Astrometric Orbit Estimation and Prediction for Exoplanets using Unscented Filters

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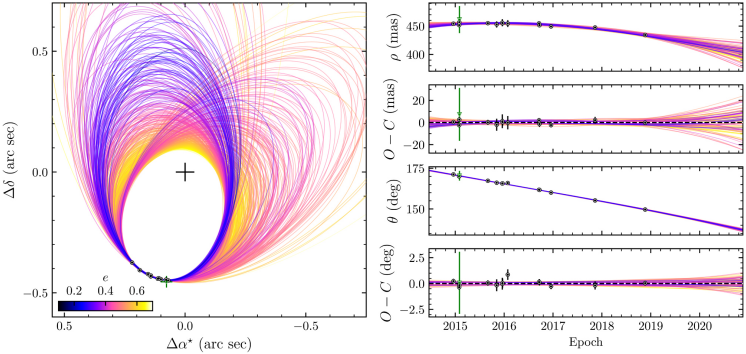
Motivation

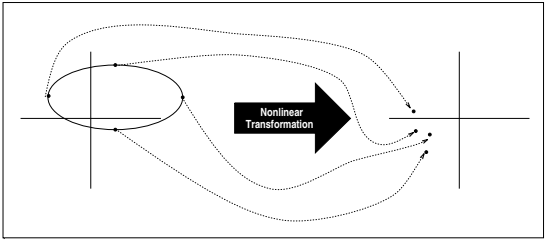
- Instruments such as the Gemini Planet Imager have enabled direct imaging and astrometric measurements on exoplanets
- Orbit fitting remains challenging and computationally expensive



De Rosa et al. (2020)

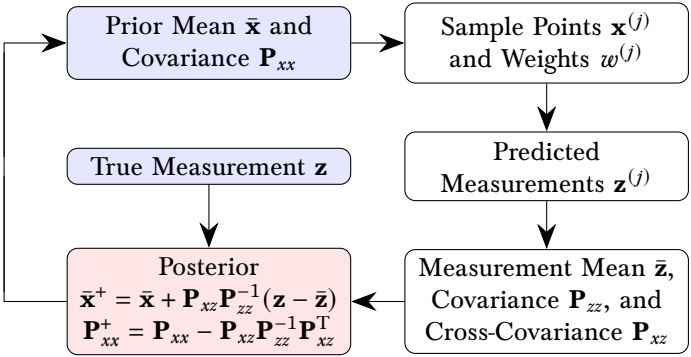
The Unscented Kalman Filter (UKF)

- A recursive, nonlinear estimation method
- Introduced by Julier and Uhlmann (1997)
- Approximates a distribution using a finite, deterministic set of points and weights
- Fast enough for real-time state estimation



Julier and Uhlmann (1997)

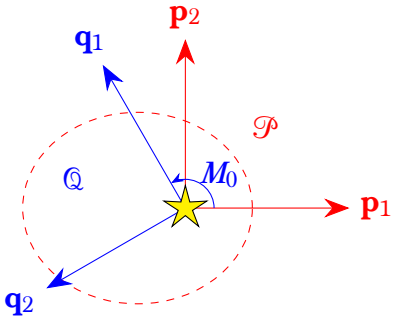
UKF Update Procedure



Can run multiple passes over same measurements — “smoothing”

Nonsingular Orbital Elements

- No singularities at $e = 0$, $i = 0$, etc.
- Any values in $\mathbb{R}^7$ describe an orbit with $e < e_{\max}$
- Based on the reference frame $\mathbb{Q}$
- Combine features of the Thiele-Innes constants and the nonsingular elements due to Cohen and Hubbard (1962)



Perifocal frame $\mathcal{P}$ and auxiliary frame $\mathbb{Q}$

Nonsingular Orbital Elements: Definitions

$$\Xi_{11} = \pi a(\cos(\omega + M_0)\cos\Omega - \sin(\omega + M_0)\sin\Omega\cos i)$$

$$\Xi_{21} = \pi a(\cos(\omega + M_0)\sin\Omega + \sin(\omega + M_0)\cos\Omega\cos i)$$

$$\Xi_{12} = \pi a(-\sin(\omega + M_0)\cos\Omega - \cos(\omega + M_0)\sin\Omega\cos i)$$

$$\Xi_{22} = \pi a(-\sin(\omega + M_0)\sin\Omega + \cos(\omega + M_0)\cos\Omega\cos i)$$

$$\eta_1 = \frac{e\cos M_0}{\sqrt{e_{\max} - e^2}}$$

$$\eta_2 = -\frac{e\sin M_0}{\sqrt{e_{\max} - e^2}}$$

$$\lambda = \log\left(P/P_{\text{scal}}\right)$$

Measurement Model

$$\mathbf{z} = \Xi\zeta(\boldsymbol{\eta}, \lambda, t) + \mathbf{w}$$

- ζ is the position in the orbital plane in $\mathbb{Q}$, scaled by $1/a$
- $\mathbf{w}$ is the measurement noise
- Measurements are linear with respect to Ξ (4 of 7 elements)

Modifications to the UKF

Square Root Sigma Point Filter

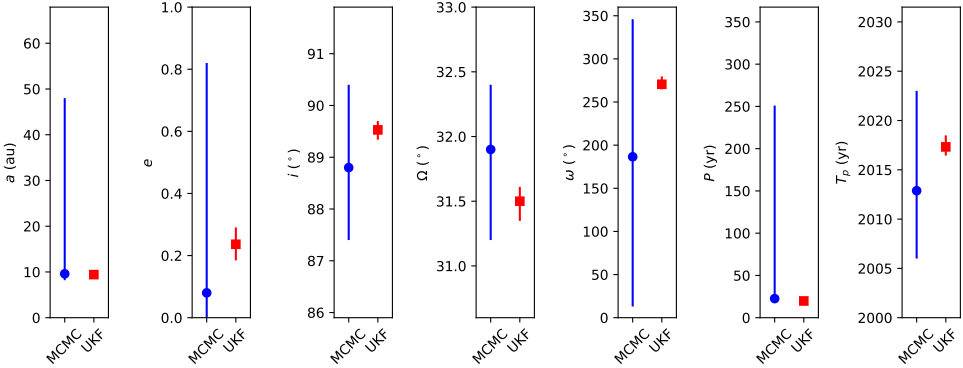
- Introduced by Brunke and Campbell (2004)
- Propagates $\sqrt{\mathbf{P}_{xx}}$ (as Cholesky decomposition) rather than $\mathbf{P}_{xx}$
- May improve numerical stability
- Currently used in our work

Quasilinear UKF

- Takes advantage of linearity of $\mathbf{z}$ with respect to $\boldsymbol{\Xi}$
- Reduces dimension of sample points required
- Work in progress

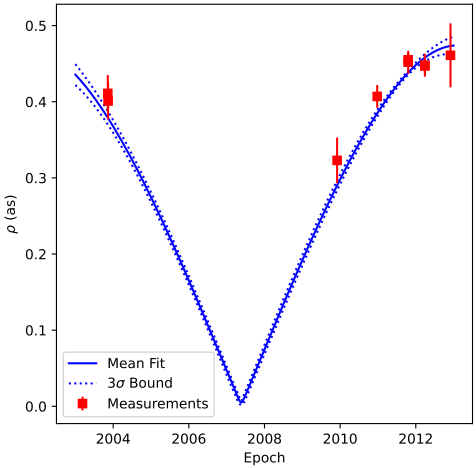
Comparison with Markov Chain Monte Carlo (MCMC): β Pictoris b

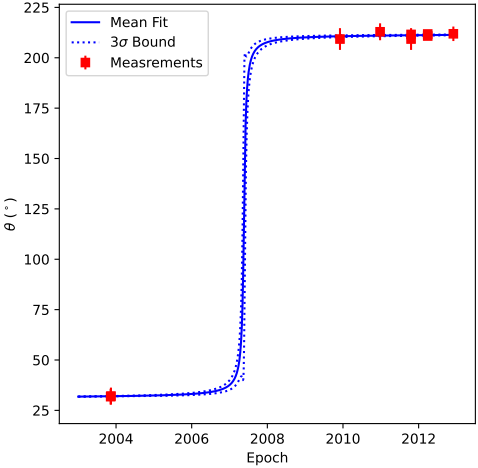
Maximum probability values and 95% credible intervals



Astrometric data from Lagrange et al. (2009) and Nielsen et al. (2014)
MCMC fit by Nielsen et al. (2014)

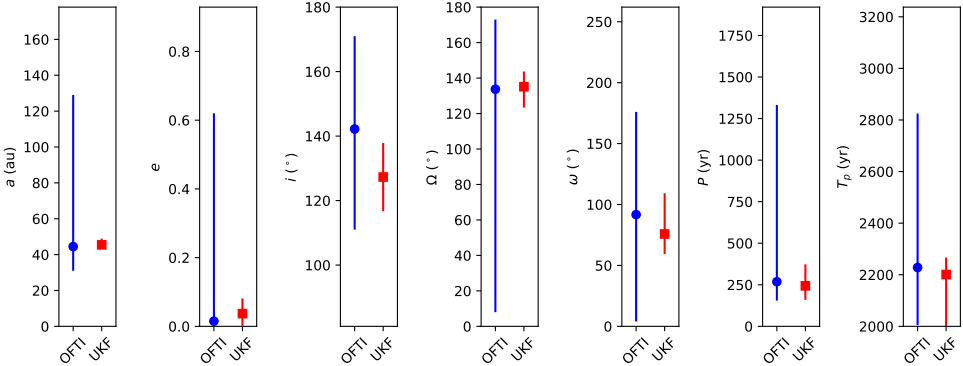
UKF Orbit Fit: β Pictoris b





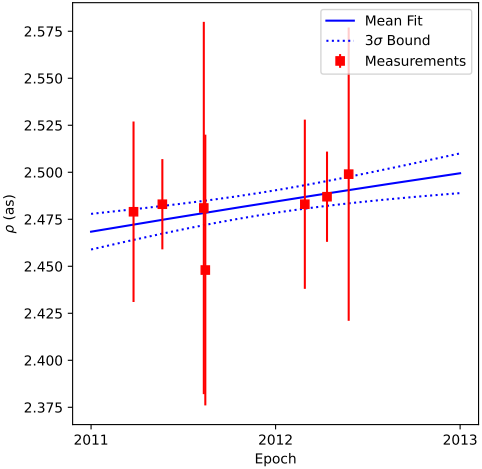
Comparison with Orbits for the Impatient (OFTI): GJ 504 b

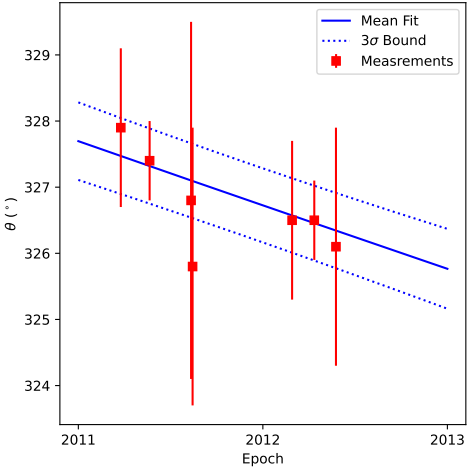
Maximum probability values and 95% credible intervals



Astrometric data from Kuzuhara et al. (2013)
OFTI fit by Blunt et al. (2017)

UKF Orbit Fit: GJ 504 b





Implementation

- Filter written in Fortran for efficiency
- Simple Python interface via F2PY and NumPy
- Typically fits an orbit in less than 1 second
- Soon to be available on GitHub

Future Work

Characterizing prior effects and convergence

- Filter convergence appears to be sensitive to prior distributions and number of filter passes
- We plan to investigate more rigorous methods for choosing priors and number of passes

Further testing

- Tests with more datasets
- Comparisons with other orbit fitting methods, e.g., least-squares Monte Carlo

References

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Recovering the Classical Elements

$$\cos i = \gamma - \operatorname{sgn}(\gamma)\sqrt{\gamma^2 - 1}, \quad \text{where} \quad \gamma = \frac{\|\boldsymbol{\Xi}\|^2}{2\det(\boldsymbol{\Xi})}$$

$$a = \frac{\|\boldsymbol{\Xi}\|}{2\pi\sqrt{1+\cos^2 i}}$$

$$e = \frac{e_{\max}\|\boldsymbol{\eta}\|}{\sqrt{1+\|\boldsymbol{\eta}\|^2}}$$

$$M_0 = \operatorname{atan2}(-\eta_2, \eta_1)$$

$$\Omega + \omega + M_0 = \operatorname{atan2}(\Xi_{21} - \Xi_{12}, \Xi_{11} + \Xi_{22})$$

$$\Omega - \omega - M_0 = \operatorname{atan2}(\Xi_{21} + \Xi_{12}, \Xi_{11} - \Xi_{22})$$

$$P = P_0 \exp \lambda$$