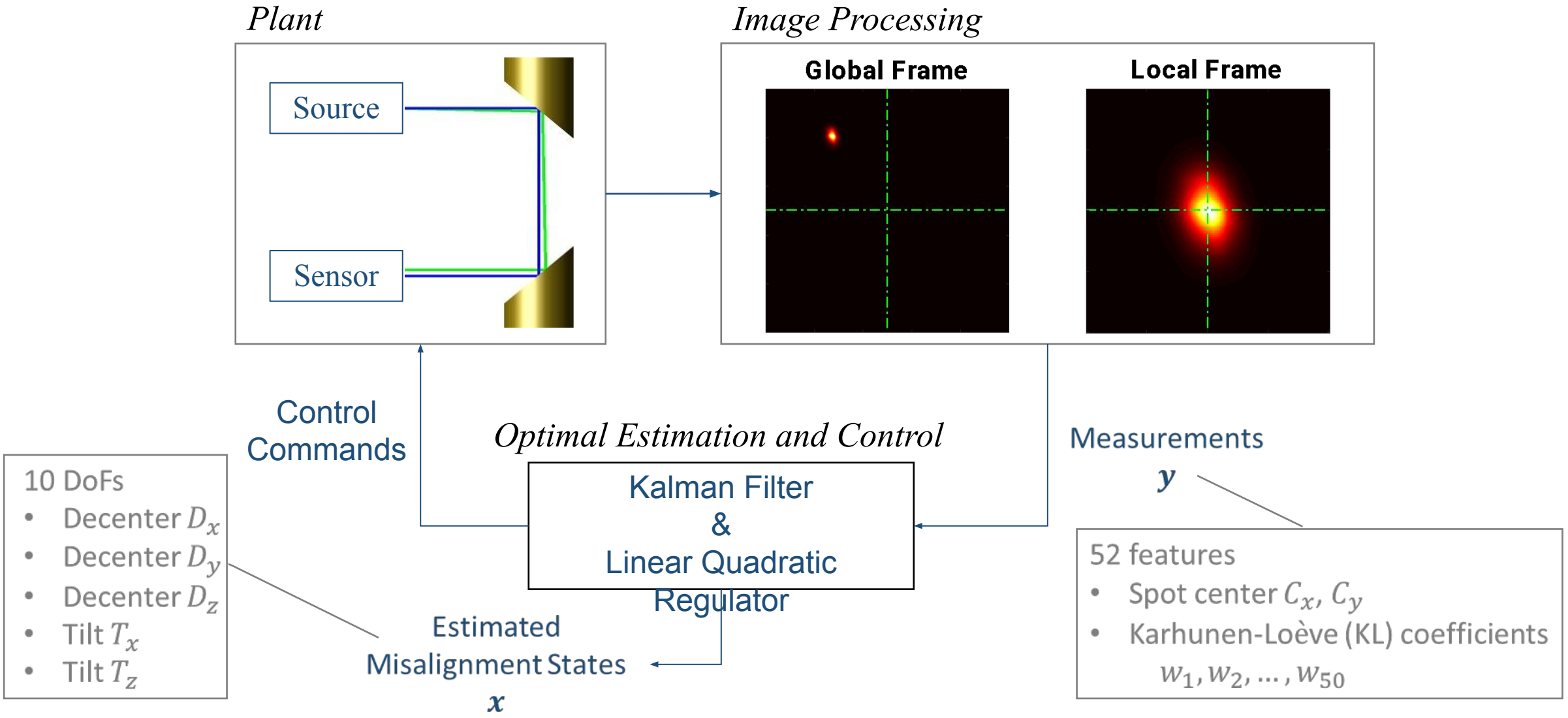
Focal plane sensing

Optimal state estimation

Optical Model



Our Approach



Simulation Result

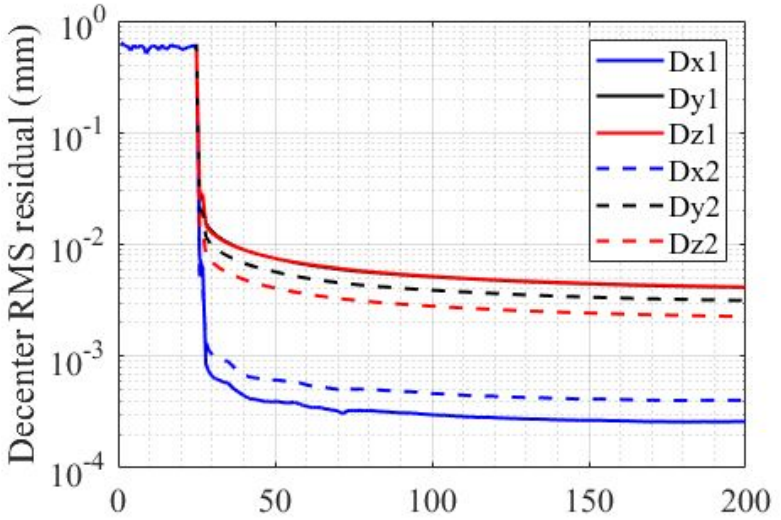
State residual

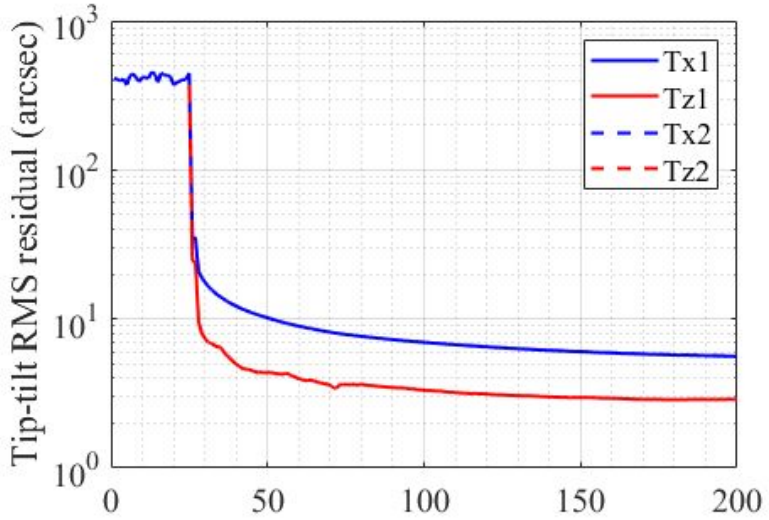
Linear misalignments

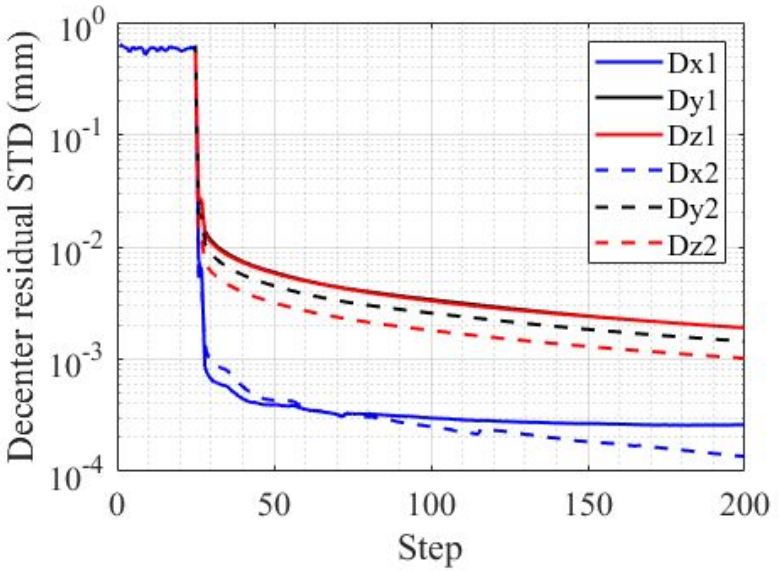
$1\ mm \rightarrow < 5\ \mu m$

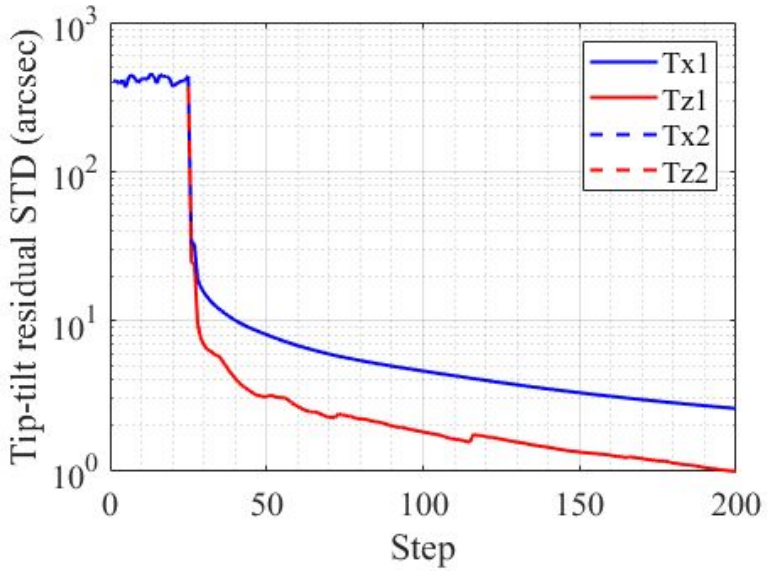
Angular misalignment

$0.2° \rightarrow < 6\ arcsec$



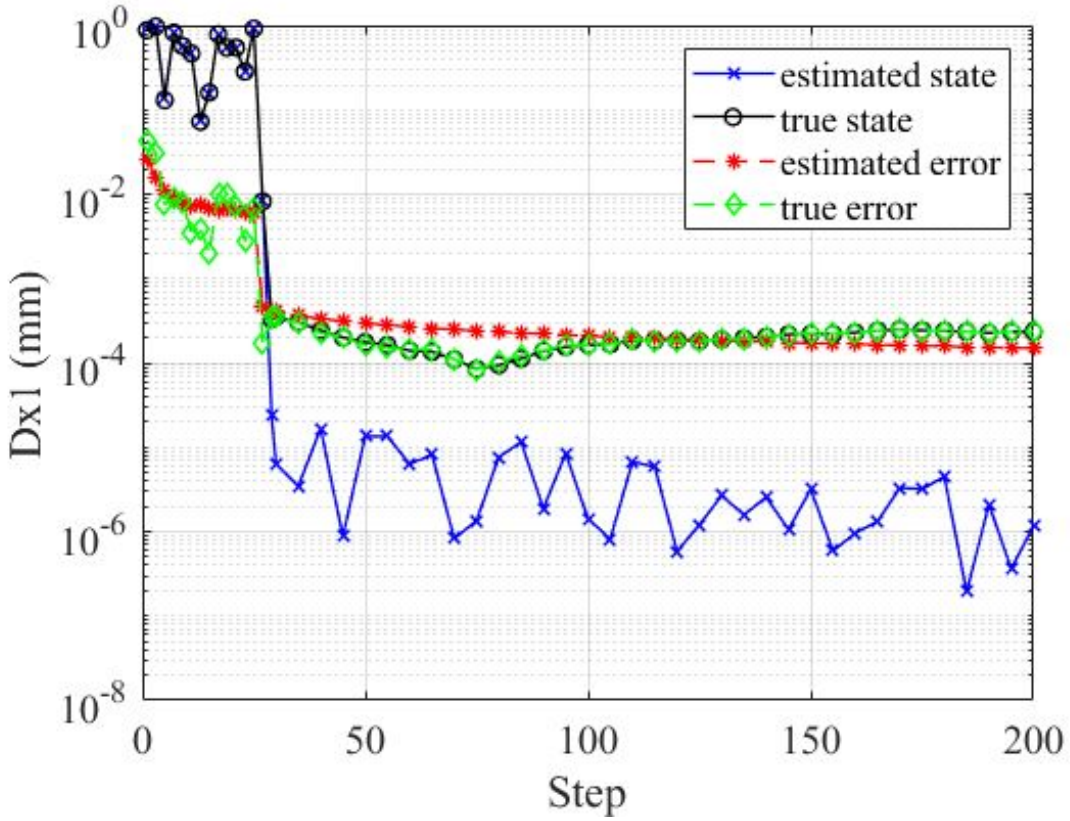


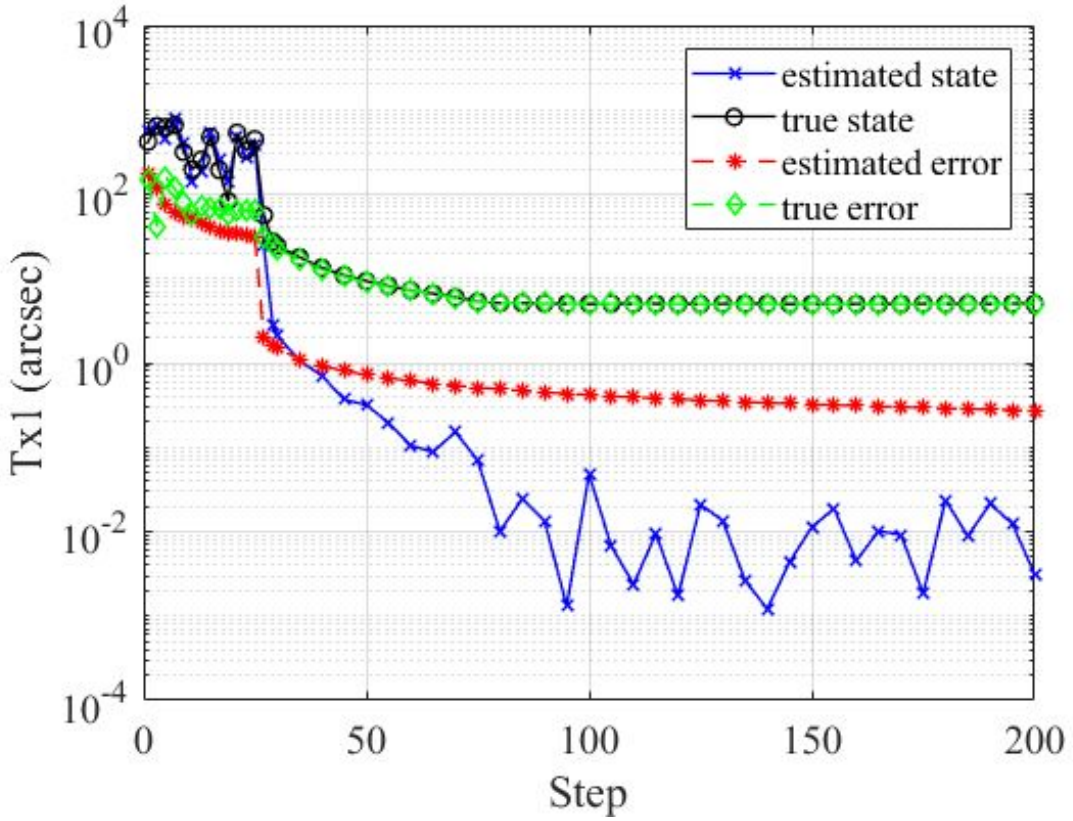




Multi-State Coupling

Estimation error





Observability Analysis

Information filter

- **Predict**
  - Predicted state:
    $\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k\mathbf{u}_k$
  - Predicted information matrix:
    $\boldsymbol{\mathcal{I}}_{k|k-1} = \mathbf{Q}_k^{-1} - \mathbf{Q}_k^{-1}\mathbf{F}_k\left(\boldsymbol{\mathcal{I}}_{k-1|k-1} + \mathbf{F}_k^{\mathrm{T}}\mathbf{Q}_k^{-1}\mathbf{F}_k\right)^{-1}\mathbf{F}_k^{\mathrm{T}}\mathbf{Q}_k^{-1}$
- **Update**
  - Updated information matrix : $\boldsymbol{\mathcal{I}}_{k|k} = \boldsymbol{\mathcal{I}}_{k|k-1} + \mathbf{H}_k^{\mathrm{T}}\mathbf{R}_k^{-1}\mathbf{H}_k$
  - Kalman gain: $\mathbf{K}_k = \boldsymbol{\mathcal{I}}_{k|k}^{-1}\mathbf{H}_k^{\mathrm{T}}\mathbf{R}_k^{-1}$
  - Updated state: $\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\left(\tilde{\mathbf{y}}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1}\right)$

Control input $\mathbf{u}_k$
State transition matrix $\mathbf{F}_k$
Control input matrix $\mathbf{B}_k$
Process noise covariance $\mathbf{Q}_k$
Measurement error covariance $\mathbf{R}_k$
Mapping function $\mathbf{y} = \mathbf{h}(\mathbf{x})$
Mapping Jacobian $\mathbf{H}_k = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k|k-1}}$

Observability Analysis

Degree of observability

- **Observability matrix**

$$\boldsymbol{O}_k = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2\mathbf{F}_1 \\ \cdots \\ \mathbf{H}_k\mathbf{F}_{k-1}\ldots\mathbf{F}_1 \end{bmatrix}$$

- **Degree of observability**

$$OD_k = \sqrt{\frac{\lambda_{min}(\bar{\boldsymbol{O}}_k)}{\lambda_{max}(\bar{\boldsymbol{O}}_k)}}$$

Scaled observability Gramian $\bar{\boldsymbol{O}}_k = \boldsymbol{O}_k^T \boldsymbol{R}_k^{-1} \boldsymbol{O}_k$

- **Eigenvector with lowest/highest observability**

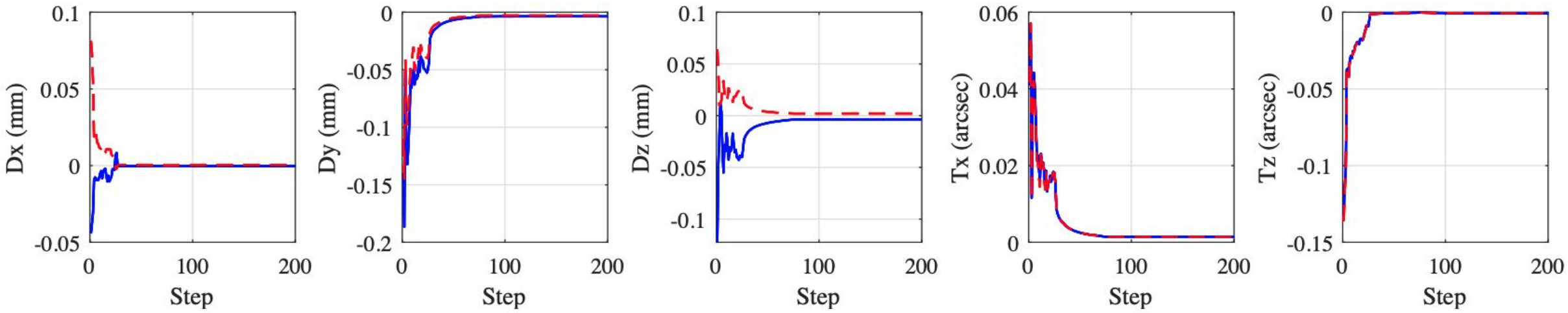
$$eig(\bar{\boldsymbol{O}}_k^{-1})$$

Multi-State Coupling

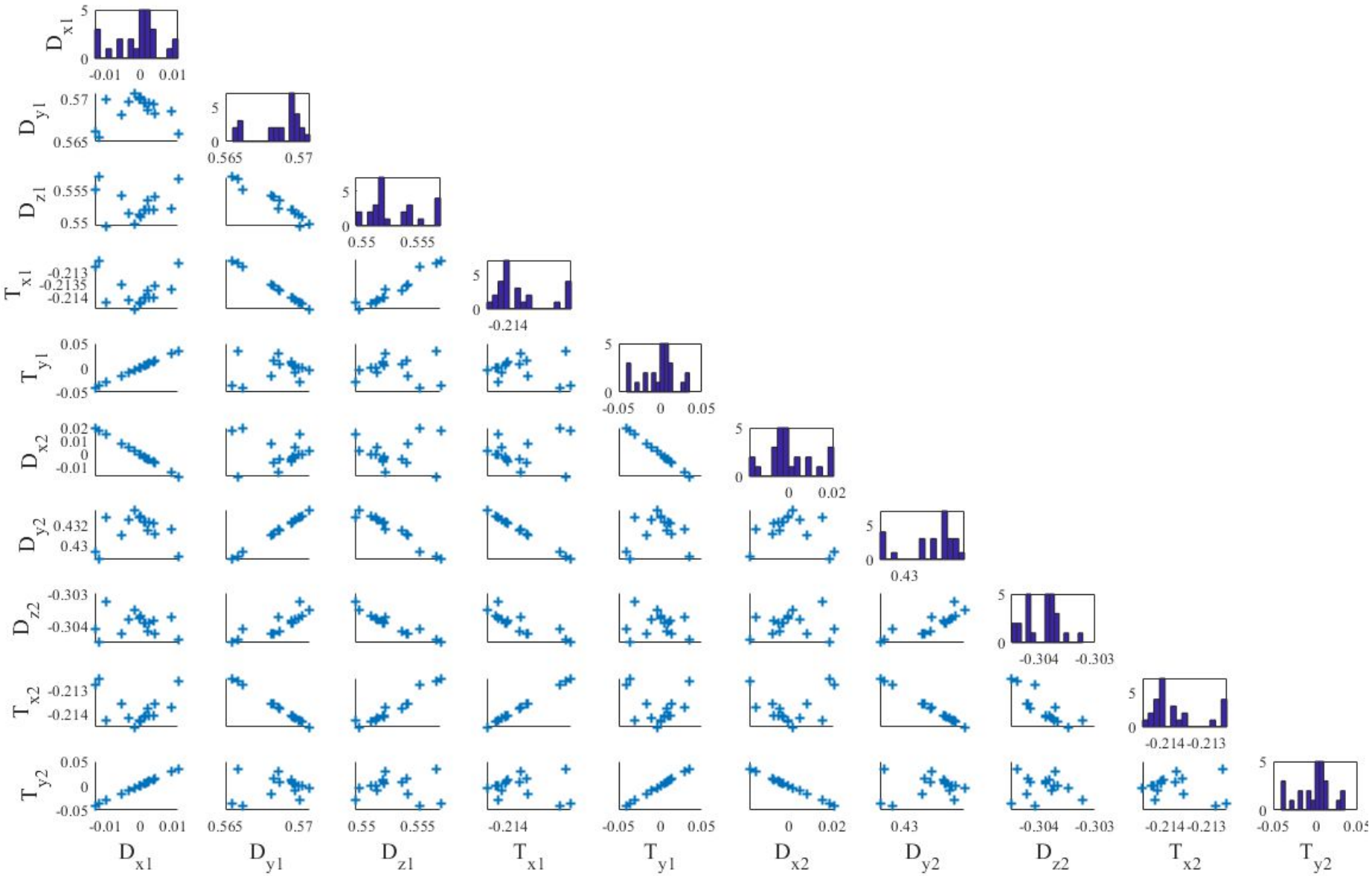
Estimation error

- Worst observed eigenvector

$\delta < 0.012$

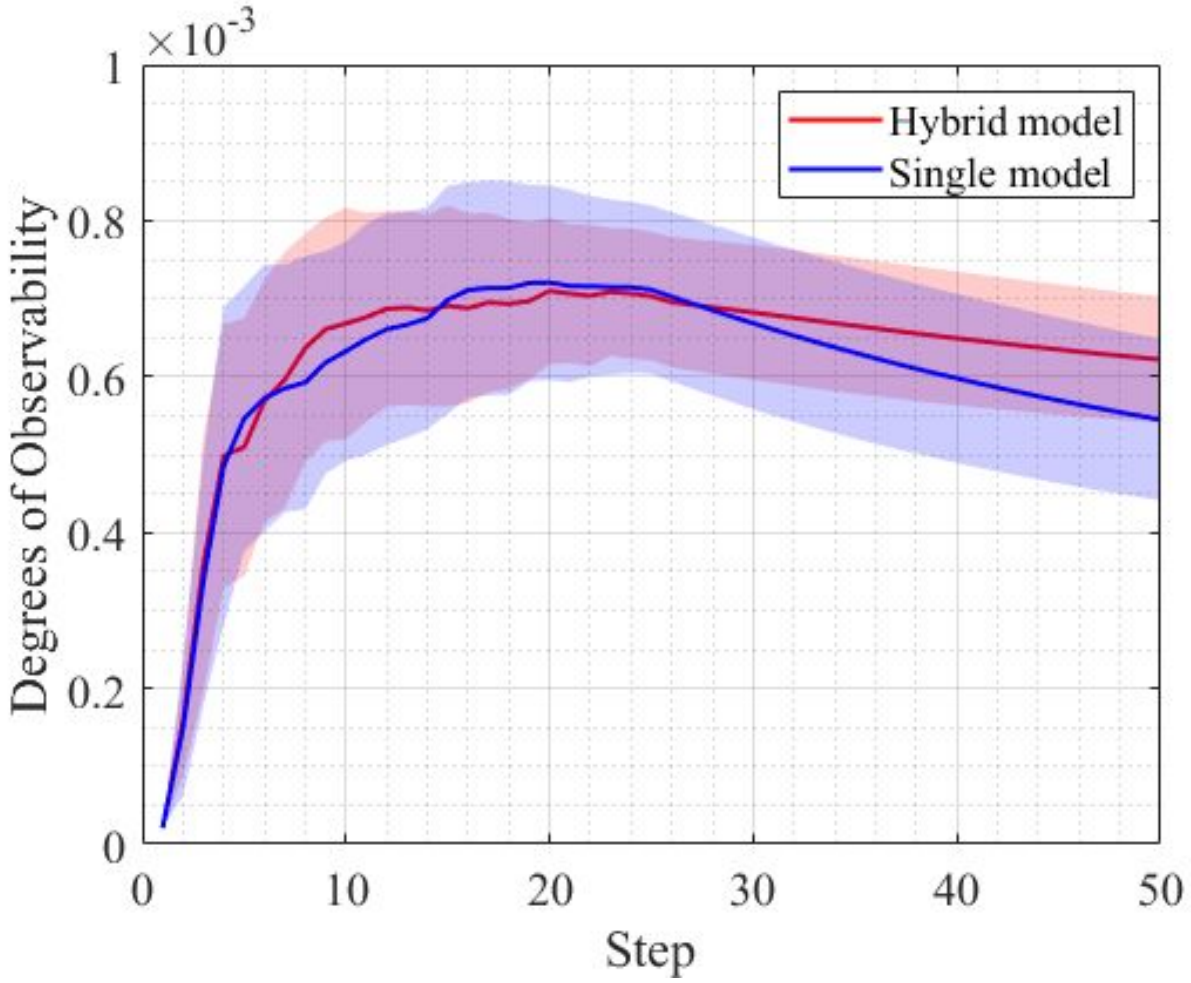


Multi-State Coupling

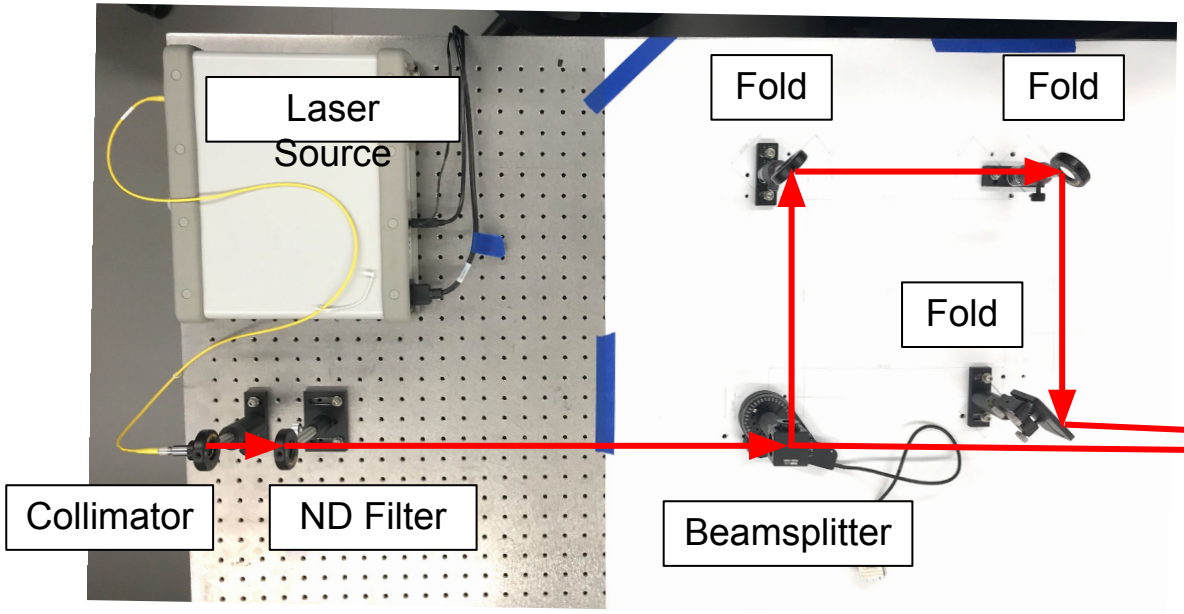


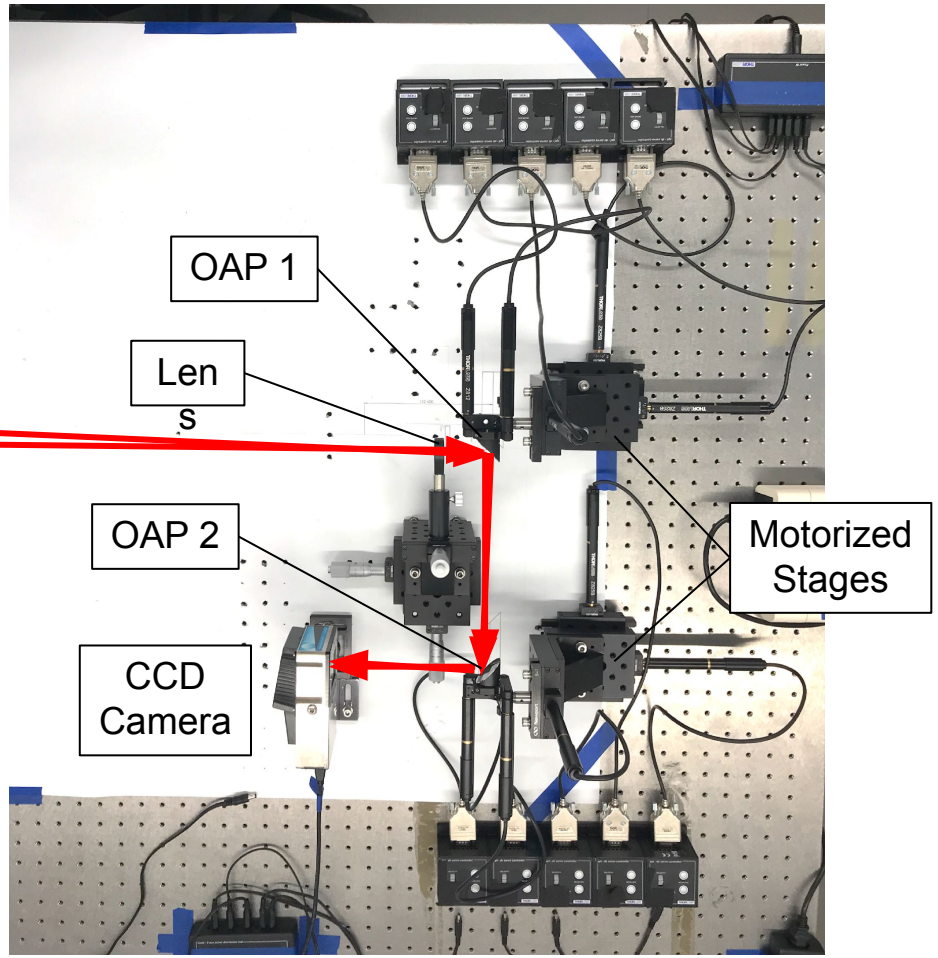
Observability Analysis

Algorithm Design



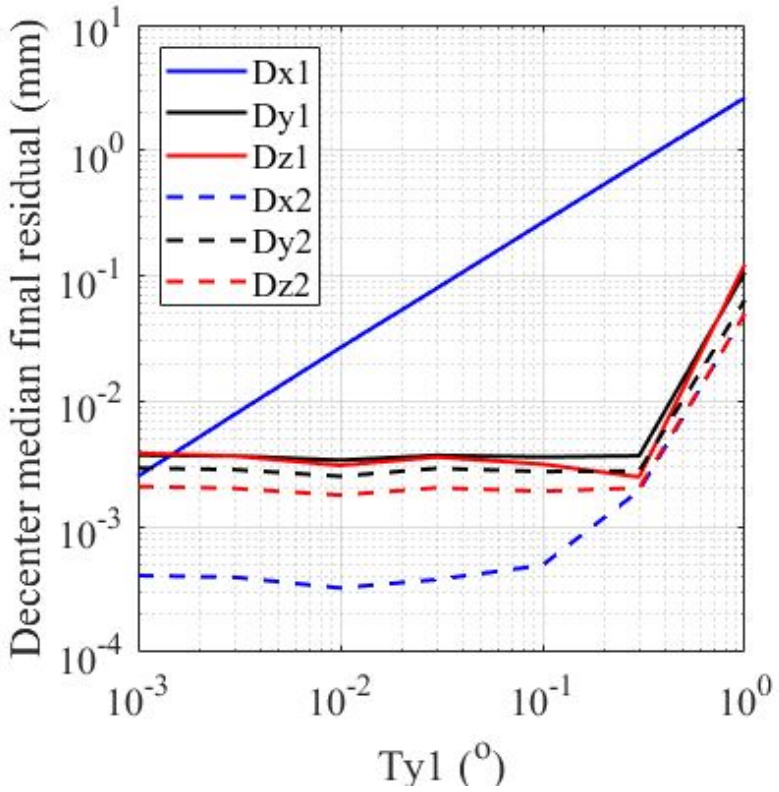
Experiments

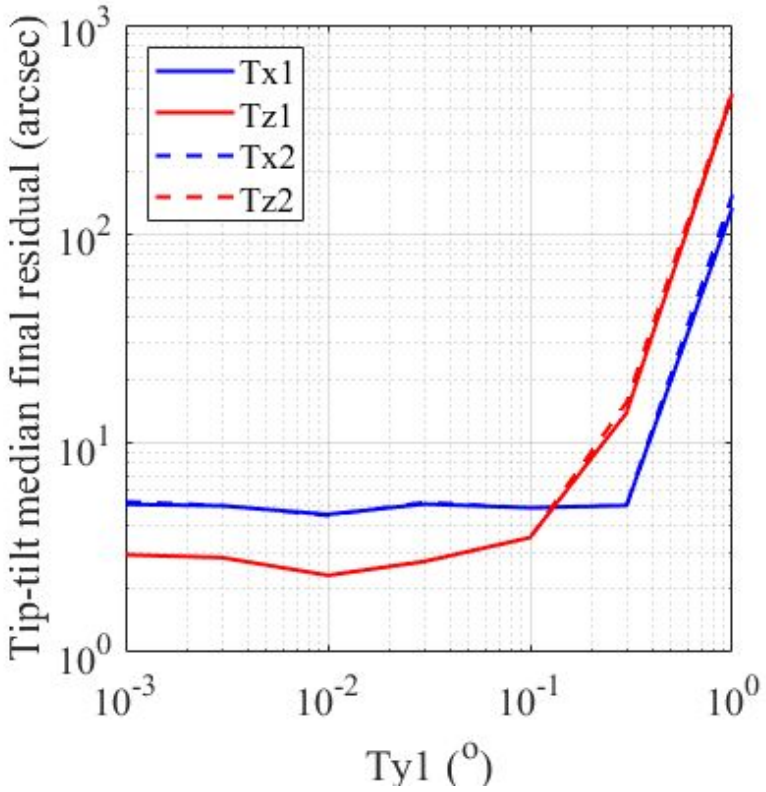


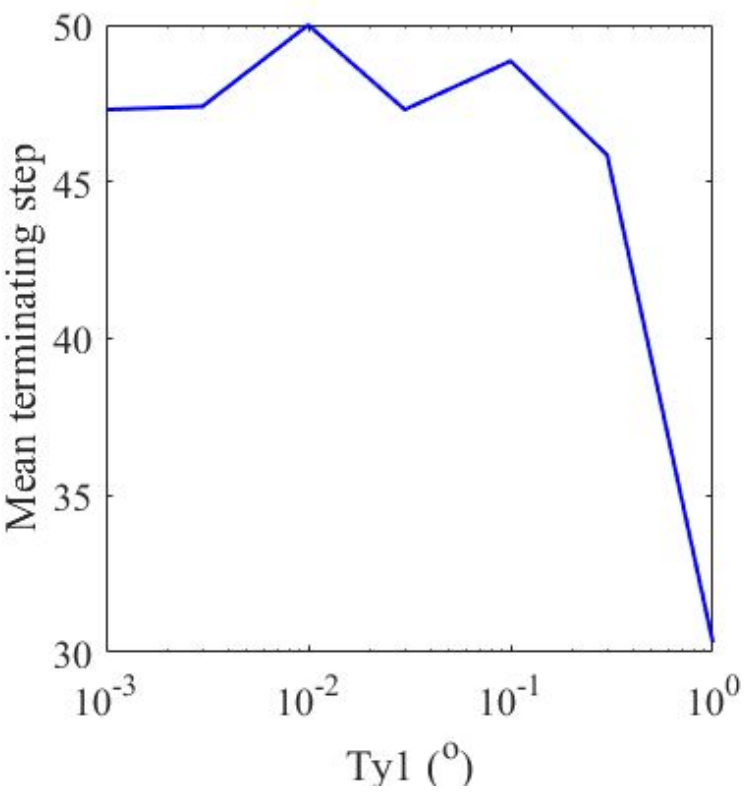


Experiments

Analysis



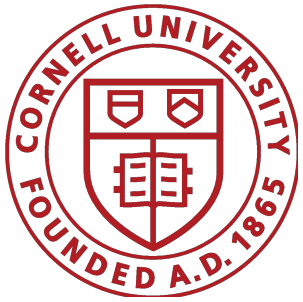




Conclusion

Automated alignment of a reflective system using pure focal plane sensing

- Continuation on simulation
- Analysis
  - Observability
  - Algorithm design
  - Multi-state coupling effect
- Preliminary Experiments
- Future work
  - Higher manual alignment accuracy in experiments
  - Algorithm optimization
  - Control strategy: well/poorly observed eigenvectors



Thank You!

Contact: dl943@cornell.edu