



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Nonlinear Filtering with Applications to Astrodynamics

B-Exam Presentation

Zvonimir Stojanovski

April 19, 2023



Nonlinear Filtering in Aerospace and Astronomy



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

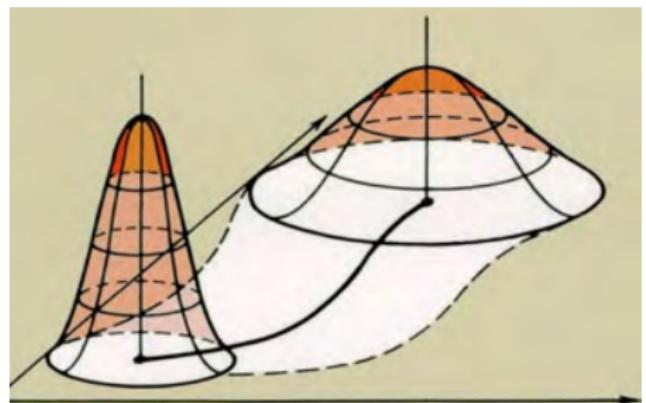
Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Obtaining state estimates from incomplete and noisy measurements
- Propagating state uncertainty in time



Stengel (1986)



Filtering Methods



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- The Kalman filter forms the basis of most modern recursive estimators
- Various extensions for nonlinear systems have been developed
 - Extended Kalman filter (EKF)
 - Unscented Kalman filter (UKF) or sigma point filter (SPF)



Goals



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Development and improvement of nonlinear filtering methods
- Applications to challenging problems in astrodynamics



Overview



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Higher-Order Unscented Estimator (HOUSE)

- An extension of the unscented Kalman filter that accounts for 3rd and 4th order moments

Autonomous Cross-Calibration for Imaging Satellites (ACCIS)

- A method for estimating dynamical states and camera parameters for constellations of imaging satellites using conventional and image-based measurements

Nonsingular Estimator for Exoplanet Orbits (NEXO)

- A method for fitting directly-observed exoplanet orbits using a Gaussian mixture sigma point filter and nonsingular orbital elements



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

The Higher-Order Unscented Estimator



Motivation



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Challenges of filtering with non-Gaussian noise

- Most filters compute only mean and covariance
- Outliers can be problematic

Examples of non-Gaussian (Pearson type IV) noise sources

- Wind shear fluctuations (Ramsdell, 1978)
- Fluctuating pressure on aircraft skin panels (Steinwolf and Rizzi, 2012)
- Solar wind intensity (Krafft et al., 2019)

How can we extend the UKF to better handle non-Gaussian distributions?



HOUSE Overview



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- An extension of the UKF
- Accounts for 3rd and 4th order moments
- $2n + 1$ sigma points for n -dimensional augmented state
- Asymmetric sigma points and weights
- Matches marginal skewness and kurtosis in n directions

*Methodology and preliminary tests presented in Z.S.'s
A-Exam*



HOUSE Testing



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Simulated dynamical systems with nonlinear dynamics and measurements
- Gaussian and non-Gaussian process and measurement noise
- Compared with original UKF and Conjugate Unscented Transform (CUT) filters (Adurthi et al., 2018)



Projectile Example: Dynamics



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- State consists of position (x, y, z) and velocity $(\dot{x}, \dot{y}, \dot{z})$
- Ballistic motion with drag and disturbance forces

$$\ddot{x} = -bv\dot{x} + f_x$$

$$\ddot{y} = -bv\dot{y} + f_y$$

$$\ddot{z} = -bv\dot{z} + f_z - g$$

where

$$v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \quad b = \frac{AC_D\rho}{2m} = 0.001 \text{ m}^{-1}$$



Projectile Example: Measurements



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

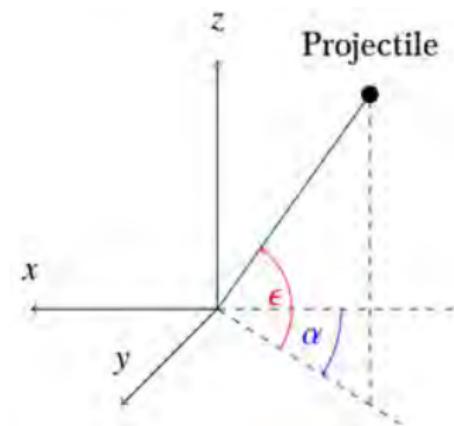
References

- Line-of-sight only:
Azimuth & Elevation

$$\alpha = \text{atan2}(y, -x) + n_\alpha$$

$$\epsilon = \text{atan2}\left(z, \sqrt{x^2 + y^2}\right) + n_\epsilon$$

- Measurements at 5 Hz





Projectile Example: Process Noise



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

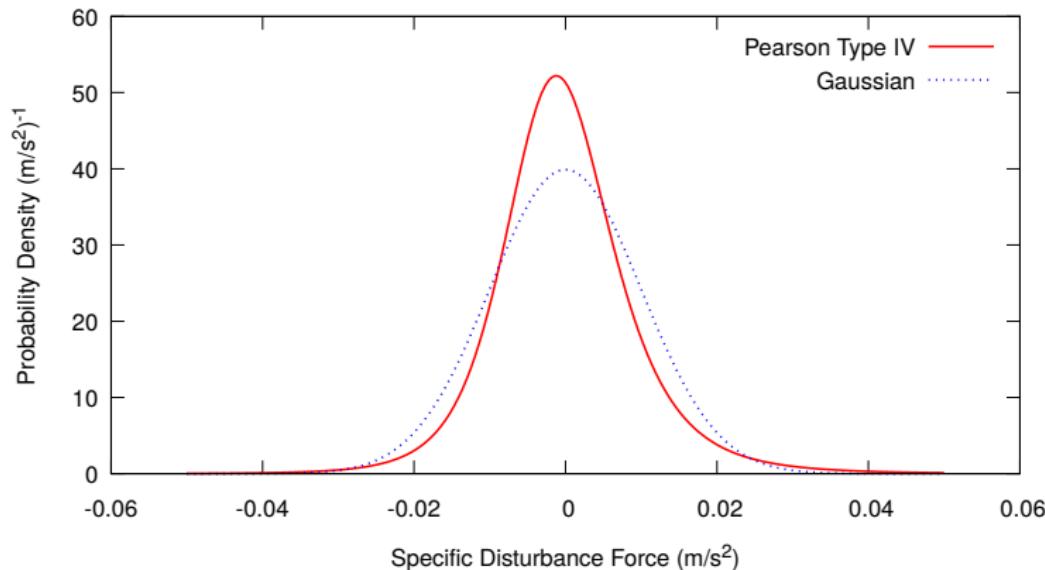
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Projectile Example: Measurement Noise



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

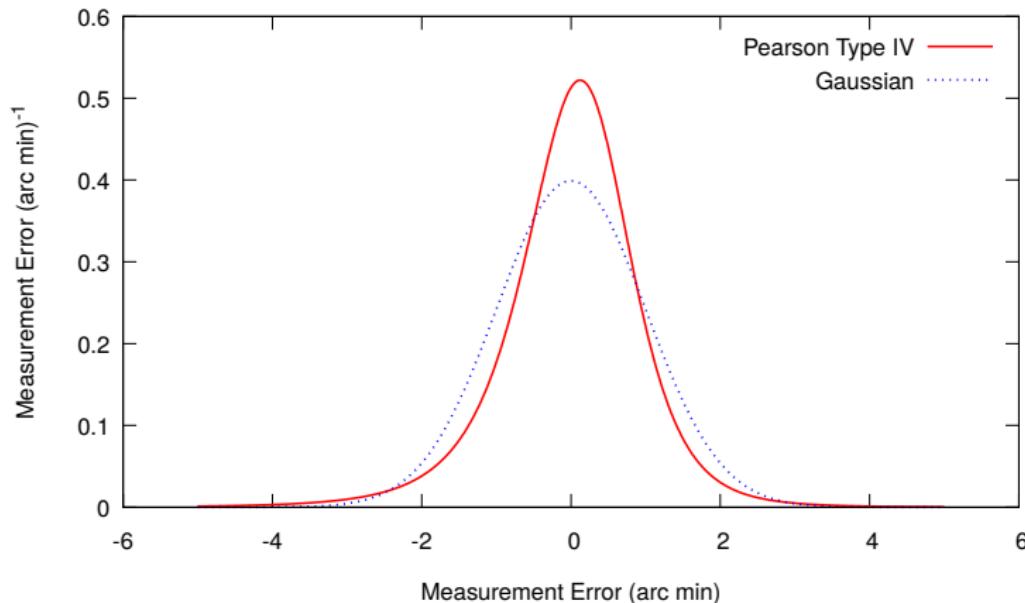
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Projectile Example: Root-Mean-Square Errors



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

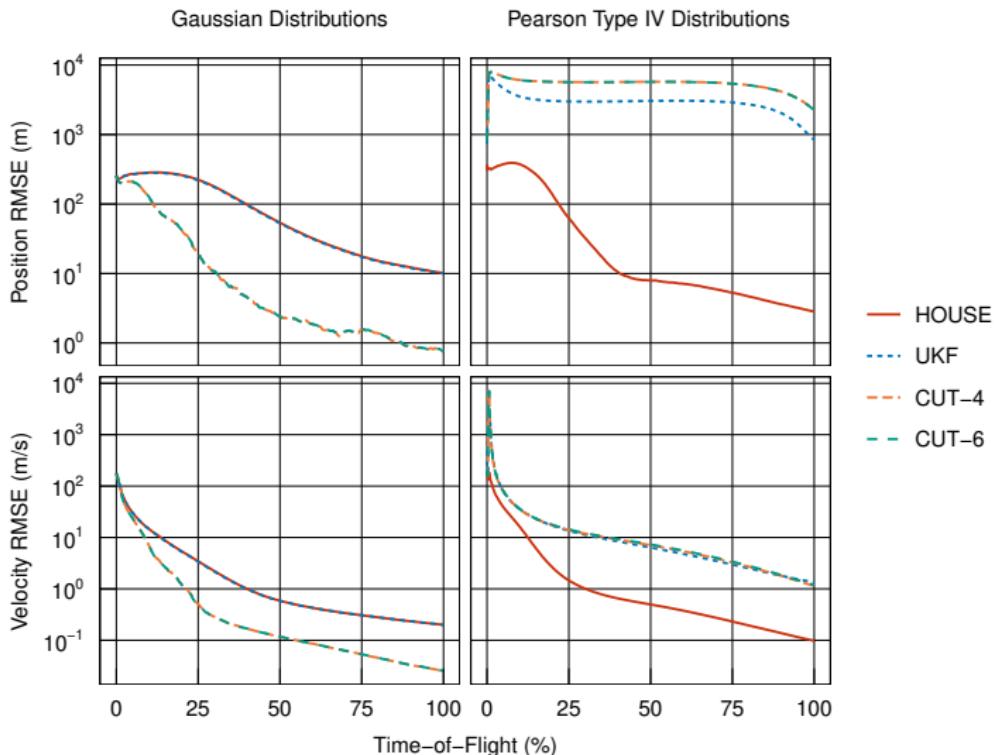
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Projectile Example: Error Distributions



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

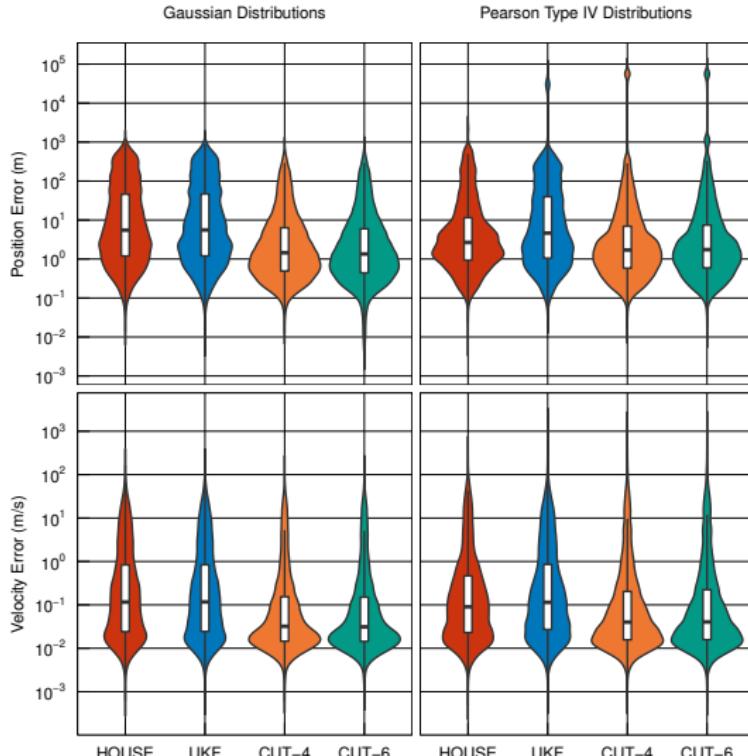
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Projectile Example: Error Distributions



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

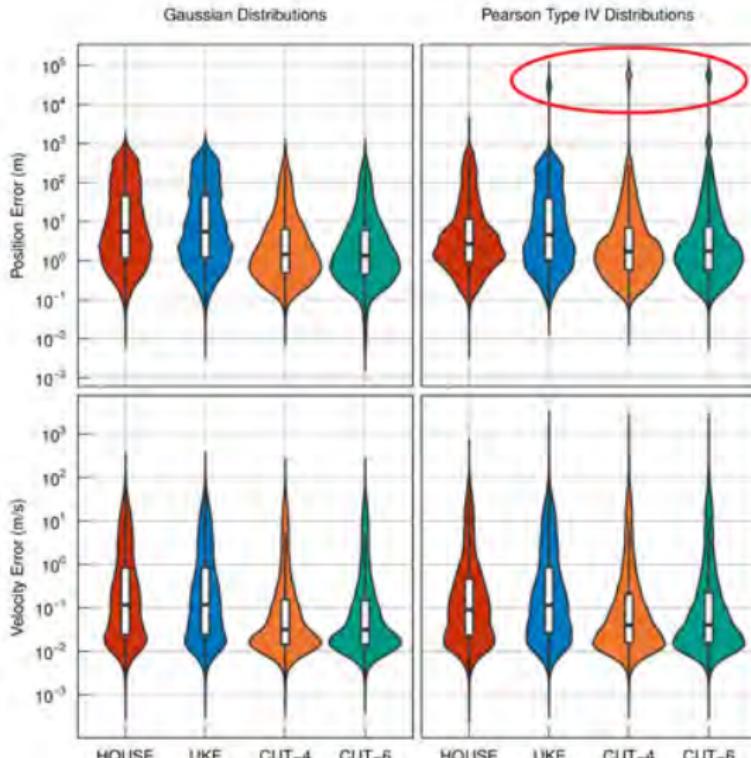
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Summary



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Developed a new extension of the UKF that accounts for skewness and kurtosis using asymmetric sigma points and weights
- Provides more accurate estimates than UKF or CUT filters when noise contains outliers



Future Work



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Extending HOUSE to account for other higher-order moments
- Analyzing trade-off between number of sigma points and filter accuracy



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Autonomous Cross-Calibration for Imaging Satellites



Motivation



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Cross-calibration is increasingly important for constellations of imaging satellites
- Current methods require humans-in-the loop, uplinks/downlinks, and dedicated calibration measurements
- How can we make the process more efficient and scalable?



ACCIS Concept of Operations



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

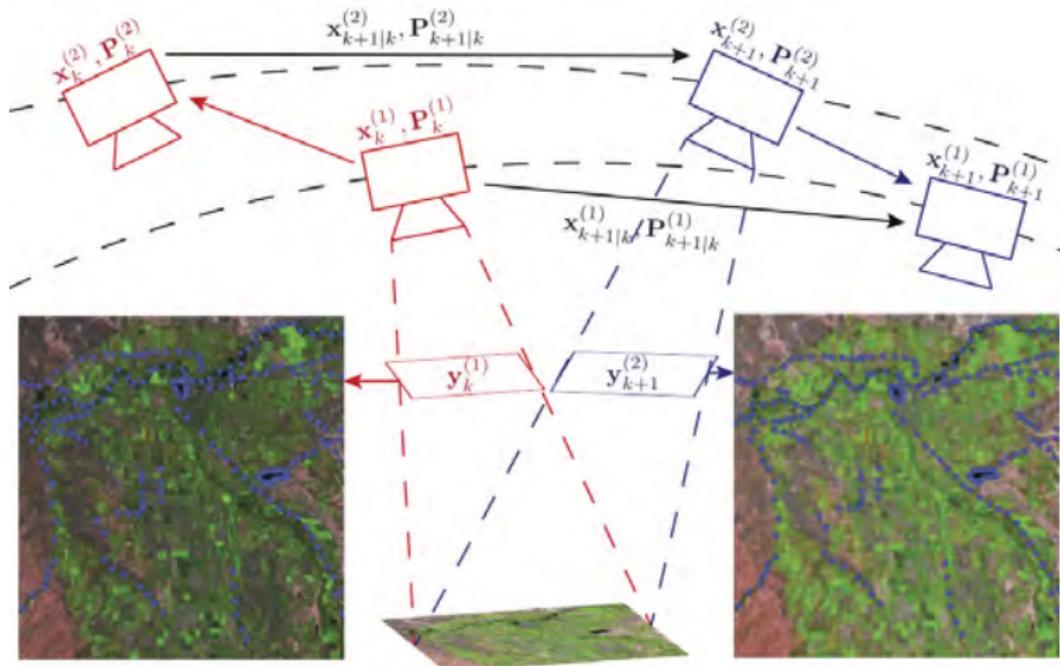
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References



D. Savransky (2020)



Dynamical Model



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Each satellite is a rigid body in a perturbed Keplerian orbit
- Earth gravity model to practically arbitrary order using EGM2008
- Atmospheric drag treated as disturbance by filter
 - Difficult to predict
 - Jacchia-Bowman 2008 atmospheric model used for “ground truth” state propagation
- PD law for control torques to maintain nadir-pointing attitude



Camera Model



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Attitude of camera frame \mathcal{C} with respect to body frame \mathcal{B}
- Defocus (blurring due to error in focal length f)
- Radial distortion model

$$\begin{aligned}\|\mathbf{r}_{P''}\| = & \left(1 - c_1 - c_2 - c_3 + c_1 \frac{\|\mathbf{r}_{P'}\|}{R} \right. \\ & \left. + c_2 \frac{\|\mathbf{r}_{P'}\|^2}{R^2} + c_3 \frac{\|\mathbf{r}_{P'}\|^3}{R^3} \right) \|\mathbf{r}_{P'}\|\end{aligned}$$





Filtering



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

$$\mathbf{x} = \begin{bmatrix} \mathbf{r}_{G/O'} \\ \mathbf{v}_{G/O'}^{\mathcal{G}} \\ \boldsymbol{\omega}_{\mathcal{B}/\mathcal{G}} \\ \boldsymbol{\rho}_{\mathcal{B}/\mathcal{A}} \\ \boldsymbol{\rho}_{\mathcal{C}/\mathcal{B}} \\ f \\ \mathbf{c} \end{bmatrix}$$

- Combined state

- State estimated using Square Root Sigma Point Filter (Brunke and Campbell, 2004)



Conventional Measurements



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

GPS

- Position and velocity
- Additive Gaussian noise model

Gyroscope

- Angular velocity
- Additive Gaussian noise model
- In the future, may add bias, drift, etc.

Star Tracker

- Attitude
- Normal and boresight errors (Markley and Crassidis, 2014)



The Scale Invariant Feature Transform (SIFT)



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

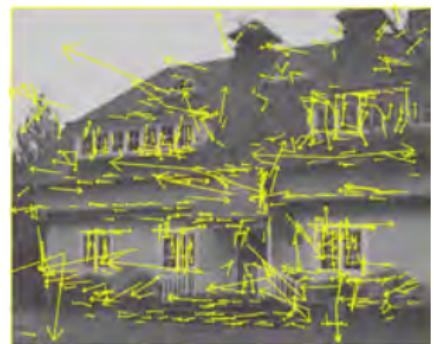
The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Extracts features, or key points,
from images (Lowe, 2004)

- Position, orientation, and scale
- Gradient-based descriptor invariant under translation, rotation, and scaling
- Descriptors can be matched between images



Lowe (2004)



Example of SIFT Key Point Matching



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

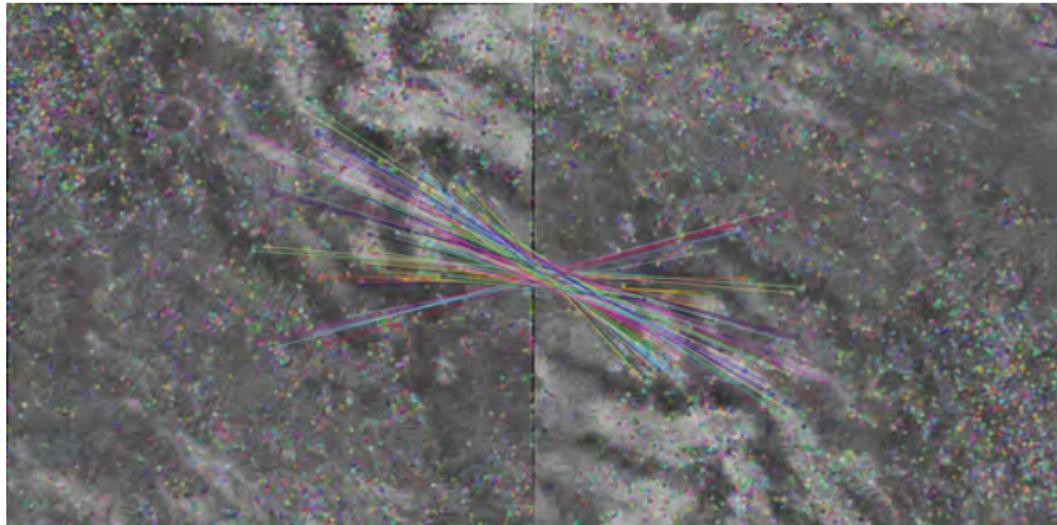
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Representation of SIFT Key Points



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Conventional representation consists of position \mathbf{r}_K , orientation angle θ , and scale S
- We compute two positions $\mathbf{r}_{K'_1}$ and $\mathbf{r}_{K'_2}$, given by

$$\mathbf{r}_{K'_{1,2}} = \mathbf{r}_K \pm \frac{1}{2} S \mathbf{u}$$

$$\text{where } \mathbf{u} = [\cos \theta \quad \sin \theta]^T$$

- Key point can be represented as

$$\mathbf{K} = \begin{bmatrix} \mathbf{r}_{K'_1}^T & \mathbf{r}_{K'_2}^T \end{bmatrix}^T$$



SIFT-Based Measurement Model



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Given matched key points \mathbf{K}_1 and \mathbf{K}_2 from images taken by two different satellites
- Using states \mathbf{x}_1 , \mathbf{x}_2 and times t_1 , t_2 :
 - Map \mathbf{K}_1 to points on Earth's surface
 - Map points on Earth to predicted key point $\hat{\mathbf{K}}_2$
- Update estimate of \mathbf{x}_2 based on difference between $\hat{\mathbf{K}}_2$ and \mathbf{K}_2



Synthetic Image Generation



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- For simulation and testing
- Cut from mosaic of Landsat images using NASA World Wind library
- Projected, distorted, and defocused using OpenCV library





Simulation Setup



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Two satellites in coplanar circular orbits
 - 500 km altitude
 - 30° inclination
 - Satellite 1 ahead of Satellite 2 by 4°
- Each satellite takes one image per minute
- Run for 4,000 min (~42 orbital periods)



Results: Dynamical State Estimates



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

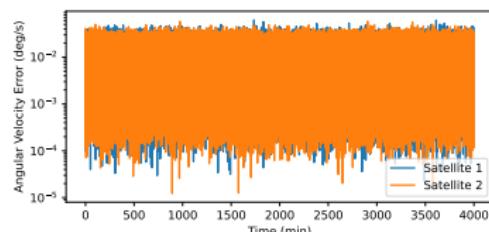
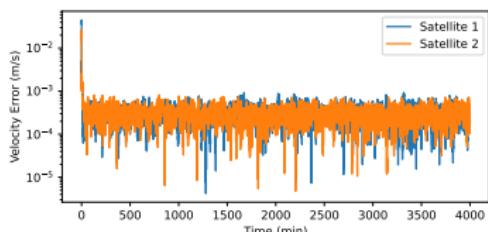
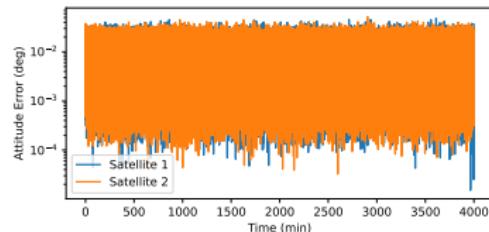
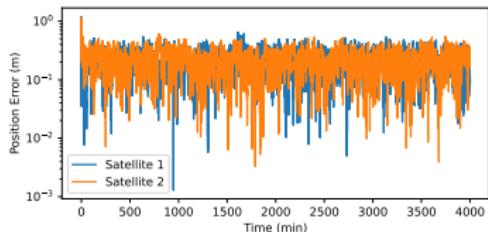
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Results: Camera Attitude and Focal Length Estimates



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

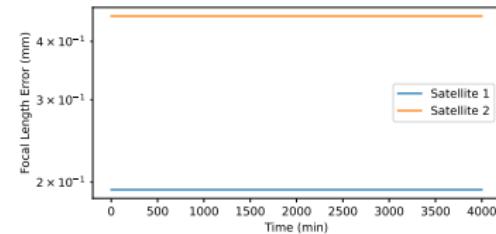
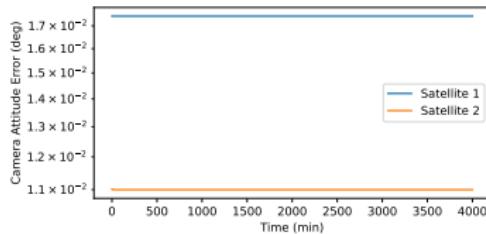
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Results: Camera Distortion Parameter Estimates



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

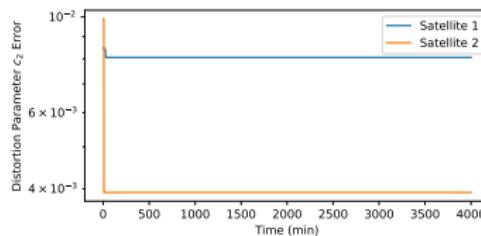
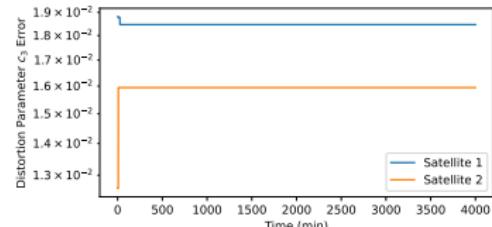
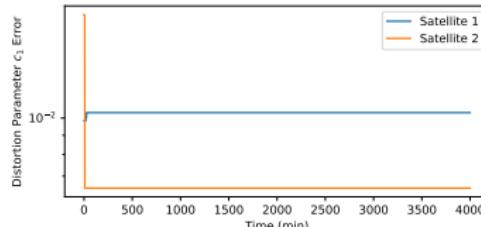
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Summary



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Developed detailed simulation framework for a constellation of imaging satellites
- Demonstrated feasibility of cross-calibration using measurements based on image features



Future Work



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Running simulations with larger constellations over longer times
- Refinement of image-based measurement model



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

The Nonsingular Estimator for Exoplanet Orbits

Motivation

Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

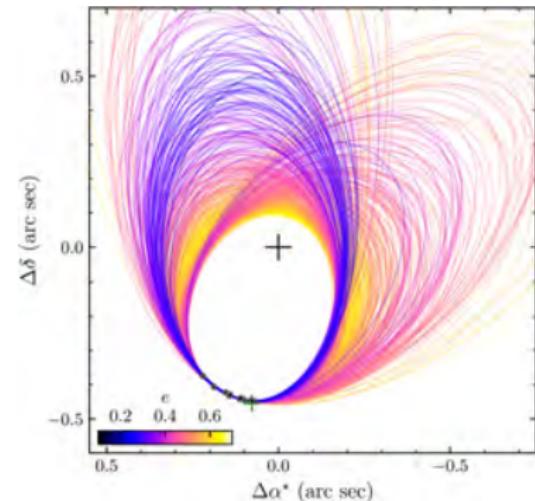
Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Instruments such as the Gemini Planet Imager have enabled direct imaging and astrometric measurements on exoplanets
- Orbit fitting remains challenging and computationally expensive



De Rosa et al. (2020)

Can we improve orbit estimates using nonlinear filtering methods?



Direct Observations



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

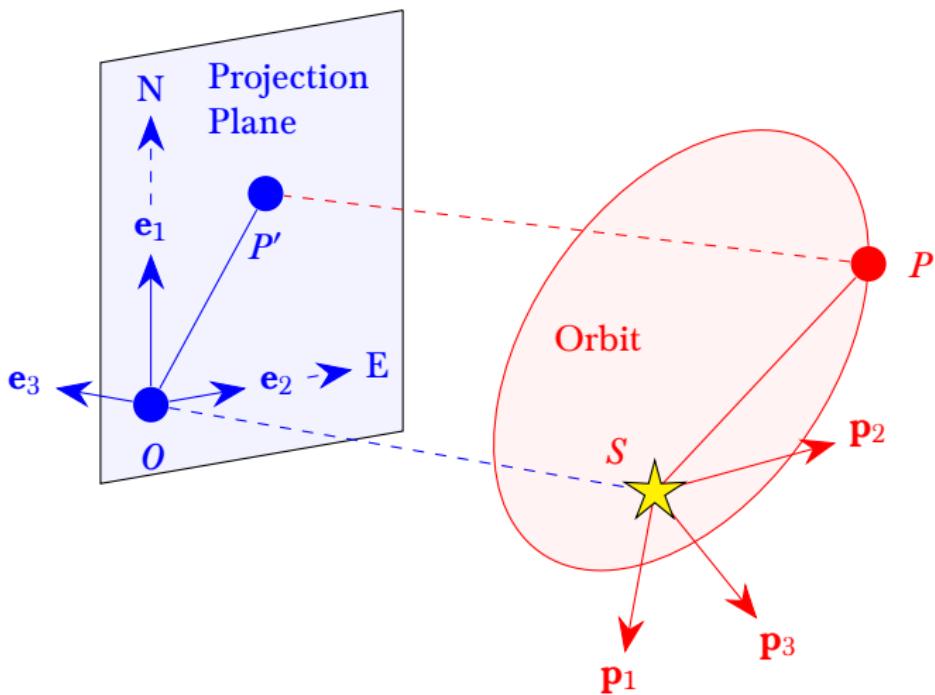
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Exoplanet Orbital Elements



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

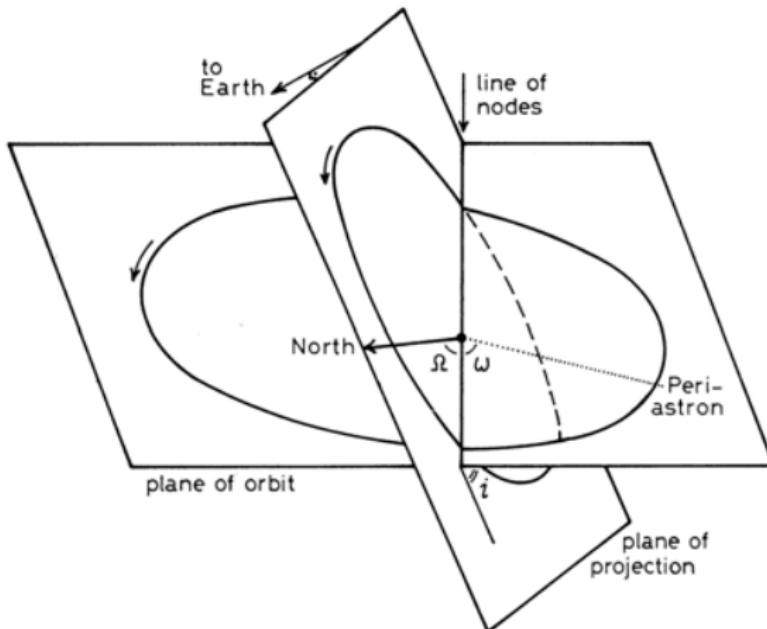
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References



Heintz (1978)



Nonsingular Orbital Elements



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

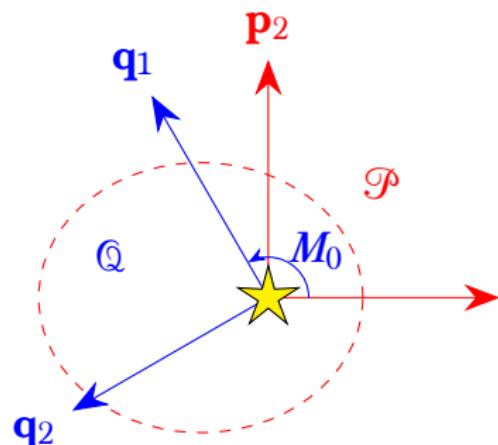
Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- No singularities at $e = 0$,
 $I = 0$, etc.
- Any values in \mathbb{R}^7
describe an elliptic orbit
- Based on the reference
frame \mathcal{Q}
- Combine features of the
Thiele-Innes constants
and the nonsingular
elements due to Cohen
and Hubbard (1962)



Perifocal frame \mathcal{P} and
auxiliary frame \mathcal{Q}



Measurement Model



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

$$\mathbf{z} = \Xi\boldsymbol{\zeta}(\boldsymbol{\eta}, \lambda, t) + \mathbf{w}$$

- $\boldsymbol{\zeta}$ is the position in the orbital plane in \mathbb{Q} , scaled by $1/a$
- λ is a function of P
- $\boldsymbol{\eta}$ is a function of e and M_0
- \mathbf{w} is the measurement noise



Nonsingular Orbital Elements: Definitions



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

$$\Xi_{11} = \varpi a(\cos(\omega + M_0) \cos \Omega - \sin(\omega + M_0) \sin \Omega \cos I)$$

$$\Xi_{21} = \varpi a(\cos(\omega + M_0) \sin \Omega + \sin(\omega + M_0) \cos \Omega \cos I)$$

$$\Xi_{12} = \varpi a(-\sin(\omega + M_0) \cos \Omega - \cos(\omega + M_0) \sin \Omega \cos I)$$

$$\Xi_{22} = \varpi a(-\sin(\omega + M_0) \sin \Omega + \cos(\omega + M_0) \cos \Omega \cos I)$$

$$\eta_1 = \frac{e \cos M_0}{\sqrt{1 - e^2}}$$

$$\eta_2 = -\frac{e \sin M_0}{\sqrt{1 - e^2}}$$

$$\lambda = \ln(P/P_0)$$



Filtering



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

State Vector

$$\mathbf{x} = [\lambda \quad \eta_1 \quad \eta_2 \quad \Xi_{11} \quad \Xi_{21} \quad \Xi_{12} \quad \Xi_{22}]^T$$

- Constant in time (negligible perturbations)
- No filter prediction step required

Measurements

- Astrometry
- Mass (from mass-luminosity relations)



Gaussian Mixtures



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Problem

- Prior distributions are very diffuse
- SPF works well with η and Ξ , but not with λ

Solution

- Use Gaussian mixture to cover wide range of λ
- Each mixture component has a small σ_λ
- Run SPF with each component of mixture



Improving Orbit Fitting Accuracy



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Filter Setup

- Use square root SPF
- Use 5th-order Gaussian cubature rule for sigma points

Repeated Filtering

- Run filter 10 times with same measurements, going forward in time
- More accurate than single filter pass or back-and-forth smoothing



Validation



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Setup

- 100 random exoplanet orbits
- Population model based on detections by the Kepler space telescope
- For each orbit, generated 5 measurements over a two-year time span
- Added random errors with $\sigma = 5$ mas

Fitting Methods

- NEXO
- Markov Chain Monte Carlo (MCMC)
- Orbits for the Impatient (OFTI), introduced by Blunt et al. (2017)



Validation Results: Semi-Major Axis



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

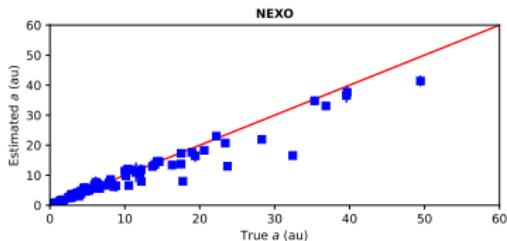
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

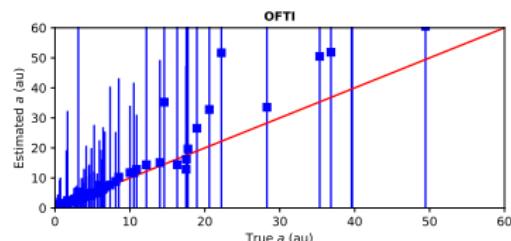
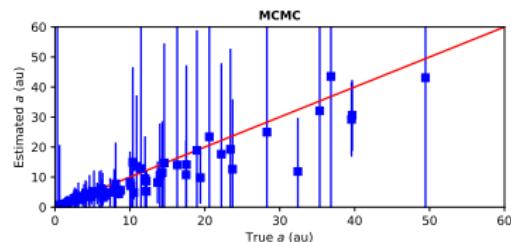
The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References



Mean Estimates and 95%
Credible Intervals





Validation Results: Period



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

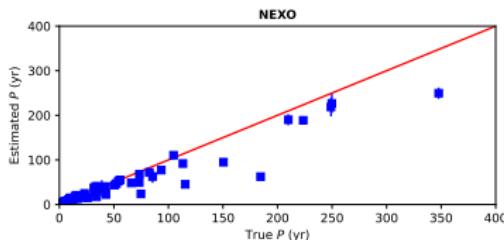
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

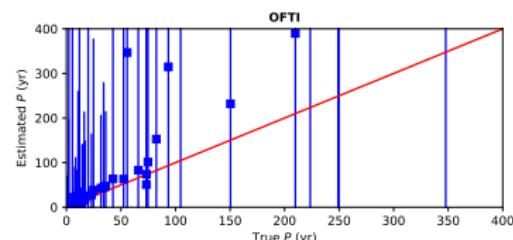
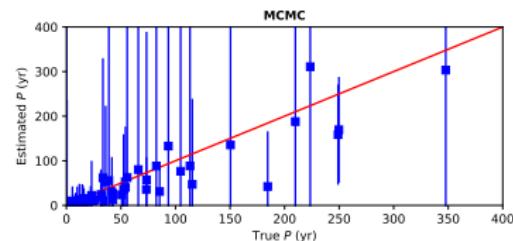
The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References



Mean Estimates and 95%
Credible Intervals





Validation Results: Eccentricity



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

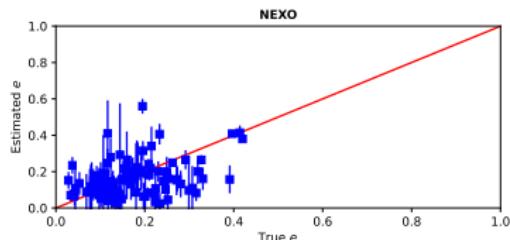
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

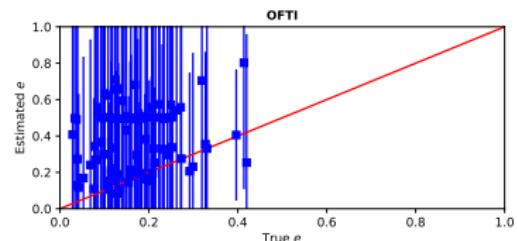
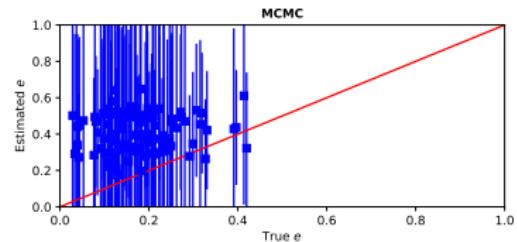
The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References



Mean Estimates and 95%
Credible Intervals





Validation Results: Inclination



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

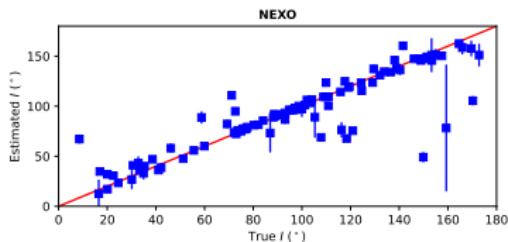
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

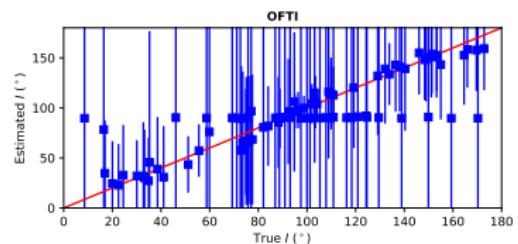
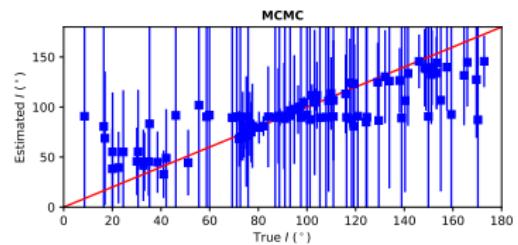
The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References



Mean Estimates and 95%
Credible Intervals





Root-Mean-Square Errors



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

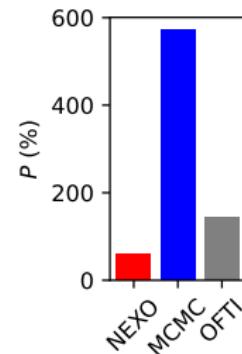
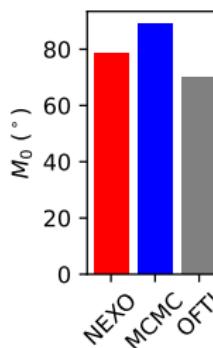
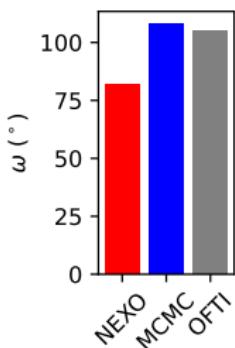
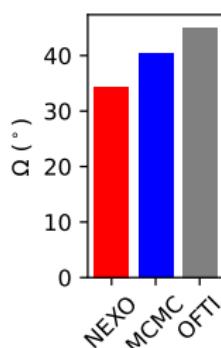
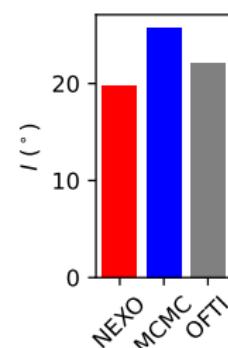
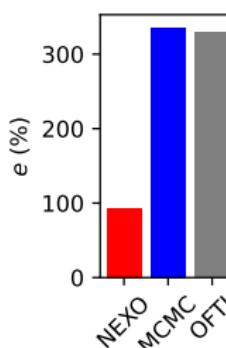
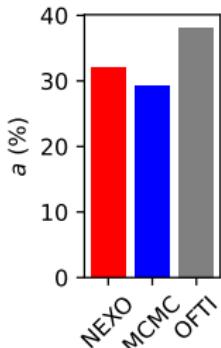
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Accuracy of Credible Intervals



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

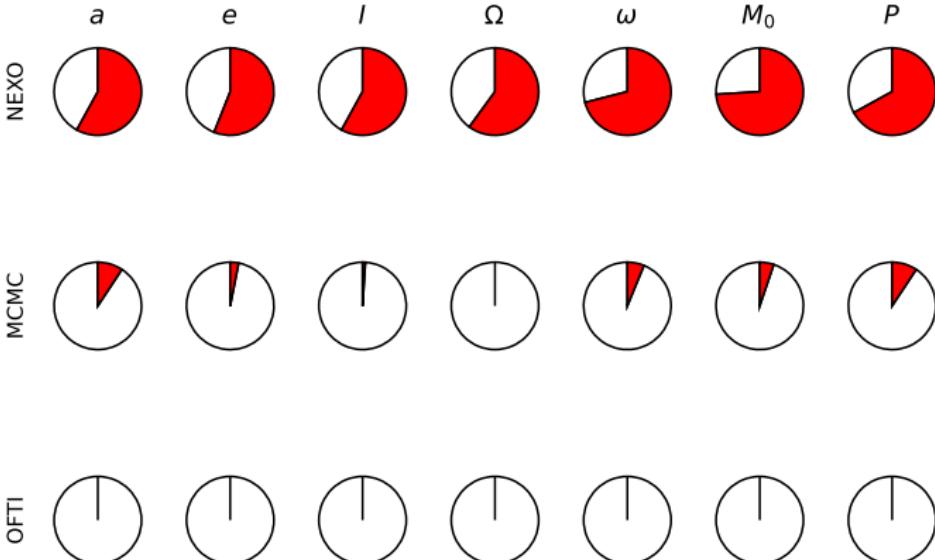
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion
References

Fractions outside of 95% credible intervals shown in red





Estimator Run Times



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

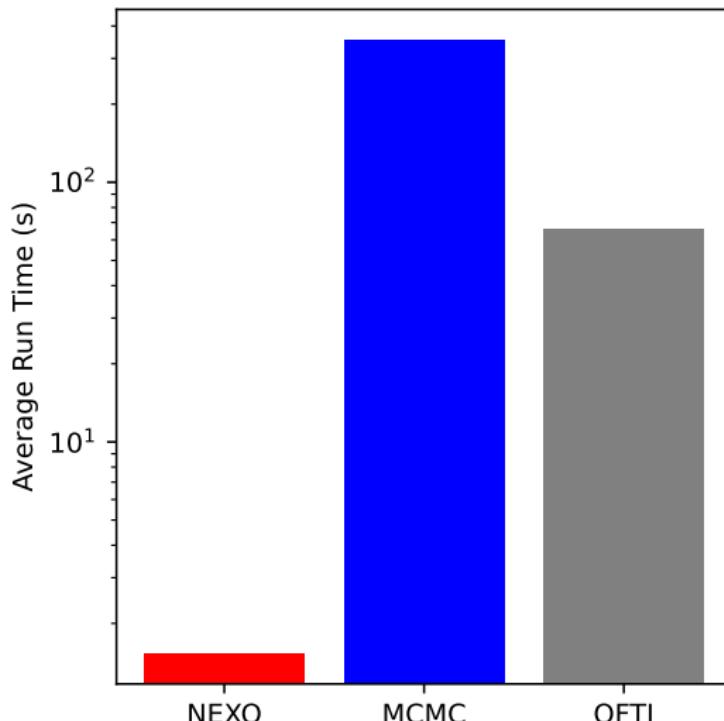
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References



Comparison MCMC: β Pictoris b

Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

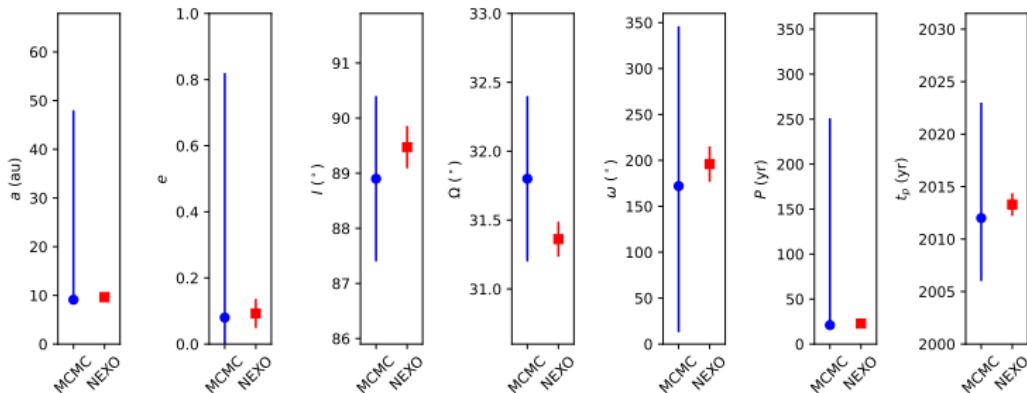
Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Estimates and 95% Credible Intervals



Astrometric data from Lagrange et al. (2009) and Nielsen et al. (2014)

MCMC fit by Nielsen et al. (2014)



NEXO Orbit Fit: β Pictoris b



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

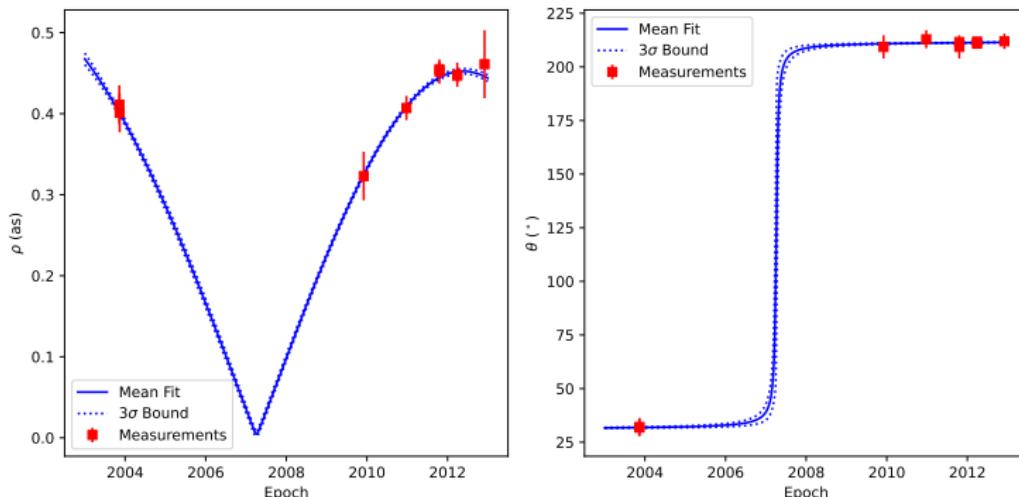
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Comparison with OFTI: GJ 504 b



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

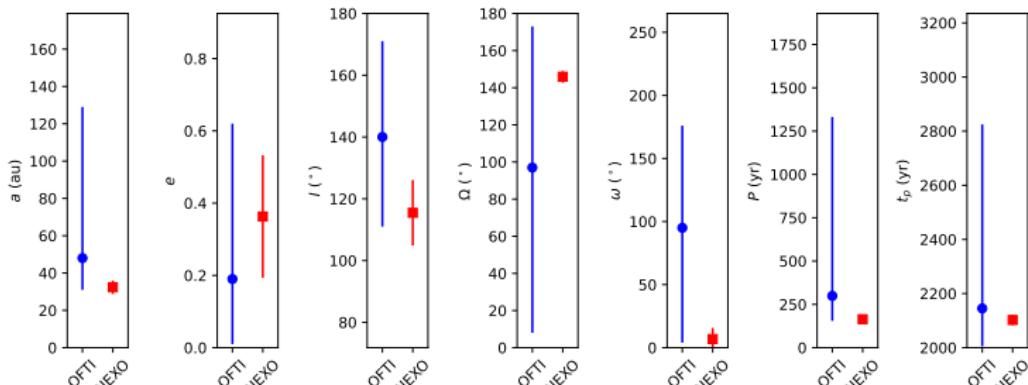
Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Estimates and 95% Credible Intervals



Astrometric data from Kuzuhara et al. (2013)
OFTI fit by Blunt et al. (2017)



NEXO Orbit Fit: GJ 504 b



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

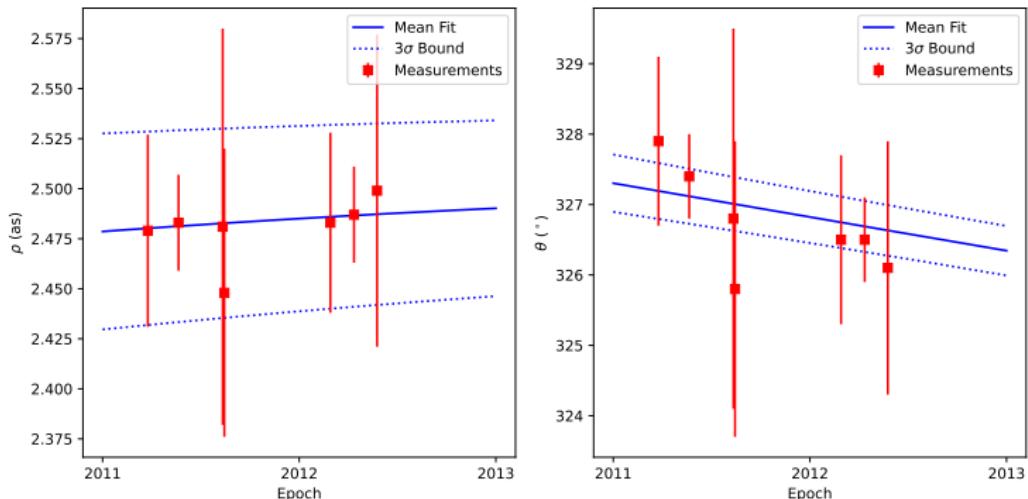
The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References





Summary



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Developed new set of nonsingular orbital elements for filtering
- Applied Gaussian mixtures and square root SPF to exoplanet orbit fitting
- Estimate accuracy comparable to or higher than MCMC and OFTI



Future Work



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- Improving reliability of error estimates and credible intervals
- Fine-tuning filter
- Testing other filtering methods



Nonlinear Filtering with Applications to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Conclusion



Summary



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Showed that nonlinear filtering methods can be successfully applied to new dynamical systems

- Careful modeling and parametrization of the system state and measurements
- Development of new filtering techniques



Future Work



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Plenty of room for further development

- New dynamical systems
- Higher-fidelity models
- More accurate filtering methods



Contributions



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

Journal Papers

- Z. Stojanovski and D. Savransky, "Higher-Order Unscented Estimator," *Journal of Guidance, Control, and Dynamics*, Vol. 44, No. 12, 2021.
- —, "Orbit Fitting for Directly-Observed Exoplanets using Nonsingular Elements and Gaussian Mixture Sigma Point Filtering," *in preparation*.
- —, "Autonomous Cross-Calibration for Imaging Satellites," *in preparation*.

Conference Papers

- —, "Autonomous Cross-Calibration for Imaging Satellites," AAS/AIAA Astrodynamics Specialist Conference, 2021.
- —, "Unscented filtering for directly-observed exoplanet orbits," SPIE Astronomical Telescopes and Instrumentation, 2022.

Presentations

- —, "Astrometric Orbit Estimation and Prediction for Exoplanets using Unscented Filters," 241st Meeting of the American Astronomical Society, 2023.



Acknowledgements



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

The authors acknowledge support for this work from the NASA Space Technology Research Grants Early Career Faculty program under NASA grant 80NSSC20K0068.



References I



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- N. Adurthi, P. Singla, and T. Singh. Conjugate unscented transformation: Applications to estimation and control. *Journal of Dynamic Systems, Measurement, and Control*, 140(3):1–22, 2018. ISSN 00220434. doi: 10.1115/1.4037783.
- S. Blunt, E. L. Nielsen, R. J. D. Rosa, et al. Orbits for the Impatient: A Bayesian Rejection-sampling Method for Quickly Fitting the Orbits of Long-period Exoplanets. *The Astronomical Journal*, 153(5), 2017.
- S. Brunke and M. E. Campbell. Square Root Sigma Point Filtering for Real-Time, Nonlinear Estimation. *Journal of Guidance, Control, and Dynamics*, 27(2): 314–317, 2004.
- C. J. Cohen and E. C. Hubbard. A Nonsingular Set of Orbital Elements. *The Astronomical Journal*, 67(1), 1962.
- R. J. De Rosa, E. L. Nielsen, J. J. Wang, et al. An Updated Visual Orbit of the Directly Imaged Exoplanet 51 Eridani b and Prospects for a Dynamical Mass Measurement with *Gaia*. *The Astronomical Journal*, 159(1), 2020.
- W. D. Heintz. *Double Stars*. D. Reidel Publishing Company, 1st edition, 1978.
- C. Krafft, A. Volokitin, and G. Gauthier. Turbulence and microprocesses in inhomogeneous solar wind plasmas. *Fluids*, 4(2):43–45, 2019. ISSN 23115521. doi: 10.3390/fluids4020069.



References II



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- M. Kuzuhara, M. Tamura, T. Kudo, et al. Direct Imaging of a Cold Jovian Exoplanet in Orbit Around the Sun-Like Star GJ 504. *The Astrophysical Journal*, 774(1), 2013.
- A.-M. Lagrange, D. Gratadour, G. Chauvin, et al. A probable giant planet imaged in the β Pictoris disk: VLT/NaCo deep L' -band imaging. *Astronomy & Astrophysics*, 493(2):L21–L25, 2009.
- D. G. Lowe. Distinctive Image Features from Scale-Invariant Keypoints. *International Journal of Computer Vision*, 60(2):91–110, 2004. ISSN 0920-5691.
- F. L. Markley and J. L. Crassidis. *Fundamentals of Spacecraft Attitude Determination and Control*. Springer, 2014.
- E. L. Nielsen, M. C. Liu, Z. Wahhaj, et al. The Gemini/NICI Planet-Finding Campaign: The Orbit of the Young Exoplanet β Pictoris b. *The Astrophysical Journal*, 794(2), 2014.
- J. V. Ramsdell. Wind shear fluctuations downwind of large surface roughness elements. *Journal of Applied Meteorology*, 17(4):436–443, 1978. doi: 10.1175/1520-0450(1978)017<0436:WSFDOL>2.0.CO;2.



References III



Nonlinear Filtering
with Applications
to Astrodynamics

Zvonimir
Stojanovski

Introduction

The Higher-Order
Unscented
Estimator

Autonomous
Cross-Calibration
for Imaging
Satellites

The Nonsingular
Estimator for
Exoplanet Orbits

Conclusion

References

- A. Steinwolf and S. A. Rizzi. Non-Gaussian Analysis of Turbulent Boundary Layer Fluctuating Pressure on Aircraft Skin Panels. *AIAA Journal of Aircraft*, 43(6): 1662–1675, 2012. doi: 10.2514/1.18294.
- R. F. Stengel. *Optimal Control and Estimation*. Dover, 1986.